JEE Adv. May 2026

Question Paper With Text Solution

17 May | Paper-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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SECTION 1 (Maximum Marks: 12)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is the correct answer.
- For each question, choose the option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

  Full Marks : +3 If **ONLY** the correct option is chosen;

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);

  Negative Marks : –1 In all other cases.

1. Consider the function $f : (0, \infty) \to (-\infty, \infty)$ given by

$$f(x) = \sqrt{x}\log_e(x) - x + 1.$$

Then which one of the following statements is **TRUE**?

(A) The derivative of the function f is decreasing in the interval (0, 1)

(B) The function f has a local maximum at some point $a \in (0, \infty)$

(C) The function f has a local minimum at some point $b \in (0, \infty)$

(D) The function f has NEITHER a point of local maximum NOR a point of local minimum in the interval $(0, \infty)$

Ans. D

Sol. $f(x) = \sqrt{x}\ln(x) - x + 1$ for $x > 0$.

$$f'(x) = \frac{\ln(x) + 2}{2\sqrt{x}} - 1$$

$$f'(x) = \frac{\ln(x) - 2\sqrt{x} + 2}{2\sqrt{x}}$$

Option (A)

To check if the derivative f'(x) is decreasing or increasing in the interval (0, 1), we find the second derivative

f"(x) : $f'(x) = \frac{1}{2}x^{-1/2}\ln(x) + x^{-1/2} - 1$

$$f''(x) = \frac{1}{2}\left(-\frac{1}{2}x^{-3/2}\ln(x) + x^{-1/2}\cdot\frac{1}{x}\right) - \frac{1}{2}x^{-3/2}$$

$$f''(x) = -\frac{\ln(x)}{4x^{3/2}}$$

Thus, $f''(x) > 0$ for all $x \in (0, 1)$, which means $f'(x)$ is increasing in the interval (0, 1).

Therefore, statement (A) is FALSE.

Finding Local Extrema (Options B, C, and D)

Local extrema occur at critical points where $f'(x) = 0$. Let's set the numerator of $f'(x)$ to 0:

$$\ln(x) - 2\sqrt{x} + 2 = 0$$

Let us define a function $g(x) = \ln(x) - 2\sqrt{x} + 2$ and find its behavior

$$g'(x) = \frac{1}{x} - \frac{1}{\sqrt{x}} = \frac{1-\sqrt{x}}{x}$$

Let's analyze the sign of $g'(x)$ for $x > 0$:

For $x \in (0, 1)$, $\sqrt{x} < 1$ $\Rightarrow g'(x) > 0$ (the function g(x) is increasing).

For $x = 1$, $g'(1) = 0$.

For $x \in (1, \infty)$, $\sqrt{x} > 1$ $\Rightarrow g'(x) < 0$ (the function g(x) is decreasing).

This shows that g(x) attains its absolute maximum at $x = 1$. and $g(1) = \ln(1) - 2\sqrt{1} + 2 = 0 - 2 + 2 = 0$

Since the maximum value of g(x) is 0 at $x = 1$, for all other values of $x \neq 1$, $g(x) < 0$.

For $x = 1$, $f'(1) = \dfrac{g(1)}{2\sqrt{1}} = 0$

For all $x \in (0, \infty)$ where $x \neq 1$, $f'(x) < 0$

Since $f'(x) \leq 0$ for the entire domain, the function f(x) is strictly decreasing everywhere except at the stationary point $x = 1$. Because $f'(x)$ does not change its sign (it remains negative before and after $x = 1$), the point $x = 1$

is a point of inflection, not a local maximum or minimum.

Therefore, f(x) has neither a point of local maximum nor a point of local minimum in $(0, \infty)$.

2. Let P be the point on the parabola $y = x^2$ such that the slope of the tangent to the parabola at the point P is 4. Let Q be the point in the first quadrant lying on the circle $x^2 + y^2 = 2$ such that the slope of the tangent to the circle at the point Q is –1. Let R be the point in the first quadrant lying on the ellipse $x^2 + 4y^2 = 8$ such that the slope of the tangent to the ellipse at the point R is $-\frac{1}{2}$.

Then the radius of the circle passing through the points P, Q and R is :

(A) $\sqrt{10}$ (B) $\sqrt{5}$ (C) $\sqrt{\frac{5}{2}}$ (D) $2\sqrt{5}$

Ans. C

Sol. The given curve is a parabola: $y = x^2 \quad \Rightarrow \frac{dy}{dx} = 2x$

the slope of the tangent at point $P(x_1, y_1)$ is 4: $2x_1 = 4 \quad \Rightarrow x_1 = 2$

Since point P lies on the parabola $y = x^2$: $y_1 = (2)^2 = 4$

Thus, the coordinates of point P are (2, 4).

The given curve is a circle: $x^2 + y^2 = 2$

$$2x + 2y\frac{dy}{dx} = 0 \qquad \Rightarrow \frac{dy}{dx} = -\frac{x}{y}$$

the slope of the tangent at point $Q(x_2, y_2)$ is –1: $-\frac{x_2}{y_2} = -1 \quad \Rightarrow x_2 = y_2$

$$x_2^2 + x_2^2 = 2 \qquad \Rightarrow 2x_2^2 = 2 \qquad \Rightarrow x_2^2 = 1$$

Since Q lies in the first quadrant $x_2 = 1 \quad \Rightarrow y_2 = 1$

Thus, the coordinates of point Q are (1, 1).

The given curve is an ellipse: $x^2 + 4y^2 = 8$

$2x + 8y\frac{dy}{dx} = 0 \qquad \Rightarrow \frac{dy}{dx} = -\frac{2x}{8y} = -\frac{x}{4y}$

the slope of the tangent at point $R(x_3, y_3)$ is $-\frac{1}{2}$: $\quad -\frac{x_3}{4y_3} = -\frac{1}{2} \qquad$ Þ $x_3 = 2y_3$

$(2y_3)^2 + 4y_3^2 = 8 \qquad \Rightarrow 4y_3^2 + 4y_3^2 = 8 \qquad \Rightarrow 8y_3^2 = 8 \qquad \Rightarrow y_3^2 = 1$

Since R lies in the first quadrant, : $\quad y_3 = 1 \quad \Rightarrow x_3 = 2(1) = 2$

Thus, the coordinates of point R are (2, 1).

