JEE Main January 2026

Question Paper With Text Solution

21 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2026 | 21TH JANUARY SHIFT-1

SECTION – A

Question ID : 8606541138

1. The value of $\text{cosec } 10° - \sqrt{3}\sec 10°$ is equal to :

$\text{cosec } 10° - \sqrt{3}\sec 10°$ का मान बराबर है :

(1) 4 (2) 6 (3) 2 (4) 8

Ans. Official answer NTA (1)

Sol. $\text{cosec } 10° - \sqrt{3}\sec 10° = \frac{1}{\sin 10°} - \frac{\sqrt{3}}{\cos 10°} = \frac{1}{\sin 10^{\circ}} - \frac{\sin 60°}{\cos 60° \cos 10°}$

$= \frac{\cos 60° \cos 10° - \sin 60° \sin 10°}{\sin 10° \cos 10° \cos 60} = \frac{\cos(70°)}{\frac{1}{2}(2\sin 10° \cos 10°)\left(\frac{1}{2}\right)}$

$= \frac{4\cos 70°}{\sin 20°} = 4$

Question ID : 8606541128

2. The sum of all the roots of the equation $(x-1)^2 - 5|x-1| + 6 = 0$, is :

समीकरण $(x-1)^2 - 5|x-1| + 6 = 0$ के सभी मूलों का योग है :

(1) 5 (2) 4 (3) 3 (4) 1

Ans. Official answer NTA (2)

Sol. Let $|x-1| = t$

$t^2 - 5t + 6 = 0$

$(t-2)(t-3) = 0$

$t = 2$ or $t = 3$

$|x-1| = 2$ or $|x-1| = 3$

$x - 1 = 2$, $x - 1 = -2$ $x - 1 = 3$ $x - 1 = -3$

$x = 3$ $x = -1$ $x = 4$ $x = -2$

Sum of all roots = 4

Question ID : 8606541126

3. The number of relations, defined on the set {a, b, c, d}, which are both reflexive and symmetric, is :

समुच्चय {a, b, c, d} पर परिभाषित संबंधों, जो कि स्वतुल्य और सममित दोनों हैं, की संख्या है :

(1) 1024 (2) 64 (3) 256 (4) 16

Ans. Official answer NTA (2)

Sol. Number of relations which are both which are both reflexive and symmetric = $2^{\frac{n^2-n}{2}}$

(put n = 4)

$= 2^6 = 64$

Question ID : 8606541145

4. Let $f : R \to (0, \infty)$ be a twice differentiable function such that f(3) = 18, f '(3) = 0 and f "(3) = 4. Then

$$\lim_{x\to 1}\left(\log_e\left(\frac{f(2+x)}{f(3)}\right)^{\frac{18}{(x-1)^2}}\right)$$ is equal to :

माना दो बार अवकलनीय फलन $f : R \to (0, \infty)$ के लिए f(3) = 18, f '(3) = 0 और f "(3) = 4 हैं, तो

$$\lim_{x\to 1}\left(\log_e\left(\frac{f(2+x)}{f(3)}\right)^{\frac{18}{(x-1)^2}}\right)$$ बराबर है :

(1) 9 (2) 18 (3) 2 (4) 1

Ans. Official answer NTA (3)

Sol. $$L = \lim_{x\to 1}\frac{18}{(x-1)^2}\ell n\left(\frac{f(2+x)}{f(3)}\right) = \frac{18}{(x-1)^2}\left(\frac{f(2+x)}{f(3)}-1\right)$$

put x = 1 + h

$$L = \lim_{h\to 0}\frac{18\left(f(3+h)-f(3)\right)}{f(3)h^2} = \lim_{h\to 0}\frac{f(3+h)-f(3)}{h^2}$$

Use L'H

$$= \lim_{h\to 0}\frac{f'(3+h)}{2h}$$

Use L'H

$$L = \lim_{h \to 0} \frac{f''(3+h)}{2} = \frac{4}{2} = 2$$

Question ID : 8606541133

5. Let the mean and variance of 7 observations 2, 4, 10, x, 12, 14, y, x > y, be 8 and 16 respectively. Two numbers are chosen from $\{1, 2, 3, x-4, y, 5\}$ one after another without replacement, then the probability, that the smaller number among the two chosen numbers is less than 4 , is :

माना 7 प्रेक्षणों 2, 4, 10, x, 12, 14, y, x > y, के मध्य और प्रसरण क्रमशः 8 और 16 हैं। {1, 2, 3, x – 4, y, 5} में से बिना प्रतिस्थापना के पहले एक और फिर दूसरी संख्या चुनी जाती है, तो चुनी गई दो संख्याओं में से छोटी संख्या के 4 से कम होने की प्रायिकता है :

