JEE Main January 2026

Question Paper With Text Solution

22 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN JANUARY 2026 | 22TH JANUARY SHIFT-1

SECTION – A

Question ID : 444792155

1. If the sum of the first four terms of an A.P. is 6 and the sum of its first six terms is 4, then the sum of its first twelve terms is :

यदि एक A.P. के प्रथम चार पदों का योग 6 है और इसके प्रथम छह पदों का योग 4 है, तो इसके प्रथम बारह पदों का योग है :

(1) –26 (2) –20 (3) –24 (4) –22

Ans. Official answer NTA (4)

Sol. $S_4 = 2(2a + 3d) = 6 \Rightarrow 2a + 3d = 3$

$$S_6 = 3(2a + 5d) = 4 \Rightarrow 2a + 5d = \frac{4}{3}$$

$$\Rightarrow a = \frac{11}{4},\ d = \frac{-5}{6}$$

$$S_{12} = 6\left(\frac{11}{2} + 11\left(\frac{-5}{6}\right)\right) = -22$$

Question ID : 444792166

2. Let $f(x) = x^{2025} - x^{2000}, x \in [0,1]$ and the minimum value of the function f(x) in the interval [0, 1] be $(80)^{80}(n)^{-81}$. Then n is equal to :

माना $f(x) = x^{2025} - x^{2000}, x \in [0,1]$ हैं और अंतराल [0, 1] में f(x) का न्यूनतम मान $(80)^{80}(n)^{-81}$ है। तो n बराबर होगा :

(1) –41 (2) –80 (3) –81 (4) –40

Ans. Official answer NTA (3)

Sol. $f'(x) = 2025x^{2024} - 2000\,x^{1999} = 0$

$$x = 0 \text{ or } x = \left(\frac{2000}{2025}\right)^{\frac{1}{25}} = \left(\frac{80}{81}\right)^{\frac{1}{25}}$$

$$f\left(\left(\frac{80}{81}\right)^{\frac{1}{25}}\right) = \left(\left(\frac{80}{81}\right)^{\frac{1}{25}}\right)^{2000}\left(\frac{80}{81} - 1\right)$$

$$= \left(\frac{80}{81}\right)^{80}\left(\frac{-1}{81}\right)$$

Question ID : 444792157

3. Two distinct numbers a and b are selected at random from 1,2,3, ..., 50. The probability, that their product ab is divisible by 3, is :

1,2,3, ..., 50 में से भिन्न संख्याएँ a और b यादृच्छया चुनी जाती हैं। उनके गुणनफल ab के 3 से विभाज्य होने की प्रायिकता है:

(1) $\frac{272}{1225}$ (2) $\frac{561}{1225}$ (3) $\frac{8}{25}$ (4) $\frac{664}{1225}$

Ans. Official answer NTA (4)

Sol. $1-\frac{^{34}C_2}{^{50}C_2}$ $\Rightarrow 1-\frac{34.33}{50.49}=\frac{664}{1225}$

Question ID : 444792156

4. The coefficient of x^{48} in $(1+x)+2(1+x)^2+3(1+x)^3+......+100(1+x)^{100}$ is equal to :

$(1+x)+2(1+x)^2+3(1+x)^3+......+100(1+x)^{100}$ में x^{48} का गुणांक बराबर होगा :

(1) $100\cdot{}^{100}C_{49}-{}^{100}C_{50}$ (2) ${}^{100}C_{50}+{}^{101}C_{49}$

(3) $100\cdot{}^{100}C_{49}-{}^{100}C_{48}$ (4) $100\cdot{}^{101}C_{49}-{}^{101}C_{50}$

Ans. Official answer NTA (4)