ΔPQR is a right–angled triangle with the right angle at vertex R ($\Delta PRQ = 90°$).

Therefore, the line segment PQ is the diameter of the circle.

Diameter (PQ) = $\sqrt{(2-1)^2 + (4-1)^2} = \sqrt{1^2 + 3^2} = \sqrt{1+9} = \sqrt{10}$

The radius r of the circle is half of the diameter: $r = \frac{\sqrt{10}}{2} = \sqrt{\frac{10}{4}} = \sqrt{\frac{5}{2}}$

3. Which one of the following matrices can be obtained by performing elementary row transformations on the 3×3 identity matrix?

(A) $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (B) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}$ (C) $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2 & 5 & 8 \end{bmatrix}$ (D) $\begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix}$

Ans. B

Sol. A matrix can be obtained by performing elementary row transformations on the identity matrix if and only if it is row–equivalent to the identity matrix.

According to matrix theory, a square matrix is row–equivalent to the identity matrix if and only if it is invertible (non–singular). Therefore, we simply need to find which of the given matrices has a non–zero determinant ($\det(A) \neq 0$).

$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \Rightarrow \qquad \det(A) = 0$

$$B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix} \Rightarrow \quad \det(B) = -5 - (-2) + 1 = -5 + 2 + 1 = -2$$

Since $\det(B) = -2 \neq 0$, the matrix is invertible. This means it can be transformed into the identity matrix, and conversely, the identity matrix can be transformed into this matrix using elementary row operations.

$$C = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 2 & 5 & 8 \end{bmatrix} \Rightarrow \det(C) = 4 - 8 + 4 = 0$$

Since $\det(C) = 0$, it is singular.

$$D = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 2 \\ 0 & 2 & 3 \end{bmatrix} \Rightarrow \quad D \sim \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 3 \\ 0 & 2 & 3 \end{bmatrix} \Rightarrow \quad \det(D) = 0$$

Only the matrix in Option (B) has a non–zero determinant and is invertible.

4. Considering only the principal values of the inverse trigonometric functions, the value of $\cot^{-1}\left(\cot(-11)\right) + 10\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) + 10\sin\left(2\tan^{-1}(2)\right)$ is :

(A) $3\pi + 7$ (B) 7 (C) $4\pi + 7$ (D) $3\pi - 5$

Ans. C

Sol. $T_1 = \cot^{-1}(\cot(-11)) = 4\pi - 11$

$$T_2 = 10\sin\left(2\cos^{-1}\left(\frac{1}{\sqrt{2}}\right)\right) \Rightarrow T_2 = 10\sin\left(2\cdot\frac{\pi}{4}\right) = 10\sin\left(\frac{\pi}{2}\right) = 10$$

$$T_3 = 10\sin(2\tan^{-1}(2))$$

Let $\alpha = \tan^{-1}(2)$, which means $\tan\alpha = 2$.

$$\sin(2\alpha)=\frac{2\tan(\alpha)}{1+\tan^2(\alpha)} \Rightarrow \sin(2\tan^{-1}(2))=\frac{2(2)}{1+2^2}=\frac{4}{1+4}=\frac{4}{5}$$

$$T_3=10\cdot\frac{4}{5}=2\cdot4=8$$

Value = $T_1+T_2+T_3$

Value = $(4\pi-11)+10+8=4\pi+7$

SECTION 2 (Maximum Marks: 16)

- This section contains **FOUR (04)** questions.
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four option(s) is(are) correct answer(s).
- For each question, choose the option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 **ONLY** if (all) the correct option(s) is(are) chosen;
  Partial Marks : +3 If all the four options are correct but **ONLY** three options are chosen;
  Partial Marks : +2 If three or more options are correct but **ONLY** two options are chosen, both of which are correct;
  Partial Marks : +1 If two or more options are correct but **ONLY** one option is chosen and it is a correct option;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : –1 In all other cases.
- For example, in a question, if (A), (B) and (D) are the **ONLY** three options corresponding to correct answers, then
  choosing ONLY (A), (B) and (D) will get +4 marks;
  choosing ONLY (A) and (B) will get +2 marks;
  choosing ONLY (A) and (D) will get +2 marks;
  choosing ONLY (B) and (D) will get +2 marks;
  choosing ONLY (A) will get +1 mark;
  choosing ONLY (B) will get +1 mark;
  choosing ONLY (D) will get +1 mark;
  choosing no option (i.e. the question is unanswered) will get 0 marks; and
  choosing any other combination of options will get –1 marks.

5. Suppose that Box I contains 6 red balls and 9 green balls, and Box II contains 8 red balls and 12 green balls. All the balls of Box I and Box II are mixed together and a ball is chosen at random from them. Let E_1 be the event that the ball chosen belonged to Box I and let E_2 be the event that the ball chosen belonged to Box II. Let F_1 be the event that the ball chosen is red and let F_2 be the event that the ball chosen is green.

   Then which of the following statements is (are) **TRUE**?

   (A) The events E_1 and F_1 are independent

   (B) The events E_2 and F_2 are dependent

(C) The conditional probability $P(F_1 | E_1)$ is equal to the conditional probability $P(F_1 | E_2)$

(D) The conditional probability $P(F_1 | E_1)$ is greater than the conditional probability $P(F_2 | E_2)$

Ans. AC

Sol.

