JEE Main April 2026

Question Paper With Text Solution

08 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2026 | 8TH APRIL SHIFT-2

SECTION – A

Question ID : 691121526

1. Consider the relation R on the set {–2,–1,0,1,2} defined by (a, b) in ∈ R if and only if $1 + ab > 0$. Then, among the statements:

I. The number of elements in R is 17

II. R is an equivalence relation

(1) Only I is true (2) Only II is true (3) Both I and II are true (4) Neither I nor II is true

Ans. (1)

Sol. $(a, a) \in R$ because $1 + a^2 > 0$

$\therefore$ Reflexive

If $1 + ab > 0 \Rightarrow 1 + ba > 0$

$\therefore$ $(a, b) \in R$ and $(b, a) \in R$

$(-2, 0) \in R (0, 1) \in R$

But $(-2, 1) \notin R$

Hence it is not transitive

$\therefore$ It is not equivalence

Total number of elements in the relation R is 17

Because Total elements = 25

But (–2, 1)(–2, 2)(–1, 2)(1, –2)(2, –2)(2, –1)

(–1,1) and (1, –1) are not in relation.

Question ID : 691121527

2. The number of values of $z \in C$, satisfying the equations

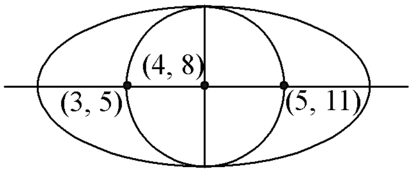
$|z-(4+8i)| = \sqrt{10}$ and $|z-(3+5i)| + |z-(5+11i)| = 4\sqrt{5}$, is:

(1) 0 (2) 2 (3) 1 (4) 4

Ans. (2)

Sol. $(x-4)^2+(y-8)^2=10 \rightarrow$ circle

$$\sqrt{(x-3)^2+(y-5)^2}+\sqrt{(x-5)^2+(y-11)^2}=4\sqrt{5} \rightarrow \text{ellipse}$$



$$2a=4\sqrt{5}, 2ae=\sqrt{4+36}=2\sqrt{10}$$

$$e=\frac{1}{\sqrt{2}}$$

$$b^2=a^2\left(1-e^2\right)=20\left(1-\frac{1}{2}\right)=10$$

$b=r \Rightarrow$ circle touches the ellipse internally at end of minor axis.

Question ID : 691121528

3. If the system of linear equations:

$x+y+z=6,$

$x+2y+5z=10,$

$2x+3y+\lambda z=\mu$

has infinitely many solutions, then the value of $\lambda+\mu$ equals:

(1) 12 (2) 16 (3) 22 (4) 28

Ans. (3)

Sol. for Infinite Many-solution

$\Delta=0$ and $\Delta_1=\Delta_2=\Delta_3=0$

$$\Delta=\begin{vmatrix}1 & 1 & 1\\ 1 & 2 & 5\\ 2 & 3 & \lambda\end{vmatrix}=0$$

$(2\lambda-15)-(\lambda-10)+(-1)=0$

$\lambda=\mathbf{6}$

Sum of equation 1 and 2

$2x+3y+6z=16$

equation (3) is $2x + 3y + 6z = \mu$

for ∞ many So $\mu = 16$

$\therefore\ \lambda + \mu = 6 + 16$

$= 22$

Question ID : 691121529

4. Let $A = \begin{bmatrix} \alpha & 1 & 2 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5\alpha & 0 \\ 0 & 4\alpha & -2\alpha \end{bmatrix}$+adj (A). If det(B) = 66, then det(adj (A)) equals :

(1) 289 (2) 361 (3) 441 (4) 529

Ans. (3)

Sol. $A = \begin{bmatrix} \alpha & 1 & 2 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{bmatrix}$

$$\text{adj}(A) = \begin{bmatrix} 15 & 3 & -6 \\ -10 & 5\alpha & 4 \\ 8 & -4\alpha & 3\alpha - 2 \end{bmatrix}$$

$$\text{Now } B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -5\alpha & 0 \\ 0 & 4\alpha & -2\alpha \end{bmatrix} + \begin{bmatrix} 15 & 3 & -6 \\ -10 & 5\alpha & 4 \\ 8 & -4\alpha & 3\alpha - 2 \end{bmatrix}$$

$$= \begin{bmatrix} 16 & 3 & -6 \\ -10 & 0 & 4 \\ 8 & 0 & \alpha - 2 \end{bmatrix}$$