Sol. $S=(1+x)+2(1+x)^2+3(1+x)^3+..........+100(1+x)^{100}$

$(1+x)S= \quad (1+x)^2+2(1+x)^3 ....+99(1+x)^{100}+100(1+x)^{101}$

$-xS=\left((1+x)+(1+x)^2+.............+(1+x)^{100}\right)-100(1+x)^{101}$

$-xS=(1+x)\left(\frac{(1+x)^{100}-1}{x}\right)-100(1+x)^{101}$

$S=\frac{-(1+x)^{101}}{x^2}+\frac{1+x}{x^2}+\frac{100(1+x)^{101}}{x}$

Coeff of $x^{48}=-{}^{101}C_{50}+100.\ {}^{101}C_{49}$

Question ID : 444792152

5. If the domain of the function $f(x)=\sin^{-1}\left(\frac{5-x}{3+2x}\right)+\frac{1}{\log_e(10-x)}$ is $(-\infty,\alpha]\cup[\beta,\gamma)-\{\delta\}$ then $6(\alpha+\beta+\gamma+\delta)$ is equal to :

यदि फलन $f(x)=\sin^{-1}\left(\frac{5-x}{3+2x}\right)+\frac{1}{\log_e(10-x)}$ का प्रांत $(-\infty,\alpha]\cup[\beta,\gamma)-\{\delta\}$ है, तो $6(\alpha+\beta+\gamma+\delta)$ बराबर है :

(1) 66 (2) 68 (3) 70 (4) 67

Ans. Official answer NTA (3)

Sol. $-1\le\frac{5-x}{3+2x}\le 1,\quad 10-x>0,\quad 10-x\ne 1$

$\frac{x+8}{3+2x}\ge 0 \quad \& \quad \frac{-3x+2}{3+2x}\le 0,\quad x<10,\quad x\ne 9$

$x\in(-\infty,-8]\cup\left[\frac{2}{3},10\right)-\{9\}$

Question ID : 444792168

6. Let the line $x=-1$ divide the area of the region $\{(x, y): 1+x^2\le y\le 3-x\}$ in the ratio $m:n$, $\gcd(m,n)=1$. Then $m+n$ is equal to :

माना रेखा $x=-1$, क्षेत्र $\{(x, y): 1+x^2\le y\le 3-x\}$ के क्षेत्रफल को $m:n$, $\gcd(m,n)=1$, के अनुपात में विभाजित करती है। तो $m+n$ बराबर होगी :

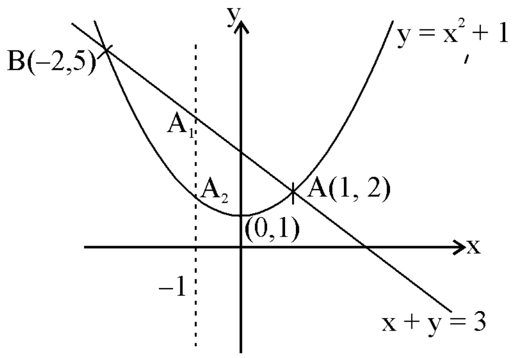
(1) 27 (2) 26 (3) 28 (4) 25

Ans. Official answer NTA (1)

Sol. $A_1=\int_{-2}^{-1}\left((3-x)-(x^2+1)\right)dx=\frac{7}{6}$

$A_2=\int_{-1}^{1}\left((3-x)-(x^2+1)\right)dx=\frac{10}{3}$

$\frac{A_1}{A_2}=\frac{7}{20}$



Question ID : 444792161

7. Let the set of all values of r, for which the circles $(x+1)^2+(y+4)^2=r^2$ and $x^2+y^2-4x-2y-4=0$ intersect at two distinct points be the interval (α, β). Then $\alpha\beta$ is equal to :

माना r के सभी मानों, जिनके लिए वृत्तों $(x+1)^2+(y+4)^2=r^2$ और $x^2+y^2-4x-2y-4=0$ के दो भिन्न प्रतिच्छेदन बिन्दु हैं, का समुच्चय अंतराल (α, β) है :

(1) 20 (2) 24 (3) 21 (4) 25

Ans. Official answer NTA (4)