Box	Red Balls (F_1)	Green Balls (F_2)	Total Balls per Box
Box I (E_1)	6	9	6 + 9 = 15
Box II (E_2)	8	12	8 + 12 = 20
Total (Mixed)	14	21	35

Probability of picking a ball from Box I (E_1): $P(E_1) = \frac{15}{35} = \frac{3}{7}$

Probability of picking a ball from Box II (E_2): $P(E_2) = \frac{20}{35} = \frac{4}{7}$

Probability of picking a Red ball (F_1): $P(F_1) = \frac{14}{35} = \frac{2}{5}$

Probability of picking a Green ball (F_2): $P(F_2) = \frac{21}{35} = \frac{3}{5}$

Checking Option (A):

$P(E_1 \cap F_1) = \frac{6}{35}$

$P(E_1) \cdot P(F_1) = \frac{3}{7} \times \frac{2}{5} = \frac{6}{35}$

Since $P(E_1 \cap F_1) = P(E_1) \cdot P(F_1)$, the events E_1 and F_1 are independent.

Checking Option (B):

$P(E_2 \cap F_2)$ is the probability that the ball is from Box II and is Green: $P(E_2 \cap F_2) = \frac{12}{35}$

$P(E_2) \cdot P(F_2) = \frac{4}{7} \times \frac{3}{5} = \frac{12}{35}$

Since $P(E_2 \cap F_2) = P(E_2) \cdot P(F_2)$, the events E_2 and F_2 are independent (not dependent).

Checking Option (C):

Conditional probability $P(F_1 | E_1)$: $P(F_1 | E_1) = \frac{6}{15} = \frac{2}{5} = 0.4$

Conditional probability $P(F_1 | E_2)$: $P(F_1 | E_2) = \frac{8}{20} = \frac{2}{5} = 0.4$

Checking Option (D):

We know $P(F_1 | E_1) = \frac{2}{5} = 0.4$

Conditional probability $P(F_2 | E_2)$: $P(F_2 | E_2) = \frac{12}{20} = \frac{3}{5} = 0.6$

6. Let P be the plane such that it contains the straight line $\frac{x-1}{2} = \frac{y-3}{3} = \frac{z+2}{1}$ and is perpendicular to the plane $x + 2y + 3z = 4$. Let P_1 be the plane which passes through the point (4, 2, 2) and is parallel to P.

Then which of the following statements is (are) **TRUE** ?

(A) The equation of the plane P is $7x - 5y + z = -10$

(B) The distance between the planes P and P_1 is 30

(C) The distance of the plane P from the origin is $2\sqrt{3}$

(D) The acute angle between the plane P and the plane $2x + 2y + z = 3$ is $\cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$

Ans. AD

Sol. plane P: its normal vector $\vec{n}_P$ is perpendicular to the direction vector of the line, $\vec{d} = (2, 3, 1)$

It is perpendicular to the plane $x + 2y + 3z = 4$. The normal vector of this given plane is $\vec{n}_Q = (1, 2, 3)$. Since the two planes are perpendicular, their normal vectors must also be perpendicular $(\vec{n}_P \perp \vec{n}_Q)$

$$\vec{n}_P = \vec{d} \times \vec{n}_Q = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 1 \\ 1 & 2 & 3 \end{vmatrix} \Rightarrow \quad \vec{n}_P = 7\hat{i} - 5\hat{j} + \hat{k}$$

the equation of plane P

$7(x-1)-5(y-3)+1(z+2)=0 \Rightarrow 7x-5y+z+10=0 \Rightarrow 7x-5y+z=-10$

the equation of plane P_1: $7x-5y+z=k$

As it passes through the point (4, 2, 2), $7(4)-5(2)+2=k \Rightarrow 28-10+2=k \Rightarrow k=20$

So, the equation of plane P_1 is $7x-5y+z-20=0$

the distance between the parallel planes

$$\text{Distance}=\frac{|D_1-D_2|}{\sqrt{A^2+B^2+C^2}} \Rightarrow \text{Distance}=\frac{|10-(-20)|}{\sqrt{7^2+(-5)^2+1^2}}=\frac{30}{\sqrt{49+25+1}}=\frac{30}{\sqrt{75}}=\frac{30}{5\sqrt{3}}=2\sqrt{3}$$

the distance of plane P from the origin: $\text{Distance}=\frac{|10|}{\sqrt{75}}=\frac{10}{5\sqrt{3}}=\frac{2}{\sqrt{3}}=\frac{2\sqrt{3}}{3}$

the acute angle between plane P and the plane $2x+2y+z=3$

$$\cos\theta=\frac{|\vec{n}_1\cdot\vec{n}_2|}{|\vec{n}_1||\vec{n}_2|} \Rightarrow \cos\theta=\frac{5}{(5\sqrt{3})(3)}=\frac{1}{3\sqrt{3}} \Rightarrow \theta=\cos^{-1}\left(\frac{1}{3\sqrt{3}}\right)$$

- Therefore, Statement (D) is TRUE

7. Let **R** denote the set of all real numbers. Let $f : \mathbf{R} \to \mathbf{R}$ be an arbitrary function and let $g : \mathbf{R} \to \mathbf{R}$ be the function defined by

$$g(x) = xf(x), \text{ for all } x \in \mathbf{R}.$$

Then which of the following statements is (are) **TRUE**?

(A) The function g is always continuous at $x = 0$

(B) If f is continuous at $x = 0$, then g is differentiable at $x = 0$

(C) If g is differentiable at $x = 0$, then f is continuous at $x = 0$

(D) If g is differentiable at $x = 0$, then $\lim_{x\to 0} f(x)$ exists

Ans. BD

Sol. option (A)

Statement: The function g is always continuous at $x = 0$

Let $f(x)=\frac{1}{x^2}$ for $x \neq 0$, and $f(0)=0$

Then, for $x \neq 0$ $g(x)=x\cdot\frac{1}{x^2}=\frac{1}{x}$

As $x \to 0, \lim_{x\to 0} g(x)=\lim_{x\to 0}\frac{1}{x}$, which does not exist

Therefore, g is not always continuous at $x=0$

Option (B),(C) and (D)

LHD $g'(0^-)=\lim_{x\to 0}\frac{(0-h).f(0-h)-0}{-h}=\lim_{x\to 0} f(0-h)$

RHD $g'(0^+)=\lim_{x\to 0}\frac{(0+h).f(0+h)-0}{(0+h)}=\lim_{x\to 0} f(0+h)$

(B) $\therefore$ $f(0+h)=f(0-h)$ {f is continous a $x=0$} $\Rightarrow g'(0^-)=g'(0^+)$

g is differentiable at $x=0$

option (C) and (D)