(1) $\frac{4}{5}$ (2) $\frac{3}{5}$ (3) $\frac{2}{5}$ (4) $\frac{1}{3}$

Ans. Official answer NTA (1)

Sol. Mean = 8 $\Rightarrow \frac{\sum x_i}{7} = 8$

$\Rightarrow 2 + 4 + 10 + x + 12 + 14 + y = 56$

$\Rightarrow x + y = 14$ ______(1)

Variance = 16 $\Rightarrow \frac{\sum x_i^2}{7} - (8)^2 = 16$

$\Rightarrow \frac{\sum x_i^2}{7} = 80 \Rightarrow 4 + 16 + 100 + x^2 + 144 + 196 + y^2 = 560$

$x^2 + y^2 = 100$ ______(2)

Solve (1) and (2)

$x = 8, y = 6$

$\{1, 2, 3, 4, 5, 6\}$

Probability $= \frac{5+4+3}{^6C_2} = \frac{12}{15} = \frac{4}{5}$

Question ID : 8606541131

6. The number of strictly increasing functions f from the set {1, 2, 3, 4, 5, 6} to the set {1, 2, 3, ..., 9} such that $f(i) \neq i$ for $1 \leq i \leq 6$, is equal to :

समुच्चय {1, 2, 3, 4, 5, 6} से समुच्चय {1, 2, 3, ..., 9} में निरंतर वर्धमान फलनों f, जब कि $1 \leq i \leq 6$ के लिए $f(i) \neq i$ है,

की संख्या बराबर है :

(1) 22 (2) 28 (3) 27 (4) 21

Ans. Official answer NTA (2)

Sol. $f(1) \neq 1$ and function should be one-one and increasing.

select 6 elements from co-domain (excluding 1)

Ans. = ${}^8C_6 = 28$

Question ID : 8606541143

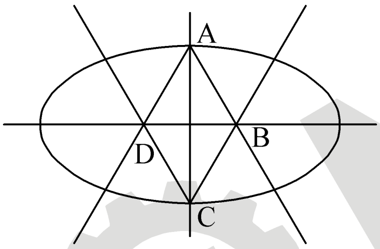
7. The area of the region, inside the ellipse $x^2 + 4y^2 = 4$ and outside the region bounded by the curves $y = |x| - 1$ and $y = 1 - |x|$, is :

दीर्घवृत्त $x^2 + 4y^2 = 4$ के अंदर और वृत्तों $y = |x| - 1$ तथा $y = 1 - |x|$ से परिबद्ध क्षेत्र के बाहर के क्षेत्र का क्षेत्रफल है :

(1) $3(\pi - 1)$ (2) $2\pi - \frac{1}{2}$ (3) $2(\pi - 1)$ (4) $2\pi - 1$

Ans. Official answer NTA (3)

Sol.



Required Area = Area of ellipse – area of rhombus ABCD

$= \pi(2)(1) - \frac{1}{2}(2)(2)$

$= 2\pi - 2$

Question ID : 8606541144

8. Let $y = y(x)$ be the solution curve of the differential equation $(1 + x^2)dy + (y - \tan^{-1} x)dx = 0$, $y(0) = 1$. Then the value of $y(1)$ is :

माना अवकल समकरण $(1 + x^2)dy + (y - \tan^{-1} x)dx = 0$, $y(0) = 1$ का हल वक्र $y = y(x)$ है, तो $y(1)$ का मान है :

(1) $\frac{2}{e^{\pi/4}} + \frac{\pi}{4} - 1$ (2) $\frac{2}{e^{\pi/4}} - \frac{\pi}{4} - 1$ (3) $\frac{4}{e^{\pi/4}} - \frac{\pi}{2} - 1$ (4) $\frac{4}{e^{\pi/4}} + \frac{\pi}{2} - 1$

Ans. Official answer NTA (1)

Sol. $(1 + x^2)dy + (y - \tan^{-1} x)dx = 0 \Rightarrow \frac{dy}{dx} + \frac{y}{1+x^2} = \frac{\tan^{-1}(x)}{1+x^2}$

$$\text{I.F.} = e^{\int \frac{dx}{1+x^2}} = e^{\tan^{-1}(x)}$$

$$\int d\left(ye^{\tan^{-1}(x)}\right) = \int \frac{\tan^{-1}(x)}{1+x^2} e^{\tan^{-1}(x)} dx$$