Sol. $C_1(-1,-4),\ r_1=r$

$C_2(2, 1), r_2 = 3$

$|r_1 - r_2| < |C_1C_2| < r_1 + r_2$

$|r-3| < \sqrt{34} < r + 3$

$r \in \left(\sqrt{34} - 3, \sqrt{34} + 3\right)$

Question ID : 444792164

8. Let P(α, β, γ) be the point on the line $\frac{x-1}{2} = \frac{y+1}{-3} = z$ at a distance $4\sqrt{14}$ from the point (1, –1, 0) and nearer to the origin. Then the shortest distance, between the lines $\frac{x-\alpha}{1} = \frac{y-\beta}{2} = \frac{z-\gamma}{3}$ and $\frac{x+5}{2} = \frac{y-10}{1} = \frac{z-3}{1}$, is equal to :

माना रेखा $\frac{x-1}{2} = \frac{y+1}{-3} = z$ पर, बिन्दु (1, –1, 0) से $4\sqrt{14}$ की दूरी पर तथा मूल बिन्दु के पास वाला बिन्दु P(α, β, γ) है। तो रेखाओं $\frac{x-\alpha}{1} = \frac{y-\beta}{2} = \frac{z-\gamma}{3}$ और $\frac{x+5}{2} = \frac{y-10}{1} = \frac{z-3}{1}$ के बीच की न्यूनतम दूरी है :

(1) $2\sqrt{\frac{7}{4}}$ (2) $4\sqrt{\frac{5}{7}}$ (3) $4\sqrt{\frac{7}{5}}$ (4) $7\sqrt{\frac{5}{4}}$

Ans. Official answer NTA (3)

Sol. P (2λ +1, –3λ–1, λ)

Q (1, –1, 0)

$PQ = 4\sqrt{14} \Rightarrow \lambda = \pm 4$

P(9, –13, 4) or P(–7, 11, –4)

Rejcted α = –7, β=11, γ = –4

$$\text{Shortest distance} = \frac{\overrightarrow{AB}.\left(\vec{p} \times \vec{q}\right)}{|\vec{p} \times \vec{q}|} = \frac{28}{\sqrt{35}}$$

Question ID : 444792163

9. Let $\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\overrightarrow{AD} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \lambda \in \mathbb{R}$. Let the projection of the vector $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ on the diagonal $\overrightarrow{AC}$ of the parallelogram ABCD be of length one unit. If α, β, where α > β, be the roots of the equation $\lambda^2x^2 - 6\lambda x + 5 = 0$, then 2α – β is equal to :

माना $\overrightarrow{AB} = 2\hat{i} + 4\hat{j} - 5\hat{k}$ और $\overrightarrow{AD} = \hat{i} + 2\hat{j} + \lambda\hat{k}, \lambda \in \mathbb{R}$ है। माना समांतर चतुर्भुज ABCD के विकर्ण $\overrightarrow{AC}$ पर सदिश $\vec{v} = \hat{i} + \hat{j} + \hat{k}$ के प्रक्षेप की लंबाई एक इकाई है। यदि समीकरण $\lambda^2x^2 - 6\lambda x + 5 = 0$ के मूल α, β हैं, जहाँ $\alpha > \beta$ है, तो $2\alpha - \beta$ बराबर है :

(1) 3 (2) 1 (3) 6 (4) 4

Ans. Official answer NTA (1)

Sol. $\overrightarrow{AC} = 3\hat{i} + 6\hat{j} + (\lambda - 5)\hat{k}$

$$\frac{\overrightarrow{AC}.(\hat{i} + \hat{j} + \hat{k})}{|\overrightarrow{AC}|} = 1 \qquad \Rightarrow \lambda = 3$$

Eq. $9x^2 - 18x + 5 = 0$

$$x = \frac{1}{3}, \frac{5}{3}$$
(β) (α)

Question ID : 444792165

10. If the image of the point P(1, 2, a) in the line $\frac{x-6}{3} = \frac{y-7}{2} = \frac{7-z}{2}$ is Q(5, b, c), then $a^2 + b^2 + c^2$ is equal to:

यदि रेखा $\frac{x-6}{3} = \frac{y-7}{2} = \frac{7-z}{2}$ में बिन्दु P(1, 2, a) का प्रतिबिम्ब Q(5, b, c) है, तो $a^2 + b^2 + c^2$ बराबर है :

(1) 298 (2) 264 (3) 283 (4) 293

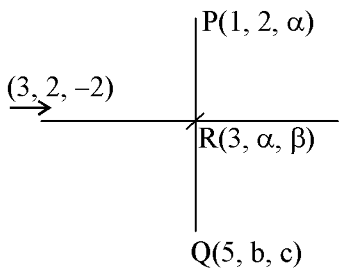
Ans. Official answer NTA (1)

Sol. $\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2} = \lambda$

$x = 3\lambda + 6 = 3$

$\lambda = -1$

R(3, 5, 9)



$\frac{b+2}{2} = 4 \quad \Rightarrow b = 8$

$a + c = 18$

& $\overrightarrow{PR} \cdot (3, 2, -2) = 0$

$6 + 6 + (9 - a)(-2) = 0$

$12 - 18 + 2a = 0$

$a = 3, c = 15$

Question ID : 444792167

11. Let $f:[1,\infty)\rightarrow\mathbb{R}$ be a differentiable function. If $6\int_1^x f(t)dt = 3xf(x)+x^3-4$ for all $x \ge 1$, then the value of $f(2)-f(3)$ is :

माना $f:[1,\infty)\rightarrow\mathbb{R}$ एक अवकलनीय फलन है। यदि सभी $x \ge 1$ के लिए $6\int_1^x f(t)dt = 3xf(x)+x^3-4$ है, तो $f(2)-f(3)$ बराबर है :

(1) 3 (2) –4 (3) –3 (4) 4

Ans. Official answer NTA (1)

Sol. Differentiate both sides w.r.t. 'x'

$6f(x) = 3xf'(x)+3f(x)+3x^2$

$x\frac{dy}{dx} = y - x^2$

$\frac{dy}{dx} - \frac{y}{x} = -x$

$\text{I.F.} = e^{\int -\frac{1}{x}dx} = \frac{1}{x}$

Solution of D.E. : $y\left(\frac{1}{x}\right) = \int -dx + c$

$y = -x^2 + cx$

At $x = 1 \Rightarrow 0 = 3f(1) - 3 \Rightarrow f(1)=1$

$c = 2$

$y = x^2 + 2x$

$f(2) - f(3) = 3$

Question ID : 444792160

12. If the chord joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ on the parabola $y^2 = 12x$ subtends a right angle at the vertex of the parabola, then $x_1x_2 - y_1y_2$ is equal to :

यदि परवलय $y^2 = 12x$ पर बिन्दुओं $P_1(x_1, y_1)$ और $P_2(x_2, y_2)$ को मिलाने वाली जीवा, परवलय के शीर्ष पर एक समकोण बनाती है, तो $x_1x_2 - y_1y_2$ बराबर है :

(1) 280 (2) 284 (3) 292 (4) 288

Ans. Official answer NTA (4)

Sol. $P_1(x_1, y_1) \equiv (3t_1^2, 6t_1)$

$P_2(x_2) \equiv (3t_2^2, 6t_2)$

$t_1 t_2 = -4$

$x_1 x_2 - y_1 y_2 = 144 + 144 = 288$

Question ID : 444792153

13. The number of distinct real solutions of the equation $x|x+4|+3|x+2|+10=0$ is :

समीकरण $x|x+4|+3|x+2|+10=0$ के भिन्न वास्तविक हलों की संख्या है :

(1) 0 (2) 3 (3) 2 (4) 1

Ans. Official answer NTA (4)

Sol. Case - I $x \leq -4$

$-x^2 - 4x - 3x - 6 + 10 = 0$

$x^2 + 7x - 4 = 0$

$x = \dfrac{-7 \pm \sqrt{65}}{2}$

$x \simeq -3 \cdot 5 \pm 4$

one sol,

Case II : $-4 < x \leq -2$

$x^2 + 4x - 3x - 6 + 10 = 0$

$x^2 + x + 4 = 0$

$D < 0$

No Sol.