$g'(0)=\lim_{x\to 0} f(x)$

This tells us that the limit $\lim_{x\to 0} f(x)$ must exist and equal the finite value g'(0). However, for f(x) to be continuous at $x=0$, we additionally require that this limit equals the functional value at that point, i.e., $\lim_{x\to 0} f(x)=f(0)$

8. Consider the matrix

$$M=\begin{bmatrix}2 & -1\\ 1 & 0\end{bmatrix}.$$

Let p, q, r, s, a, b, c and d be integers such that

$$M^{26} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} \text{ and } \sum_{k=1}^{26} M^k = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Then which of the following statements is (are) **TRUE**?

(A) There exists a 2×2 invertible matrix N with real entries such that $MN = N\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.

(B) The value of a is 378.

(C) For any two given integers m and n, there exist unique integers x and y such that $px + qy = m$ and $rx + sy = n$

(D) For each positive real number t, the system of linear equations $(a + t)x + by = 1$, $cx + (d + t)y = -1$ has a unique solution

Ans. ACD

Sol. the characteristic equation of matrix M

$$\det(M - \lambda I) = 0 \Rightarrow \det\begin{bmatrix} 2-\lambda & -1 \\ 1 & -\lambda \end{bmatrix} = 0 \Rightarrow \quad \lambda^2 - 2\lambda + 1 = 0 \Rightarrow (\lambda - 1)^2 = 0$$

The eigenvalues of M are $\lambda = 1, 1$ (repeated twice)

We can express M as $M = I + A$ where $A = M - I = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}$

$$A^2 = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Since I and A c•mmute (IA = AI), we can apply the Binomial Theorem to find M^k

$$M^k = (I + A)^k = I^k + kI^{k-1}A + \frac{k(k-1)}{2!}I^{k-2}A^2 + \ldots$$

Since $A^2 = 0$, all higher powers of A are also zero

$$M^k = I + kA = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + k\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 1+k & -k \\ k & 1-k \end{bmatrix}$$

(A) Since the matrix M has a repeated eigenvalue $\lambda = 1$ with an algebraic multiplicity of 2, By definition of similarity, there exists an invertible matrix N such that

$N^{-1}MN = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \Rightarrow MN = N\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ Statement (A) is TRUE

For k = 26: $M^{26} = \begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} 1+26 & -26 \\ 26 & 1-26 \end{bmatrix} = \begin{bmatrix} 27 & -26 \\ 26 & -25 \end{bmatrix}$

The given system of equations can be written in matrix form as $\begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} m \\ n \end{bmatrix}$

The determinant of this coefficient matrix is $\det(M^{26}) = (\det M)^{26} = (2(0)-(-1)(1))^{26} = 1^{26} = 1$

Since the determinant is 1, the inverse matrix will have purely integer entries

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 27 & -26 \\ 26 & -25 \end{bmatrix}^{-1}\begin{bmatrix} m \\ n \end{bmatrix} = \begin{bmatrix} -25 & 26 \\ -26 & 27 \end{bmatrix}\begin{bmatrix} m \\ n \end{bmatrix}$$

For any given integers m and n, $x = -25m + 26n$ and $y = -26m + 27n$ will always yield unique integer solutions

Checking option (B)

We are given $\sum_{k=1}^{26} M^k = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

From our formula $M^k = \begin{bmatrix} 1+k & -k \\ k & 1-k \end{bmatrix}$, the entry a is obtained by summing the top-left entries

$$a = \sum_{k=1}^{26}(1+k) = \sum_{k=1}^{26}1 + \sum_{k=1}^{26}k \Rightarrow a = 26 + \frac{26 \times 27}{2} = 26 + 351 = 377$$

Checking option (D)

Let's find the values of all components of the summed matrix $\sum_{k=1}^{26} M^k$

$a = 377$; $b = \sum_{k=1}^{26}(-k) = -351$; $c = \sum_{k=1}^{26}k = 351$; $d = \sum_{k=1}^{26}(1-k) = 26 - 351 = -325$

The system of equations in statement (D) has a unique solution if and only if the determinant of its coefficient matrix is non-zero

$$\Delta = \det\begin{bmatrix} a+t & b \\ c & d+t \end{bmatrix} \neq 0$$

$$\Delta = (a+t)(d+t) - bc = t^2 + (a+d)t + (ad - bc)$$

$$\Delta = t^2 + 52t + 676 = (t+26)^2$$

For a unique solution, we require $\Delta \neq 0 \Rightarrow (t+26)^2 \neq 0 \Rightarrow t \neq -26$

Since t is specified to be a positive real number ($t > 0$), t can never equal –26. Therefore, Δ is always positive and non-zero, ensuring a unique solution

Statement (D) is TRUE

SECTION 3 (Maximum Marks: 16)

- This section contains **FOUR (04)** questions.
- The answer to each question is a **NUMERICAL VALUE.**
- For each question, enter the correct integer corresponding to the answer using the mouse and the onscreen virtual numeric keypad in the place designated to enter the answer.
- If the numerical value has more than two decimal places, **truncate/round-off** the value to **TWO** decimal places.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 If **ONLY** the correct integer is entered;
  Zero Marks : 0 In all other cases.

9. Let S = {1, 2, 3, ..., 10}. Consider the set

X = {R : R is an equivalence relation on the set S such that R has exactly 42 elements.

Then the number of elements in X is _______.

Ans. 2520

Sol. A relation R on set S = {1, 2, ..., 10} has exactly 42 ordered pairs (a, b).

1. Reflexivity: For every element $a \in S$, the pair $(a, a) \in R$.

   Since there are 10 elements in S, all 10 diagonal pairs– (1,1), (2,2), ..., (10,10) must be in R.