As $|B| = 66$ $\quad\therefore\ \alpha = 1$

Now $|\text{adj } A| = |A|^{n-1} = |A|^2$

$$|A| = \begin{vmatrix} 1 & 1 & 2 \\ 2 & 3 & 0 \\ 0 & 4 & 5 \end{vmatrix} = 21$$

$\therefore\ |\text{adj } A| = 21^2 = 441$

Question ID : 691121530

5. Let $\alpha = 3 + 4 + 8 + 9 + 13 + 14 + \ldots$ upto 40 terms. If $(\tan\beta)^{\frac{\alpha}{1020}}$ is a root of the equation $x^2 + x - 2 = 0$, $\beta \in \left(0, \frac{\pi}{2}\right)$ then $\sin^2\beta + 3\cos^2\beta$ is equal to :

(1) 2 (2) $\frac{2}{7}$ (3) $\frac{5}{2}$ (4) $\frac{3}{2}$

Ans. (1)

Sol. $\alpha = (3 + 8 + 13 \ldots\ldots$ upto 20 terms$) + (4 + 9 + 14 + \ldots\ldots$ upto 20 terms$)$

$= \frac{20}{2}[6+19\times5]+\frac{20}{2}(8+19\times5) = 2040$

$(\tan\beta)^{\frac{2040}{1020}} = \tan^2\beta$

Now, roots of $x^2 + x - 2 = 0$

$x^2 + 2x - x - 2 = 0$

$x(x + 2) - (x + 2) = 0$

$\Rightarrow x = -2, 1$

$\Rightarrow \tan^2\beta = 1$

$\beta = \frac{\pi}{4}$

$\therefore \sin^2\frac{\pi}{4} + 3\cos^2\frac{\pi}{4}$

$\frac{1}{2} + \frac{3}{2} \Rightarrow 2$

Question ID : 691121531

6. A candidate has to go to the examination centre to appear in an examination. The candidate uses only one means of transportation for the entire distance out of bus, scooter and car. The probabilities of the candidate going by bus, scooter and car, respectively, are $\frac{2}{5}, \frac{1}{5}$ and $\frac{2}{5}$. The probabilities that the candidate reaches late at the examination centre are $\frac{1}{5}, \frac{1}{3}$ and $\frac{1}{4}$ if the candidate uses bus, scooter and car, respectively. Given that the candidate reached late at the examination centre, the probability that the candidate travelled by bus is:

(1) $\frac{11}{37}$ (2) $\frac{12}{37}$ (3) $\frac{13}{37}$ (4) $\frac{14}{37}$

Ans. (2)

Sol. Prob of being late

$$P(L)=\left(\frac{2}{5}\times\frac{1}{5}\right)+\left(\frac{1}{5}\times\frac{1}{3}\right)+\left(\frac{2}{5}\times\frac{1}{4}\right)$$

$$=\frac{2}{25}+\frac{1}{15}+\frac{2}{20}=\frac{37}{150}$$

$$P(\text{Requiral})=\frac{\frac{2}{25}}{\frac{37}{150}}=\frac{12}{37}$$

Question ID : 691121532

7. A set of four observations has mean 1 and variance 13. Another set of six observations has mean 2 and variance 1. Then, the variance of all these 10 observations is equal to:

(1) 5.96 (2) 6.14 (3) 6.04 (4) 6.24

Ans. (3)

Sol. Given $\bar{x}=1,\sigma_1^2=13$

$\bar{y}=2,\sigma_2^2=1$

$$\text{Combined variance}=\frac{n_1\sigma_1^2+n_2\sigma_2^2}{n_1+n_2}+\frac{n_1n_2}{(n_1+n_2)^2}(\bar{x}-\bar{y})^2$$

$$=\frac{4\times13+6\times1}{10}+\frac{4\times6}{10^2}(2-1)^2$$

$$\frac{58}{10}+\frac{24}{100}=\frac{580+24}{100}=\frac{604}{100}=6.04$$

Question ID : 691121533

8. If $26\left(\frac{2^3}{3}({}^{12}C_2)+\frac{2^5}{5}({}^{12}C_4)+\frac{2^7}{7}({}^{12}C_6)+\ldots+\frac{2^{13}}{13}({}^{12}C_{12})\right)=3^{13}-\alpha,$ then α is equal to :

(1) 45 (2) 48 (3) 51 (4) 54

Ans. (3)