Case III : $x > -2$

$x^2 + 4x + 3x + 6 + 10 = 0$

$x^2 + 7x + 16 = 0$

$D < 0$

No solution

Question ID : 444792162

14. The number of solutions of $\tan^{-1} 4x + \tan^{-1} 6x = \dfrac{\pi}{6}$, where $-\dfrac{1}{2\sqrt{6}} < x < \dfrac{1}{2\sqrt{6}}$, is equal to :

समीकरण $\tan^{-1} 4x + \tan^{-1} 6x = \dfrac{\pi}{6}$, $-\dfrac{1}{2\sqrt{6}} < x < \dfrac{1}{2\sqrt{6}}$, के हलों की संख्या होगी :

(1) 0 (2) 1 (3) 2 (4) 3

Ans. Official answer NTA (2)

Sol. $f'(x)=\dfrac{4}{1+16x^2}+\dfrac{6}{1+36x^2}>0$ (Increasing function)

$f\left(\dfrac{1}{2\sqrt{6}}\right)=\tan^{-1}\left(\dfrac{4}{2\sqrt{6}}\right)+\tan^{-1}\left(\dfrac{6}{2\sqrt{6}}\right)$

$=\tan^{-1}\left(\sqrt{\dfrac{2}{3}}\right)+\tan^{1}\left(\sqrt{\dfrac{3}{2}}\right)=\dfrac{\pi}{2}$

$\therefore$ 1 solution

Question ID : 444792159

15. If the line $\alpha x+2y=1$, where $\alpha\in\mathbb{R}$, does not meet the hyperbola $x^2-9y^2=9$, then a possible value of α is :

यदि रेखा $\alpha x+2y=1, \alpha\in\mathbb{R}$, अतिपरवलय $x^2-9y^2=9$ को नहीं मिलती है, तो α का एक संभव मान होगा :

(1) 0.7 (2) 0.8 (3) 0.5 (4) 0.6

Ans. Official answer NTA (2)

Sol. $y=\dfrac{-\alpha}{2}x+\dfrac{1}{2}$ does not meet hyperbola $\dfrac{x^2}{9}-\dfrac{y^2}{1}=1$

$c^2<a^2m^2-b^2$

$\dfrac{1}{4}<\dfrac{9\alpha^2}{4}-1$

$\alpha^2>\dfrac{5}{9}$

Question ID : 444792158

16. If a random variable x has the probability distribution

x	0	1	2	3	4	5	6	7
$P(x)$	0	2k	k	3k	$2k^2$	2k	k^2+k	$7k^2$

then $P(3<x\le 6)$ is equal to :

यदि एक यादृच्छिक चर x का प्रायिकता बंटन

x	0	1	2	3	4	5	6	7
$P(x)$	0	2k	k	3k	$2k^2$	2k	k^2+k	$7k^2$

है, तो $P(3 < x \le 6)$ बराबर है :

(1) 0.64 (2) 0.33 (3) 0.22 (4) 0.34

Ans. Official answer NTA (2)

Sol. $\sum p_i = 1 \Rightarrow k = \frac{1}{10}$

$P(3 < x \le 6) = P(4) + P(5) + P(6)$

$3k^2 + 3k = 0.33$

Question ID : 444792154

17. If $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$, then the determinant of the matrix $\left(A^{2025} - 3A^{2024} + A^{2023}\right)$ is :

यदि $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ है, तो आव्यूह $\left(A^{2025} - 3A^{2024} + A^{2023}\right)$ का सारणिक है :

(1) 16 (2) 28 (3) 12 (4) 24

Ans. Official answer NTA (1)

Sol. $A = \begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$ $|A|=1$

Char.eq. :$A^2-7A+I=0$

$\left|A^{2025} - 3A^{2024} + A^{2023}\right|$

$|A|^{2023} |A^2 - 3A + I|$

$= (1) |4A| = 16$

Question ID : 444792151

18. Let the relation R on the set M = {1,2,3, ..., 16} be given by $R = \{(x, y) : 4y = 5x - 3, x, y \in M\}$. Then the minimum number of elements required to be added in R, in order to make the relation symmetric, is equal to :