   Remaining elements to choose = 42 – 10 = 32

2. Symmetry: If $(a, b) \in R$ (where $a \neq b$), then (b, a) must also be in R.

This means the remaining 32 elements must come in pairs.

Number of symmetric pairs (a, b) with $a \neq b = \frac{32}{2} = 16$

3. Transitivity: If $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$.

Concurrently with symmetry, this means if any set of elements are related to one another, every single element in that subset must be related to every other element in that same subset. They form closed, mutually related groups.

If a mutually related group has x elements, it will contribute exactly x^2 ordered pairs to the relation R (since every element pairs with all x elements, including itself).

Let the 10 distinct elements of S be divided into k mutually related groups, with sizes $x_1, x_2, \ldots, x_k$.

1. Total number of elements: $x_1 + x_2 + \ldots\ldots + x_k = 10$

2. Total number of ordered pairs: $x_1^2 + x_2^2 + \ldots + x_k^2 = 42$

Case I: Group sizes of 6, 2, 1, 1 $\Rightarrow (6^2 + 2^2 + 1^2 + 1^2 = 36 + 4 + 1 + 1 = 42)$

Case II: Group sizes of 5, 4, 1 $\Rightarrow (5^2 + 4^2 + 1^2 = 25 + 16 + 1 = 42)$

the number of ways to divide n distinct objects into groups of sizes p, q, r, ... is given by the formula:

Number of ways $= \frac{n!}{(p!\cdot q!\cdot r!\ldots)\times a!\cdot b!\ldots}$

(where a, b, ... represent the number of times a group size repeats).

Calculation for Case I: Group sizes (6, 2, 1, 1)

$\text{Ways}_1 = \frac{10!}{6!\cdot 2!\cdot 1!\cdot 1!\times 2!} \Rightarrow \quad \text{Ways}_1 = \frac{10\times 9\times 8\times 7\times 6!}{6!\times 2\times 1\times 2} = \frac{5040}{4} = 1260$

Calculation for Case II: Group sizes (5, 4, 1)

$\text{Ways}_2 = \frac{10!}{5!\cdot 4!\cdot 1!} \Rightarrow \quad \text{Ways}_2 = \frac{10\times 9\times 8\times 7\times 6\times 5!}{5!\times 24} = \frac{30240}{24} = 1260$

Total number of relations $= \text{Ways}_1 + \text{Ways}_2$

Total = 1260 + 1260 = 2520

10. Consider the function $f : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to (-\infty, \infty)$ defined by

$f(x) = (|x| + |x - 1|)\sin x + [x \sin x]$,

where $[x \sin x]$ is the greatest integer less than or equal to $x \sin x$.

Let α be the total number of points in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ at which f is NOT continuous, and let β be the total number of points in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ at which f is NOT differentiable.

Then the value of $\alpha + \beta$ is _______.

Ans. 5

Sol. To solve this problem, let's break the function f(x) into two parts:

$f(x) = g(x) + h(x)$

where: $g(x) = (|x| + |x - 1|)\sin x$ $\quad$ $h(x) = [x \sin x]$

The given interval is $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.

Analyze $h(x) = [x \sin x]$. the range of x sin x on this interval is $\left(0, \frac{\pi}{2}\right]$.

The only integers in this range are 0 and 1. Consequently, [x sin x] can only take the values 0 or 1:

- $[x \sin x] = 0$ when $1 > x \sin x \geq 0$
- $[x \sin x] = 1$ when $\frac{\pi}{2} > x \sin x \geq 1$

Since x sin x is strictly increasing from 0 to ≈ 1.57 as x goes from 0 to $\frac{\pi}{2}$, there is exactly one value $x_0 \in \left(0, \frac{\pi}{2}\right)$ such that : $\quad x_0 \sin x_0 = 1$

Since the function is even, it also reaches 1 at $-x_0$.

Thus, h(x) has exactly two points of discontinuity : $x = x_0$ and $x = -x_0$.

The absolute value function part, $g(x) = (|x| + |x - 1|)\sin x$, is a combination of continuous functions and is therefore continuous everywhere.

Since g(x) is continuous everywhere and h(x) is discontinuous at $x = \pm x_0$, their sum f(x) is discontinuous at exactly those two points. $\quad \alpha = 2$

Any point of discontinuity is automatically a point of non-differentiability. Therefore, f(x) is non-differentiable at $x = x_0$ and $x = -x_0$.

Now, we must check the remaining critical points of the absolute value functions, which are x = 0 and x = 1.

Checking differentiability at x = 0:

For $x \in (-x_0, 0)$: $|x| = -x$ and $|x - 1| = 1 - x \Rightarrow g(x) = (1 - 2x)\sin x$

For $x \in (0, 1)$: $|x| = x$ and $|x - 1| = 1 - x \Rightarrow g(x) = 1 \cdot \sin x = \sin x$

Left-hand derivative : $f'(0^-) = \frac{d}{dx}((1-2x)\sin x)\Big|_{x=0} = -2\sin(0) + (1-0)\cos(0) = 1$

Right-hand derivative : $f'(0^+) = \frac{d}{dx}(\sin x)\Big|_{x=0} = \cos(0) = 1$

Since $f'(0^-) = f'(0^+) = 1$, the function is differentiable at x = 0.

Checking differentiability at x = 1:

For $x \in (0, 1)$: $g(x) = \sin x$

For $x \in (1, x_0)$: $|x| = x$ and $|x - 1| = x - 1 \Rightarrow g(x) = (2x - 1)\sin x$

Left-hand derivative : $f'(1^-) = \frac{d}{dx}(\sin x)\Big|_{x=1} = \cos(1)$

Right-hand derivative : $f'(1^+) = \frac{d}{dx}((2x-1)\sin x)\Big|_{x=1} = 2\sin(1) + (2(1)-1)\cos(1) = 2\sin(1) + \cos(1)$

$(f'(1^-) \neq f'(1^+))$. Thus, the function is not differentiable at x = 1.