Sol. General term

$$T_r = \frac{2^{2r+1}}{2^{(2r+1)}} \cdot {}^{12}C_{2r}$$

$$= \frac{2^{2r+1}}{13} \cdot {}^{13}C_{2r+1}$$

Now Sum of series $= \sum_{r=1}^{6} \frac{1}{13} \cdot {}^{13}C_{2r+1} \cdot 2^{2r+1}$

$$= \frac{1}{13}\left\{{}^{13}C_3 \cdot 2^3 + {}^{13}C_5 \cdot 2^5 + \ldots + {}^{13}C_{13} \cdot 2^{13}\right\}$$

Now, $(1+x)^{13} - (1-x)^{13} = 2\left\{{}^{13}C_1 \cdot x + {}^{13}C_3 x^3 + \ldots + {}^{13}C_{13} \cdot x^{13}\right\}$

At $x = 2$

$3^{13} - (-1)^{13} = 2\{13.2 + 13.5\}$

$= 2(26 + 13.5)$

$\therefore \alpha = 51$

Question ID : 691121534

9. A person has three different bags and four different books. The number of ways, in which he can put these books in the bags so that no bag is empty, is:

(1) 18 (2) 36 (3) 39 (4) 72

Ans. (2)

Sol.

B_1	B_2	B_3
1	1	2

No. of ways $= \frac{4!}{2!2!} \times 3! = 6 \times 6 = 36$

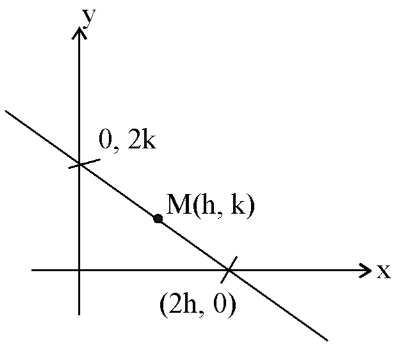
Question ID : 691121535

10. If a straight line drawn through the point of intersection of the lines $4x + 3y - 1 = 0$ and $3x + 4y - 1 = 0$, meets the co-ordinate axes at the points P and Q, then the locus of the mid point of PQ is:

(1) $x + y - 7 = 0$ (2) $x + y - 14xy = 0$ (3) $2x + y + 14xy = 0$ (4) $x + 2y - 14xy = 0$

Ans. (2)

Sol.



locus of $M \equiv \frac{x}{2h}+\frac{y}{2k}=1$ ....(1)

locus of P.O.I. of

$3x + 4y = 1$

$4x + 3y = 1$

$x=\frac{1}{7}; y=\frac{1}{7}$

$\therefore \frac{1}{7}\cdot\frac{1}{2h}+\frac{1}{7}\cdot\frac{1}{2k}=1$

$\frac{1}{x}+\frac{1}{y}=14$

$x + y - 14xy = 0$

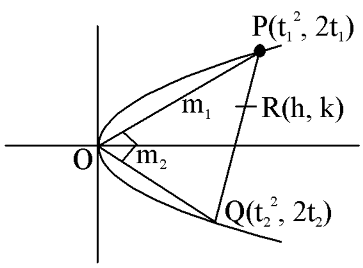
Question ID : 691121536

11. Let O be the vertex of the parabola $y^2 = 4x$ and its chords OP and OQ are perpendicular to each other. If the locus of the mid-point of the line segment PQ is a conic C, then the length of its latus rectum is:

(1) 1 (2) 2 (3) 4 (4) 8

Ans. (2)

Sol.



$t_1t_2 = -4$

$2h = t_1^2 + t_2^2 \qquad 2k = 2(t_1 + t_2)$

$2h = k^2 - 2t_1 t_2$

$2h = k^2 + 8$

$k^2 = 2h - 8$

$= 2(h - 4)$

L.R. = 2

Question ID : 691121537

12. Let $\alpha = 3\sin^{-1}\left(\frac{6}{11}\right)$ and $\beta = 3\cos^{-1}\left(\frac{4}{9}\right)$, where inverse trigonometric functions take only the principal values. Given below are two statements:

Statement I: $\cos(\alpha+\beta) > 0$

Statement II: $\cos(\alpha) < 0$

In the light of the above statements, choose the correct answer from the options given below:

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Ans. (1)

Sol. $\frac{1}{2} < \frac{6}{11} < \frac{1}{\sqrt{2}}$

$\sin^{-1}\left(\frac{1}{2}\right) < \sin^{-1}\left(\frac{6}{11}\right) < \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\frac{\pi}{6} < 3\sin^{-1}\left(\frac{6}{11}\right) < \frac{\pi}{4}$