माना समुच्चय M = {1,2,3, ..., 16} पर संबंध R, $R = \{(x, y) : 4y = 5x - 3, x, y \in M\}$ द्वारा दिया गया है। तो संबंध R को सममित बनाने के लिए इसमें जोड़े जाने वाले आवश्यक अवयवों की न्यूनतम संख्या है :

(1) 1 (2) 2 (3) 3 (4) 4

Ans. Official answer NTA (2)

Sol. R = {(3,3) (7, 8) (11, 13)}

For symmctric (8, 7), (13, 11) must be added

Question ID : 444792169

19. Let the solution curve of the differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx, x > 0, y(1) = 0$, be y = y(x). Then y(3) is equal to :

माना अवकल समीकरण $xdy - ydx = \sqrt{x^2 + y^2}dx, x > 0, y(1) = 0$ का हल वक्र $y = y(x)$ है। तो $y(3)$ बराबर है :

(1) 2 (2) 6 (3) 1 (4) 4

Ans. Official answer NTA (4)

Sol. $xdy - ydx = \sqrt{x^2 + y^2}dx$

$$\frac{dy}{dx} - \frac{y}{x} = \sqrt{1 + \frac{y^2}{x^2}}$$

Put y = vx

$$v + x\frac{dv}{dx} - v = \sqrt{1 + v^2}$$

$$\int \frac{dv}{\sqrt{1 + v^2}} = \int \frac{dx}{x}$$

$$\ln\left|v + \sqrt{1 + v^2}\right| = \ln|xc|;\ y(1) = 0$$

$v + \sqrt{1 + v^2} = xc$; x = 1, v = 0

1 = c

$$y + \sqrt{x^2 + y^2} = x^2$$

$$y - x^2 = \sqrt{x^2 + y^2}$$

$$y^2 + x^4 - 2x^2y = x^2 + y^2$$

$$\frac{x^2 - 1}{2} = y$$

y(3) = 4

Question ID : 444792170

20. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \left(\frac{1}{[x] + 4}\right)dx$, where $[\cdot]$ denotes the greatest integer function, is :

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{1}{[x]+4}\right)dx$, जहाँ [·] महत्तम पूर्णांक फलन है, का मान है :

(1) $\frac{1}{60}(21\pi-1)$ (2) $\frac{7}{60}(\pi-3)$ (3) $\frac{7}{60}(3\pi-1)$ (4) $\frac{1}{60}(\pi-7)$

Ans. Official answer NTA (3)

Sol. $\int_{-\pi/2}^{-1}\frac{1}{2}+\int_{-1}^{0}\frac{1}{3}+\int_{0}^{1}\frac{1}{4}+\int_{1}^{\pi/2}\frac{1}{5}$

$=\frac{1}{2}\left(-1+\frac{\pi}{2}\right)+\frac{1}{3}(1)+\frac{1}{4}(1)+\frac{1}{5}\left(\frac{\pi}{2}-1\right)$

$\frac{21}{60}\pi-\frac{7}{60}$

SECTION – B

Question ID : 444792173

21. Let ABC be a triangle. Consider four points p_1, p_2, p_3, p_4 on the side AB , five points p_5, p_6, p_7, p_8, p_9 on the side BC , and four points $p_{10}, p_{11}, p_{12}, p_{13}$ on the side AC . None of these points is a vertex of the triangle ABC . Then the total number of pentagons, that can be formed by taking all the vertices from the points $p_1, p_2, \ldots, p_{13}$, is _________.