The points where f(x) is not differentiable are $x = -x_0$, $x = x_0$, and x = 1. $\beta = 3$

$\alpha + \beta = 2 + 3 = 5$

11. The number of ways to distribute 10 identical red pens and 14 identical blue pens among four persons such that each person gets 6 pens, is _______.

Ans. 206

Sol. r_i & b_i are number of pens received by i^{th} person

$\therefore r_i + b_i = 6$

& $r_1 + r_2 + r_3 + r_4 = 10$

Also $b_i = 6 - r_i$

$r_i \geq 0$ & $b_i \geq 0$

$\Rightarrow r_i \leq 6$

$\therefore\ 0 \leq r_i \leq 6$

$\therefore$ Cases possible

$r_1 + r_2 + r_3 + r_4 = 10\ ;\ 0 \leq r_i \leq 6$

$\therefore$ Requirel cases $= {}^{10+4-1}C_3 - \left({}^4C_1 \times {}^6C_3\right)$

$= {}^{13}C_3 - 4 \times {}^6C_3 \qquad = 286 - 80 \qquad = 206$

12. Let $\alpha = \left(1-2\cos\left(\frac{\pi}{11}\right)\right)\left(1-2\cos\left(\frac{3\pi}{11}\right)\right)\left(1-2\cos\left(\frac{9\pi}{11}\right)\right)\left(1-2\cos\left(\frac{27\pi}{11}\right)\right)\left(1-2\cos\left(\frac{81\pi}{11}\right)\right)$. Then the value of $5 - \alpha^2$ is ________.

Ans. 4

Sol. $\alpha = \left(1-2\cos\frac{\pi}{11}\right)\left(1-2\cos\frac{3\pi}{11}\right)\left(1-2\cos\frac{9\pi}{11}\right)\left(1-2\cos\frac{27\pi}{11}\right)\left(1-2\cos\frac{81\pi}{11}\right)$

Let $\theta = \frac{\pi}{11}$

$\alpha = (1-2\cos\theta)(1-2\cos 3\theta)(1-2\cos 9\theta)(1-2\cos 27\theta)(1-2\cos 81\theta)$

Now $1 - 2\cos\theta$

$1 - 2\left\{2\cos^2\frac{\theta}{2} - 1\right\}$

$3 - 4\cos^2\theta/2$

$$\frac{\left(3-4\cos^2\theta/2\right)\cdot\cos\theta/2}{\cos\theta/2}$$

$$\Rightarrow -\frac{\cos\frac{3\theta}{2}}{\cos\frac{\theta}{2}}$$

$$\therefore\ \alpha=\left(-\frac{\cos\frac{3\theta}{2}}{\cos\frac{\theta}{2}}\right)\left(-\frac{\cos\frac{9\theta}{2}}{\cos\frac{3\theta}{2}}\right)\cdots\cdots\left(-\frac{\cos\frac{243\theta}{2}}{\cos\frac{81\theta}{2}}\right)=-\frac{\cos\frac{243\theta}{2}}{\cos\frac{\theta}{2}}$$

$$\text{Now } \theta=\frac{\pi}{11}\Rightarrow\frac{\theta}{2}=\frac{\pi}{22}\Rightarrow\ 243\frac{\theta}{2}=\frac{\pi}{22}\times 243=11\pi+\frac{\pi}{22}$$

$$\cos 243\frac{\theta}{2}=-\cos\frac{\pi}{22}=-\cos\frac{\theta}{2}$$

$$\therefore\ \alpha=1\quad\Rightarrow\quad 5-\alpha^2=4$$

SECTION 4 (Maximum Marks: 16)

- This section contains **FOUR (04)** Matching List Sets.
- Each set has **ONE** Multiple Choice Question.
- Each set has **TWO** lists: **List-I** and **List-II**.
- **List-I** has Four entries (P), (Q), (R) and (S) and **List-II** has **Five** entries (1), (2), (3), (4) and (5).
- **FOUR** options are given in each Multiple Choice Question based on **List-I** and **List-II** and **ONLY ONE** of these four options satisfies the condition asked in the Multiple Choice Question.
- Answer to each question will be evaluated according to the following marking scheme:
  Full Marks : +4 **ONLY** if the option corresponding to the correct combination is chosen;
  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered);
  Negative Marks : -1 In all other cases.

13. Match each entry in **List-I** to the correct entry in **List-II** and choose the correct option.

List-I	**List-II**
(P) If α and β are the distinct roots of the equation $x^2+x+1=0$, then the quadratic equation with roots $\frac{1}{(\alpha+1)^{2026}}$ and $\frac{1}{(\beta+1)^{2026}}$	(1) $x^2+x+1=0$
(Q) If α and β are the distinct roots of the equation $x^2+x+1=0$, then the quadratic equation with roots $\frac{1}{(\alpha+1)^{2027}}$ and $\frac{1}{(\beta+1)^{2027}}$	(2) $x^2-x+1=0$
(R) If γ and δ are the distinct roots of the equation $x^2-x+1=0$, then the value of $\frac{1}{(\gamma-1)^{2026}}+\frac{1}{(\delta-1)^{2026}}$ is	(3) $x^2+x-1=0$

(S) If p and r are the distinct roots of the equation $x^2 + x - 1 = 0$, then the value of

$$\frac{1}{(p+1)^3} + \frac{1}{(r+1)^3} \text{ is}$$

(4) -1

(5) -4

The correct option is :

(A) (P) $\rightarrow$ (1); (Q) $\rightarrow$ (2); (R) $\rightarrow$ (5); (S) $\rightarrow$ (4)

(B) (P) $\rightarrow$ (3); (Q) $\rightarrow$ (1); (R) $\rightarrow$ (4); (S) $\rightarrow$ (5)

(C) (P) $\rightarrow$ (1); (Q) $\rightarrow$ (2); (R) $\rightarrow$ (4); (S) $\rightarrow$ (5)

(D) (P) $\rightarrow$ (2); (Q) $\rightarrow$ (3); (R) $\rightarrow$ (5); (S) $\rightarrow$ (4)