$\frac{\pi}{2} < \alpha < \frac{3\pi}{4} \quad \therefore \cos\alpha < 0$

$0 < \frac{4}{9} < \frac{1}{2}$

$\frac{\pi}{3} < \cos^{-1}\left(\frac{4}{9}\right) < \frac{\pi}{2}$

$\pi < 3\cos^{-1}\left(\frac{4}{9}\right) < \frac{3\pi}{2}$

$\pi < \beta < \frac{3\pi}{2}$

Now $\frac{3\pi}{2} < \alpha + \beta < \frac{9\pi}{4}$

$\therefore \cos(\alpha + \beta) > 0$

Question ID : 691121538

13. For the function $f(x) = e^{\sin|x|} - |x|, x \in$ in R consider the following statements:

Statement I: f is differentiable for all $x \in R$

Statement II: f is increasing in $\left(-\pi, -\frac{\pi}{2}\right)$

In the light of the above statements, choose the correct answer from the options given below :

(1) Both Statement I and Statement II are true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Statement I is false but Statement II is true

Ans. (1)

Sol. $f(x) = e^{\sin|x|} - |x|;\ x \in R$

$$f(x) = \begin{cases} e^{\sin x} - x; & x > 0 \\ e^{-\sin x} + x; & x < 0 \end{cases}$$

LHD at $x = 0$

$f'(x) = e^{-\sin x}(-\cos x) + 1$

at $x = 0$

$= -1 + 1$

$= 0$

RHD at $x = 0$

$f'(x) = e^{\sin x}.\cos x - 1$

at $x = 0$

$= 1 - 1$

$= 0$

Diff at $x = 0$

Statement 1 is true

and for $x \in \left(-\pi, \frac{-\pi}{2}\right)$

$f(x) = e^{-\sin x} + x$

$f'(x) = -e^{-\sin x}\cos x + 1$

for $x \in \left(-\pi, \frac{-\pi}{2}\right)$ $\cos x < 0$

$\therefore f'(x) > 0$

$\therefore f(x)$ is increasing

Question ID : 691121539

14. Let $\vec{a} = 4\hat{i} - \hat{j} + 3\hat{k}$, $\vec{b} = 10\hat{i} + 2\hat{j} - \hat{k}$ and a vector $\vec{c}$ such that $2(\vec{a} \times \vec{b}) + 3(\vec{b} \times \vec{c}) = \vec{0}$. If $\vec{a} \cdot \vec{c} = 15$, then $\vec{c} \cdot (\hat{i} + \hat{j} - 3\hat{k})$ is equal to :

(1) –6 (2) –5 (3) –4 (4) –3

Ans. (2)

Sol. $2(\vec{a} \times \vec{b}) - 3(\vec{c} \times \vec{b}) = 0$

$(2\vec{a} - 3\vec{c}) \times \vec{b} = 0$

$|\vec{b}| = 2\vec{a} - 3\vec{c}$

$2\vec{a} - 3\vec{c} = \lambda\vec{b}$

$\vec{c} = \frac{2\vec{a} - \lambda\vec{b}}{3}$ ....(1)

$\vec{a} \cdot \vec{c} = 15$

$\left(\frac{2\vec{a} - \lambda\vec{b}}{3}\right) \cdot \vec{a} = 15$

$\frac{2|a|^2 - \lambda\vec{b} \cdot \vec{a}}{3} = 15$

$2(26) - \lambda(40 - 2 - 3) = 45$

$\lambda = \frac{1}{5}$

$$\Rightarrow \vec{c}-.(\hat{i}+\hat{j}-3\hat{k})=\frac{\left(2(4\hat{i}-\hat{j}+3k)-\frac{1}{5}(10\hat{i}+2\hat{j}-k)\cdot(\hat{i}+\hat{j}-3\hat{k})\right)}{3}$$

$$=\frac{2(4-1-9)-\frac{1}{5}(10+2+3)}{3}=-5$$

Question ID : 691121540

15. Let the foot of perpendicular from the point $(\lambda, 2, 3)$ on the line $\frac{x-4}{1}=\frac{y-9}{2}=\frac{z-5}{1}$ be the point $(1, \mu, 2)$. Then the distance between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$ and $\frac{x-\lambda}{2}=\frac{y-\mu}{3}=\frac{z+5}{6}$ is equal to :