माना ABC एक त्रिभुज है। भुजा AB पर चार बिन्दुओं p_1, p_2, p_3, p_4, भुजा BC पर पांच बिन्दुओं p_5, p_6, p_7, p_8, p_9, और भुजा AC पर चार बिन्दुओं $p_{10}, p_{11}, p_{12}, p_{13}$ का विचार कीजिए। इनमें से कोई भी बिन्दु त्रिभुज ABC का शीर्ष नहीं है। तो सभी शीर्ष बिन्दुओं को $p_1, p_2, \ldots, p_{13}$ में से लेकर बनाए जा सकने वाले पंचभुजों की संख्या _________ होगी।

Ans. Official answer NTA (660)

Sol.

AB	BC	CA	
2	2	$1\to 6\times10\times4$	= 240
2	1	$2\to 6\times5\times6$	= 180
1	2	$2\to 4\times10\times5$	= 240
			= 660

Question ID : 444792175

22. If $\int (\sin x)^{\frac{-11}{2}}(\cos x)^{\frac{-5}{2}}dx = -\frac{p_1}{q_1}(\cot x)^{\frac{9}{2}} - \frac{p_2}{q_2}(\cot x)^{\frac{5}{2}} - \frac{p_3}{q_3}(\cot x)^{\frac{1}{2}} + \frac{p_4}{q_4}(\cot x)^{\frac{-3}{2}} + C$, where p_i and q_i are positive integers with gcd(p_i, q_i) = 1 for i = 1, 2, 3, 4 and C is the constant of integration, then $\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4}$ is equal to________.

यदि $\int (\sin x)^{\frac{-11}{2}}(\cos x)^{\frac{-5}{2}}dx = -\frac{p_1}{q_1}(\cot x)^{\frac{9}{2}} - \frac{p_2}{q_2}(\cot x)^{\frac{5}{2}} - \frac{p_3}{q_3}(\cot x)^{\frac{1}{2}} + \frac{p_4}{q_4}(\cot x)^{\frac{-3}{2}} + C$ है, जहाँ p_i और q_i धन पूर्णांक हैं, i = 1, 2, 3, 4 के लिए gcd(p_i, q_i) = 1 है और C समाकलन अचर है, तो $\frac{15p_1p_2p_3p_4}{q_1q_2q_3q_4}$ बराबर__________है।

Ans. Official answer NTA (16)

Sol. $\int (\tan x)^{-11/2}\sec^8 x\,dx$

$\tan x = t$

$\int t^{-11/2}\left(1+t^2\right)^3 dt = \int t^{-11/2}\left(1+t^6+3t^2+3t^4\right)dt$

$= \int\left(t^{-11/2}+t^{1/2}+3t^{-7/2}+3t^{-3/2}\right)dt$

$= \frac{t^{-9/2}}{(-9/2)} + \frac{t^{3/2}}{3/2} + 3\frac{t^{-5/2}}{(-5/2)} + \frac{3t^{-1/2}}{(-1/2)} + c$

$= \frac{-2}{9}(\cot x)^{9/2} + \frac{2}{3}(\cot x)^{-3/2} - \frac{6}{5}(\cot x)^{5/9} - 6(\cot x)^{1/2} + c$

$15\left(\frac{2.2.6.6}{9.3.5.1}\right) = 16$

Question ID : 444792174

23. If $\frac{\cos^2 48^\circ - \sin^2 12^\circ}{\sin^2 24^\circ - \sin^2 6^\circ} = \frac{\alpha+\beta\sqrt{5}}{2}$, where $\alpha, \beta \in \mathbb{N}$, then $\alpha+\beta$ is equal to _________.

यदि $\frac{\cos^2 48^\circ - \sin^2 12^\circ}{\sin^2 24^\circ - \sin^2 6^\circ} = \frac{\alpha+\beta\sqrt{5}}{2}$ है, जहाँ $\alpha, \beta \in \mathbb{N}$ हैं, तो $\alpha+\beta$ बराबर _________है।

Ans. Official answer NTA (4)

Sol. $\frac{\left(\frac{1}{2}\right)\cos 36^\circ}{\frac{1}{2}\sin 18^\circ} = \frac{\frac{\sqrt{5}+1}{4}}{\frac{\sqrt{5}-1}{4}} = \frac{6+2\sqrt{5}}{4} = \frac{3+\sqrt{5}}{2}$

Question ID : 444792171

24. Let $\alpha = \frac{-1+i\sqrt{3}}{2}$ and $\beta = \frac{-1-i\sqrt{3}}{2}, i = \sqrt{-1}$. If

$(7-7\alpha+9\beta)^{20}+(9+7\alpha-7\beta)^{20}+(-7+9\alpha+7\beta)^{20}+(14+7\alpha+7\beta)^{20}=m^{10}$, then m is ________.