Ans. C

Sol. P] Let $\alpha = w$ & $\beta = w^2$

$$\text{So } \frac{1}{(\alpha+1)^{2026}} = \frac{1}{(1+w)^{2026}} = \frac{1}{\left(-w^2\right)^{2026}} = \frac{1}{w^{4052}} = \frac{1}{w^2} = w$$

$$\& \ \frac{1}{(\beta+1)^{2026}} = \frac{1}{\left(1+w^2\right)^{2026}} = \frac{1}{(-w)^{2026}} = \frac{1}{w^{2026}} = \frac{1}{w} = w^2$$

$\therefore$ Quadratic will be $x^2 + x + 1 = 0$

P$\rightarrow$ 1

Q] Again $\alpha = w$ & $\beta = w^2$

$$\frac{1}{(\alpha+1)^{2027}} = \frac{1}{(w+1)^{2027}} = \frac{1}{(-w^2)^{2027}} = -\frac{1}{w^{4054}} = -\frac{1}{w} = -w^2$$

$$\& \ \frac{1}{(\beta+1)^{2027}} = \frac{1}{\left(1+w^2\right)^{2027}} = \frac{1}{(-w)^{2027}} = -\frac{1}{w^{2027}} = -\frac{1}{w^2} = -w$$

$\therefore$ Quartratic Equation: $x^2 - x + 1 = 0$

Q $\rightarrow$ 2

R] Let $\gamma = -w^2$ & $\delta = -w$

$$\therefore \ \frac{1}{(\gamma-1)^{2026}} = \frac{1}{\left(-w^2\right)^{2026}} = w$$

$$\frac{1}{(\delta-1)^{2026}} = \frac{1}{(-w)^{2026}} = w^2$$

$w + w^2 = -1$

$R \rightarrow 4$

S] $x^2 + x - 1 = 0$

find $\dfrac{1}{(p+1)^3} + \dfrac{1}{(r+1)^3} = \dfrac{(r+1)^3 + (p+1)^3}{\{(p+1)(r+1)\}^3} = \dfrac{r^3+1+3r^2+3r+p^3+1+3p^2+3p}{(-1)^3}$

Now $p + r = -1$; $pr = -1$

$p^2 + r^2 = 3 \Rightarrow p^3 + r^3 = -4$

$S \rightarrow 5$

14. Match each entry in **List-I** to the correct entry in **List-II** and choose the correct option.

List-I	List-II
(P) The number of elements in the set $\left\{x \in [-\pi, \pi] : \sin^6 x + \cos^4 x = 1\right\}$	(1) is 1
(Q) The number of elements in the set $\left\{x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] : \sin^2 x + \cos^6 x = 1\right\}$	(2) is 2
(R) The number of elements in the set $\left\{x \in [-\pi, \pi] : \cos^2\left(\frac{x}{2}\right) - \sin^2 x = \frac{1}{2}\right\}$	(3) is 3
(S) The number of elements in the set $\left\{x \in [-2\pi, 2\pi] : 6\sin^2\left(\frac{x}{2}\right) - \cos 3x = 3\right\}$	(4) is 4
	(5) is 5

The correct option is :

(A) (P) $\rightarrow$ (2); (Q) $\rightarrow$ (5); (R) $\rightarrow$ (3); (S) $\rightarrow$ (4)

(B) (P) $\rightarrow$ (5); (Q) $\rightarrow$ (3); (R) $\rightarrow$ (2); (S) $\rightarrow$ (4)

(C) (P) $\to$ (5); (Q) $\to$ (4); (R) $\to$ (1); (S) $\to$ (3)

(D) (P) $\to$ (4); (Q) $\to$ (3); (R) $\to$ (2); (S) $\to$ (5)

Ans. B

Sol. P] $x \in [-\pi, \pi]$

$\sin^6 x + \cos^4 x = 1$

$\sin^6 x \le \sin^2 x$ & $\cos^4 x \le \cos^2 x$

$\sin^6 x + \cos^4 x \le \sin^2 x + \cos^2 x$

for this to hold

$\sin^6 x = \sin^2 x$ & $\cos^4 x = \cos^2 x$

$\sin^2 x = 0, 1$ & $\cos^2 x(\cos^2 x - 1) = 0$

$\cos^2 x = 0, \cos^2 x = 1$

C–1 $\sin^2 x = 0$ then $\cos^2 x = 1 \Rightarrow$ $\sin x = 0 \Rightarrow$ $x = n\pi$; $n \in Z$

C–2 $\sin^2 x = 1, \cos^2 x = 0 \Rightarrow x = \left\{-\frac{\pi}{2}, \frac{\pi}{2}\right\}$

$\therefore$ Total elements = 5

Q] $\sin^2 x + \cos^6 x = 1 \Rightarrow \sin^2 x = 1 \quad \cos^6 x = 0$

or $\sin^2 x = 0$ & $\cos^2 x = 1 \Rightarrow \quad x = \frac{-\pi}{2}, 0, \frac{\pi}{2}$ $\qquad Q \to 3$

R] $\cos^2 \frac{x}{2} - \sin^2 x = \frac{1}{2} \Rightarrow \quad \left(\frac{1+\cos x}{2}\right) - (1 - \cos^2 x) = \frac{1}{2}$

$2\cos^2 x + \cos x - 2 = 0 \Rightarrow \quad \cos x = \frac{-1+\sqrt{17}}{4}, \frac{-1-\sqrt{17}}{4}$

No. of element = 2

$R \to 2$

S] $6\sin^2 \frac{x}{2} - \cos 3x = 3 \Rightarrow \quad \cos 3x = 3\left\{2\sin^2 \frac{x}{2} - 1\right\}$

$4\cos^3 x - 3\cos x = -3 \cdot \cos x \Rightarrow \quad 4\cos^3 x = 0 \Rightarrow \quad \cos x = 0$

in $[-2\pi, 2\pi]$ $\quad x = \left\{-\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}\right\}$

$S \to 4$

15. For real numbers α, β, γ, δ and μ, consider the 3 × 3 matrix:

$$M = \begin{bmatrix} \alpha & \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{3}} & \beta & \frac{1}{\sqrt{3}} \\ \gamma & \delta & \mu \end{bmatrix}$$

Suppose that $MM^T = I$, where M^T is the transpose of matrix M, and I is the 3 × 3 identity matrix.