(1) $\frac{12}{7}$ (2) $\frac{\sqrt{145}}{7}$ (3) $\frac{\sqrt{146}}{7}$ (4) $\frac{\sqrt{143}}{7}$

Ans. (3)

Sol. $L_1 \equiv \frac{x-4}{1}=\frac{y-9}{2}=\frac{z-5}{1}$, Subsitute $(1, \mu, 2)$

$\frac{1-4}{1}=\frac{\mu-9}{2}=\frac{2-5}{1}$

$\mu - 9 = -6$

$\mu = 3$

Let $\perp$r vector $(1-\lambda, \mu-2, 2-3)$

i.e. $(1-\lambda, 1, -1)$

As Apply condition of $\perp$r $(1-\lambda).1 + 1.2 + (-1).1 = 0$

$\Rightarrow \lambda = 2$

Now dist between line $L_A \equiv \frac{x-1}{2}=\frac{y-2}{3}=\frac{z+4}{6}$

$L_B = \frac{x-2}{2}=\frac{y-3}{3}=\frac{z+5}{6}$

$$d=\left|\frac{(\vec{a}_2-\vec{a}_1)\times\vec{b}}{|b|}\right|=\frac{\sqrt{146}}{7}$$

Question ID : 691121541

16. The value of the integral $\int_0^2 \frac{\sqrt{x(x^2+x+1)}}{(\sqrt{x+1})(\sqrt{x^4+x^2+1})}dx$ is equal to :

(1) $\frac{1}{3}\log_e(3-2\sqrt{2})$ (2) $\frac{2}{3}\log_e(4+\sqrt{2})$

(3) $\frac{2}{3}\log_e(3+2\sqrt{2})$ (4) $\frac{1}{3}\log_e(1+6\sqrt{2})$

Ans. (3)

Sol. $I=\int_0^2 \sqrt{\frac{x\left(x^2+x+1\right)}{(x+1)\left(x^2-x+1\right)\left(x^2+x+1\right)}}dx$

$=\int_0^2 \sqrt{\frac{x}{x^3+1}}dx$

$\left(x^{3/2}=t \Rightarrow \frac{3}{2}x^{1/2}dx=dt\right)$

$=\frac{2}{3}\int_0^{2^{3/2}} \frac{dt}{\sqrt{t^2+1}}$

$=\frac{2}{3}\left(\ell n\left(t+\sqrt{t^2+1}\right)\right)_0^{2^{3/2}}$

$=\frac{2}{3}\ell n\left(2^{3/2}+3\right)$

Question ID : 691121542

17. Let $y=y(x)$ be the solution of the differential equation .

$x\sqrt{1-x^2}dy+\left(y\sqrt{1-x^2}-x\cos^{-1}x\right)dx=0,\ x\in(0,1)\ \lim_{x\to1^-} y\left(x\right)=1$. Then $y\left(\frac{1}{2}\right)$ equals :

(1) $3-\frac{\pi}{\sqrt{3}}$ (2) $4-\sqrt{3}\pi$ (3) $4-\frac{2\pi}{\sqrt{3}}$ (4) $3-\frac{\pi}{2\sqrt{3}}$

Ans. (1)

Sol. $x\sqrt{1-x^2}dy+\left(y\sqrt{1-x^2}-x\cos^{-1}x\right)\cdot dx=0$

$$\frac{dy}{dx}+\frac{y}{x}=\frac{\cos^{-1}x}{\sqrt{1-x^2}}$$

$$P=\frac{1}{x};Q=\frac{\cos^{-1}x}{\sqrt{1-x^2}}$$

$$\text{I.F.}=e^{\int\frac{1}{x}dx}$$

$$=e^{\ell n x}=x$$

$$y\cdot x=\int\frac{x\cos^{-1}x}{\sqrt{1-x^2}}dx+c$$

det $\cos^{-1}x=t$

$$\frac{-1}{\sqrt{1-x^2}}dx=dt$$

$$-\int t\cdot\cos t\,dt$$

$$\Rightarrow-[t\sin t+\cos t]$$

$$\therefore y\cdot x=-\sqrt{1-x^2}\cos^{-1}x-x+C$$

As $\underset{x\to 1^-}{dt}\ y(x)=1$

$$1.1=-1-\sqrt{1-1^2}\cos^{-1}1+c$$
$$c=2$$

$$\therefore x.y=2-x-\sqrt{1-x^2}\cos^{-1}x$$

At $x=\frac{1}{2}$

$$y=3-\frac{\pi}{\sqrt{3}}$$

Question ID : 691121543

18. Let $f:(1,\infty)\to R$ be a function defined as $f(x)=\frac{x-1}{x+1}$. Let $f^{i+1}(x)=f(f^{i}(x)),\ i=1,2,\dots,25,$ where $f^1(x)=f(x)$. If $g(x)+f^{26}(x)=0, x\in(1,\infty)$, then the area of the region bounded by the curves $y=g(x)$, $2y=2x-3$, $y=0$ and $x=4$ is:

(1) $\frac{1}{8}+\log_e 2$ (2) $\frac{1}{4}+\log_e 2$ (3) $\frac{5}{6}+3\log_e 2$ (4) $\frac{5}{6}+\log_e 2$

Ans. (1)

Sol. $f^2(x) = f(f(x)) = \dfrac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1} = \dfrac{-1}{x}$

$f^3(x) = f\left(f^2(x)\right) = f\left(\dfrac{-1}{x}\right) = \dfrac{1+x}{1-x}$

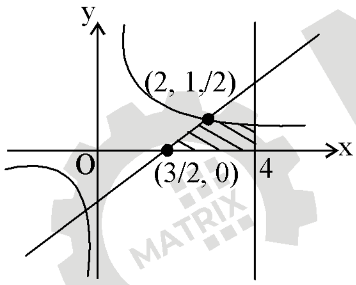
$f^4(x) = f\left(f^3(x)\right) = f\left(f\left(\dfrac{-1}{x}\right)\right) = x$

$f^5(x) = f\left(f^4(x)\right) = f(x)$

$f^6(x) = f\left(f^5(x)\right) = f(f(x)) = \dfrac{-1}{x}$

$f^{26}(x) = \dfrac{-1}{x}$

$g(x) = \dfrac{1}{x}$



$x - \dfrac{3}{2} = \dfrac{1}{x} \Rightarrow 2x^2 - 3x - 2 = 0$

$\Rightarrow x = \dfrac{-1}{2}, 2$

Req. area $= \int_2^4 \dfrac{1}{x}dx + \dfrac{1}{2}\times\dfrac{1}{2}\times\dfrac{1}{2}$

$= \dfrac{1}{8} + (\ell nx)_2^4 = \dfrac{1}{8} + \ell n2$

Question ID : 691121544

19. Let $f(x)=\begin{cases}\frac{1}{3} & ,x\le \pi/2\\ \frac{b(1-\sin x)}{(\pi-2x)^2} & x>\pi/2\end{cases}$. If f is continuous at $x=\pi/2$, then the value of $\int_0^{3b-6}|x^2+2x-3|\,dx$ is:

(1) 5 (2) 2 (3) 3 (4) 4

Ans. (4)

Sol. $\lim_{x\to\frac{\pi^-}{2}} f(x)=\lim_{x\to\frac{\pi^+}{2}} f(x)$

$\text{LHL}=\frac{1}{3}$

$\text{RHL}=\lim_{x\to\frac{\pi^+}{2}}\frac{b(1-\sin x)}{(\pi-2x)^2}$

$=\lim_{h\to 0}\frac{b\left(1-\sin\left(\frac{\pi}{2}+h\right)\right)}{\left(\pi-2\left(\frac{\pi}{2}+h\right)\right)^2}$

$=\lim_{h\to 0}\frac{b(1-\cos h)}{4h^2}$

$=\lim_{h\to 0}\frac{b\cdot 2\cdot\sin^2\frac{h}{2}}{4h^2}=\frac{2b}{16}=\frac{b}{8}$

$\therefore \frac{1}{3}=\frac{b}{8}\Rightarrow b=\frac{8}{3}$

$I=\int_0^2|x^2+2x-3|dx$

$=\int_0^1-(x^2+2x-3)\,dx+\int_1^2(x^2+2x-3)dx$

$=\left[-\frac{x^3}{3}-\frac{2x^2}{2}+3x\right]_0^1+\left[\frac{x^3}{3}+\frac{2x^2}{2}-3x\right]_1^2$

$=\left(-\frac{1}{3}-1+3\right)+\left(\frac{8}{3}+4-6-\frac{1}{3}-1+3\right)$

$=\frac{5}{3}+\frac{7}{3}=4$

Question ID : 691121545

20. Let $\frac{x^2}{f(a^2+7a+3)}+\frac{y^2}{f(3a+15)}=1$ represent an ellipse with major axis along y-axis, where f is a strictly decreasing positive function on R. If the set of all possible values of a is $R-[\alpha,\beta]$, then $\alpha^2+\beta^2$ is equal to:

(1) 28 (2) 40 (3) 61 (4) 24

Ans. (2)

Sol. $f(3a + 15) > f(a^2 + 7a + 3)$

$3a + 15 < a^2 + 7a + 3$

$a^2 + 4a - 12 > 0$

$(a + 6)(a - 2) > 0$

$a \in R - [-6, 2]$

$\alpha = -6, \beta = 2$

$\alpha^2 + \beta^2 = 36 + 4$

$= 40$

SECTION – B

Question ID : 691121546

21. The sum of squares of all the real solutions of the equation

$\log_{(x+1)}(2x^2+5x+3) = 4-\log_{(2x+3)}(x^2+2x+1)$ is equal to _________.

Ans. (2)

Sol. $\log_{x+1}\left(2x^2+5x+3\right)+\log_{2x+3}(x+1)^2 = 4$

$\log_{x+1}(2x+3)(x+1)+2\log_{2x+3}(x+1) = 4$

$\log_{x+1}\left(2x+3\right)+1+2\log_{2x+3}(x+1) = 4$

$t+\frac{2}{t}=3$

$t^2 - 3t + 2 = 0$

$(t - 1)(t - 2) = 0$

$t = 1, 2$

C-1 : $\log_{x+1} 2x+3=1$

$2x+3=x+1$

$x=-2$

C-2 : $\log_{x+1}(2x+3)=2$

$2x+3=(x+1)^2$

$x=\pm\sqrt{2}$

Check the domain $\therefore x \neq -\sqrt{2}-2$ and -2

$\therefore x=\sqrt{2}$

$\therefore \left(\sqrt{2}\right)^2=2$

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22. If $\int_{\pi/4}^{3\pi/4}(\cot(x-\frac{\pi}{3})\cot(x+\frac{\pi}{3})+1)dx=\alpha\log_e(\sqrt{3}-1)$, then $9\alpha^2$ is equal to ________.

Ans. (12)

Sol. $\int_{\pi/6}^{\pi/4}\left(\left(\cot\left(x-\frac{\pi}{3}\right)\cot\left(x+\frac{\pi}{3}\right)+1\right)dx\right.$

using $\cot(A-B)=\frac{\cot B\cot A+1}{\cot B-\cot A}$

$\Rightarrow \int_{\pi/6}^{\pi/4}-\frac{1}{\sqrt{3}}\left[\cot\left(x-\frac{\pi}{3}\right)-\cot\left(x+\frac{\pi}{3}\right)\right]dx$

$\Rightarrow \frac{-1}{\sqrt{3}}\left[\ell n\left|\frac{\sin\left(x-\frac{\pi}{3}\right)}{\sin\left(x+\frac{\pi}{3}\right)}\right|\right]_{\frac{\pi}{6}}^{\frac{\pi}{4}}$

$=\frac{-1}{\sqrt{3}}\left[\ell n\left|\left(\frac{\sin 15^\circ}{\sin 75^\circ}\right)\right|-\ell n\left|\frac{\sin 30^\circ}{\sin 90^\circ}\right|\right]$

$=\frac{-1}{\sqrt{3}}\left[\ell n\left(\tan 15^\circ\right)-\ell n\left(\frac{1}{2}\right)\right]$

$=\frac{-1}{\sqrt{3}}\left[\ell n\left(2-\sqrt{3}\right)+\ell n 2\right]$

$\Rightarrow \frac{-1}{\sqrt{3}}\ln\left(4-2\sqrt{3}\right) = \frac{-2}{\sqrt{3}}\ln\left(\sqrt{3}-1\right)$

$\therefore \alpha = \frac{-2}{\sqrt{3}} \therefore 9\alpha^2 = 12$

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23. Let a line L_1 pass through the origin and be perpendicular to the lines

$L_2 : \vec{r} = (3+t)\hat{i} + (2t-1)\hat{j} + (2t+4)\hat{k}$ and

$L_3 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$, $t, s \in R$.

If $(a, b, c), a \in Z$, is the point on L_3 at a distance of $\sqrt{17}$ from the point of intersection of L_1 and L_2, then $(a+b+c)^2$ is equal to _________.