माना $\alpha = \frac{-1+i\sqrt{3}}{2}$ और $\beta = \frac{-1-i\sqrt{3}}{2}, i = \sqrt{-1}$ हैं। यदि

$(7-7\alpha+9\beta)^{20}+(9+7\alpha-7\beta)^{20}+(-7+9\alpha+7\beta)^{20}+(14+7\alpha+7\beta)^{20}=m^{10}$ है, तो m है ________.

Ans. Official answer NTA (49)

Sol. $\alpha = \omega,\ \beta = \omega^2$

$$\therefore \left(7-7\omega+9\omega^2\right)^{20}+\left(9+7\omega-7\omega^2\right)^{20}+\left(-7+9\omega+7\omega^2\right)^{20}+\left(14+7\omega+7\omega^2\right)^{20}$$

$$\Rightarrow \omega^{40}\left(7\omega-7\omega^2+9\right)^{20}+\left(9+7\omega-7\omega^2\right)^{20}+\omega^{20}\left(-7\omega^2+9+70\right)^{20}+7^{20}$$

$$\Rightarrow \left(7\omega-7\omega^2+9\right)^{20}\left(\omega^2+\omega+1\right)+7^{20}=49^{10}$$

Question ID : 444792172

25. Let A be a 3 × 3 matrix such that $A+A^T=O$. If $A\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}3\\3\\2\end{bmatrix}, A^2\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}-3\\19\\-24\end{bmatrix}$ and

$\det(\operatorname{adj}(2\operatorname{adj}(A+I)))=(2)^{\alpha}\cdot(3)^{\beta}\cdot(11)^{\gamma}, \alpha,\beta,\gamma$ are non-negative integers, then $\alpha+\beta+\gamma$ is equal to ________.

माना एक 3 × 3 आव्यूह A इस प्रकार है कि $A+A^T=O$ है। यदि $A\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}3\\3\\2\end{bmatrix}, A^2\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}-3\\19\\-24\end{bmatrix}$ और

$\det(\operatorname{adj}(2\operatorname{adj}(A+I)))=(2)^{\alpha}\cdot(3)^{\beta}\cdot(11)^{\gamma}, \alpha,\beta,\gamma$ ऋणेत्तर पूर्णांक हैं, तो $\alpha+\beta+\gamma$ बराबर ________ है।

Ans. Official answer NTA (18)

Sol. $A=-A^T$

$$A=\begin{bmatrix}0 & a & b\\-a & 0 & c\\-b & -c & 0\end{bmatrix}$$

$$A\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}3\\3\\2\end{bmatrix}$$

$\Rightarrow a = -3$ and $-b + c = 2$

$$A^2\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}-3\\19\\-24\end{bmatrix}$$

$\Rightarrow -9 - b^2 + bc = -3$

$-9-b^2+b(2+b)=-3$

$b = 3$

$c = 5$

$$A=\begin{bmatrix}0 & -3 & 3\\3 & 0 & 5\\-3 & -5 & 0\end{bmatrix}$$

$$A+I=\begin{bmatrix}1 & -3 & 3\\3 & 1 & 5\\-3 & -5 & 1\end{bmatrix}$$

$|A + I| = 44$

Now, $|\operatorname{adj}(2\operatorname{adj}(A+I))|$

$=|2\operatorname{adj}(A+I)|^2$

$= 2^6|\operatorname{adj}(A+I|^2$

$=2^6|A+I|^4$

$=2^6(44)^4$

$=2^{14}\times 11^4$

$\alpha=14, \beta=0, \gamma=4$