Let the three column vectors be defined as :

$\vec{u} = \alpha\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \gamma\hat{k}$, $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \beta\hat{j} + \delta\hat{k}$, $\vec{w} = -\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{3}}\hat{j} + \mu\hat{k}$.

Match each entry in **List-I** to the correct entry in **List-II** and choose the correct option.

List-I	**List-II**
(P) The value of $\gamma^2 + \delta^2$ is	(1) 0
(Q) If $x\vec{u} + y\vec{v} + z\vec{w} = \hat{j}$ for some real numbers x, y and z, then the value of x is	(2) 1
(R) The value of $\left|\vec{u}\cdot(\vec{v}\times\vec{w})\right|$ is	(3) $\frac{1}{\sqrt{2}}$
(S) The value of $\left|\vec{u}\times(\vec{v}\times\vec{w})\right|$ is	(4) $\frac{1}{\sqrt{3}}$
	(5) $\frac{5}{6}$

The correct option is :

(A) (P) → (5); (Q) → (4); (R) → (2); (S) → (1)
(B) (P) → (4); (Q) → (5); (R) → (1); (S) → (2)
(C) (P) → (5); (Q) → (3); (R) → (2); (S) → (1)
(D) (P) → (5); (Q) → (4); (R) → (1); (S) → (2)

Ans. A

Sol. $MM^T = I \quad \Rightarrow$ M is orthogonal

$\alpha = 0$; $\beta = \frac{1}{\sqrt{3}}$

$\gamma^2 + \delta^2 + \mu^2 = 1 \Rightarrow \quad \gamma^2 + \delta^2 = \frac{5}{6}$ So $\quad \mu^2 = \frac{1}{6}$ & $\delta = \mu$

Also $\vec{v}\cdot\vec{v} = 1 = \vec{u}\cdot\vec{u} = \vec{w}\cdot\vec{w} \quad$ & $\vec{v}\cdot\vec{v} = \vec{v}\cdot\vec{w} = \vec{w}.\vec{v} = 0 \quad P \to 5$

$Q \to x\vec{u} + y\vec{v} + z\vec{w} = \hat{j}$

Take dot with $\vec{u}$

$\Rightarrow x = \vec{u}.j = \frac{1}{\sqrt{3}} \quad Q \to 4$

$R \to |\vec{u}.(\vec{v}\times\vec{w})| = 1$

$R \to 2$

$S \to |\vec{u}\times(\vec{v}\times\vec{w})| = 0$

then $S \to 1$

16. Match each entry in **List-I** to the correct entry in **List-II** and choose the correct option.

List-I	**List-II**
(P) The circle with centre (1, 2) and touching the straight line $3x + 4y = 1$, passes through	(1) the point (1, 1)
(Q) The common tangent to the circle $x^2 + y^2 = 2$ and the parabola $y^2 = 8x$ with positive slope, passes through	(2) the point (7, 9)
(R) Let M be the end point of the latus rectum of the ellipse $3x^2 + 4y^2 = 48$ such that M lies in the first quadrant. Then the normal to the ellipse drawn at M passes through	(3) the point (3, 2)
(S) Let H be the hyperbola whose centre is at the origin, one of the foci is at (5, 0), and one directrix is $5x + 16 = 0$. Then H passes through	(4) the point (2, 5)
	(5) the point $\left(8, 3\sqrt{3}\right)$

The correct option is :

(A) (P) $\to$ (3); (Q) $\to$ (4); (R) $\to$ (1); (S) $\to$ (2)

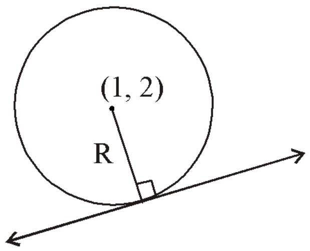
(B) (P) $\to$ (3); (Q) $\to$ (2); (R) $\to$ (1); (S) $\to$ (5)

(C) (P) $\to$ (3); (Q) $\to$ (2); (R) $\to$ (4); (S) $\to$ (5)

(D) (P) $\to$ (4); (Q) $\to$ (1); (R) $\to$ (2); (S) $\to$ (3)

Ans. B

Sol.



$R = \frac{|3+8-1|}{5} = 2$

$\therefore$ Circle $(x-1)^2 + (y-2)^2 = 4$

pt (3, 2) satisfy.

$P \to 3$

Q] $y^2 = 4x$ & $x^2 + y^2 = 2$

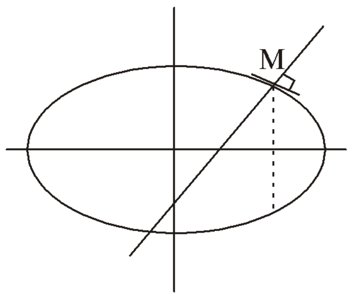
$\frac{2/m}{\sqrt{1+m^2}} = \sqrt{2}$

$m = +1$

Eq. of tangent $\equiv$ $y = x + 2$

(7, 9) satisfy.

R]



$\frac{x^2}{16} + \frac{y^2}{12} = 1 \quad \therefore e = \frac{1}{2}$

$M\left(ae, \frac{b^2}{a}\right) = \left(4 \cdot \frac{1}{2}, \frac{12}{4}\right) = (2, 3)$

Eq. of normal

$\frac{a^2x}{x_1} - \frac{b^2y}{y_1} = a^2e^2$

$2x - y = 1$

Passes through (1, 1)

S] $ae = 5;\ \frac{a}{e} = \frac{16}{5}$

$\therefore\ a = 4,\ e = \frac{5}{4}$

$\therefore\ \frac{x^2}{16} - \frac{y^2}{9} = 1$ is hyperbola

Passes through $(8, 3\sqrt{3})$

$S \to 5$