Ans. (4)

Sol. Direction of line $L_1 = \vec{v}_2 \times \vec{v}_3$

$= \begin{vmatrix} i & j & k \\ 1 & 2 & 2 \\ 2 & 2 & 1 \end{vmatrix} = -2i + 3j - 2k$

$\therefore L_1 = \vec{0} + \lambda(-2i + 3j - 2k)$

Now point of intersec line of L_2 and L_3

$-2\lambda = 3 + t$, ...(1)

$3\lambda = 2t - 1$ and ....(2)

$-2\lambda = 2t + 4$ ....(3)

from (1) and (3)

$3 + t = 2t + 4$

$t = -1$

$-2\lambda = 2$

$\lambda = -1$

$\therefore$ Point of intersection (2, –3, 2)

The point (a, b, c) lies on L_3 it has the form $(3 + 2s, 3 + 2s, 2 + s)$

Dist from (2, –3, 2) from it $\sqrt{17}$

$\therefore (3 + 2s - 2)^2 + (3 + 2s + 3)^2 + (2 + s - 2)^2 = 17$

$9s^2 + 28s + 20 = 0$

$s = -10/9, -2$

as a, b, c $\in Z$

$\therefore\ s = -2$

$a = -1$

$b = -1$

$c = 0$

$(a + b + c)^2 = 4$

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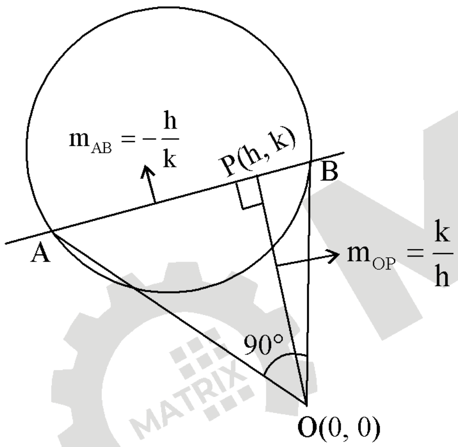
24. Consider the circle $C : x^2 + y^2 - 6x - 8y - 11 = 0$. Let a variable chord AB of the circle C subtend a right angle at the origin. If the locus of the foot of the perpendicular drawn from the origin on the chord AB is the circle $x^2 + y^2 - \alpha x - \beta y - \gamma = 0$, then $\alpha + \beta + 2\gamma$ is equal to ________.

Ans. (18)

Sol. Equation of AB



$y - k = -\frac{h}{k}(x - h)$

$hx + ky = h^2 + k^2$

H.F. : $\frac{hx + ky}{h^2 + k^2} = 1$

Now, $x^2 + y^2 - (6x + 8y)\left(\frac{hx + ky}{h^2 + k^2}\right) - 11\left(\frac{hx + ky}{h^2 + k^2}\right)^2 = 0$

Now coeff of x^2 + coeff of $y^2 = 0$

$2 - \frac{6h + 8k}{h^2 + k^2} - \frac{11}{h^2 + k^2} = 0$

$2h^2 + 2k^2 - 6h - 8k - 11 = 0$

$x^2 + y^2 - 3x - 4y - 55 = 0$

$\alpha + \beta + 2\gamma = 3 + 4 + 11$

$= 18$

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25. Let f be a polynomial function such that

$$\log_2(f(x)) = \left(\log_2\left(2+\frac{2}{3}+\frac{2}{9}+\ldots\ldots\infty\right)\right)\cdot\log_3\left(1+\frac{f(x)}{f(1/x)}\right), x>0$$ and f(6)=37. Then $\sum_{n=1}^{10} f(n)$ is equal to

________.

Ans. (395)

Sol. $S = 2+\frac{2}{3}+\frac{2}{9}+\cdots$

$= \frac{2}{1-1/3} = 3$

$\log_2\left(f'(x)\right) = \log_2 3.\log_3\left(1+\frac{f(x)}{f\left(\frac{1}{x}\right)}\right)$

$= \log_2\left(1+\frac{f(x)}{f\left(\frac{1}{x}\right)}\right)$

$\Rightarrow f(x)+f\left(\frac{1}{x}\right) = f(x)\cdot f\left(\frac{1}{x}\right)$

$\therefore$ f(x) = 1 + x^n and f(6) = 37

$\therefore$ n = 2

$\therefore$ f(x) = 1 + x^2

Now $\sum_{n=1}^{10} f(n)$

$\sum_{n=1}^{10} 1+n^2$

$\Rightarrow 10+\frac{10\times11\times21}{6} \Rightarrow 395$