Put $\tan^{-1} x = t$

$$\Rightarrow ye^{\tan^{-1}(x)} = \int te^t dt$$

$$= e^t(t - 1) + c$$

$$ye^{\tan^{-1}(x)} = e^{\tan^{-1}(x)}\left(\tan^{-1}(x) - 1\right) + c$$

$\because\ y(0) = 1 \qquad \Rightarrow c = 2$

$$ye^{\tan^{-1}(x)} = e^{\tan^{-1}(x)}\left(\tan^{-1}(x) - 1\right) + 2$$

$$y = \tan^{-1}(x) - 1 + 2e^{-\tan^{-1}(x)}$$

$$y(1) = \frac{\pi}{4} - 1 + 2e^{-\pi/4}$$

Question ID : 8606541142

9. The value of $\int_{-\pi/6}^{\pi/6} \left(\frac{\pi + 4x^{11}}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} \right) dx$ is equal to :

$\int_{-\pi/6}^{\pi/6} \left(\frac{\pi + 4x^{11}}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} \right) dx$ का मान बराबर है :

(1) 4π (2) 2π (3) 8π (4) 6π

Ans. Official answer NTA (1)

Sol. $I = \int_{-\pi/6}^{\pi/6} \frac{\pi + 4x^{11}}{1 - \sin\left(|x| + \frac{\pi}{6}\right)} dx$

$$I = \int_{-\pi/6}^{\pi/6} \frac{\pi}{1-\sin\left(|x|+\frac{\pi}{6}\right)}dx + \int_{-\pi/6}^{\pi/6} \frac{4x^{11}}{1-\sin\left(|x|+\frac{\pi}{6}\right)}dx$$

Even Odd

$$= 2\int_{0}^{\pi/6} \frac{\pi}{1-\sin(x+\pi/6)}dx + 0$$

put $x + \frac{\pi}{6} = t$

$$= 2\int_{\pi/6}^{\pi/3} \frac{\pi}{1-\sin t}dt$$

$$= 2\pi\int_{\pi/6}^{\pi/3} \frac{1+\sin t}{1-\sin^2 t}dt$$

$$= 2\pi\int_{\pi/6}^{\pi/3} \left(\sec^2 t + \sec t \tan t\right)dt$$

$$= 2\pi[\tan t + \sec t]_{\pi/6}^{\pi/3}$$

$$= 4\pi$$

Question ID : 8606541140

10. Let $\vec{c}$ and $\vec{d}$ be vectors such that $|\vec{c}+\vec{d}| = \sqrt{29}$ and $\vec{c}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{d}$. If $\lambda_1, \lambda_2 (\lambda_1 > \lambda_2)$ are the possible values of $(\vec{c}+\vec{d})\cdot(-7\hat{i}+2\hat{j}+3\hat{k})$, then the equation

$K^2x^2 + (K^2 - 5K + \lambda_1)xy + \left(3K + \frac{\lambda_2}{2}\right)y^2 - 8x + 12y + \lambda_2 = 0$ represents a circle, for K equal to :

माना सदिशों $\vec{c}$ और $\vec{d}$ के लिए $|\vec{c}+\vec{d}| = \sqrt{29}$ और $\vec{c}\times(2\hat{i}+3\hat{j}+4\hat{k}) = (2\hat{i}+3\hat{j}+4\hat{k})\times\vec{d}$ हैं। यदि $(\vec{c}+\vec{d})\cdot(-7\hat{i}+2\hat{j}+3\hat{k})$ के संभव मान $\lambda_1, \lambda_2 (\lambda_1 > \lambda_2)$ हैं, तो K के किस मान के लिए समीकरण

$K^2x^2 + (K^2 - 5K + \lambda_1)xy + \left(3K + \frac{\lambda_2}{2}\right)y^2 - 8x + 12y + \lambda_2 = 0$ एक वृत्त को निरूपित करता है :

(1) 1 (2) –1 (3) 2 (4) 4

Ans. Official answer NTA (1)

Sol. Let $2\hat{i}+3\hat{j}+4\hat{k}=\vec{a}$

$\vec{c}\times\vec{a}=\vec{a}\times\vec{d}$

$(\vec{c}+\vec{d})\times\vec{a}=0$

$\vec{c}+\vec{d}=\alpha\vec{a}$

$|\vec{c}+\vec{d}|=\sqrt{29}$ $\quad\Rightarrow \alpha=\pm 1$

$(\vec{c}+\vec{d})\cdot(-7\hat{i}+2\hat{j}+3\hat{k})=\pm(4)$

$\Rightarrow \lambda_1=4$ $\quad\Rightarrow \lambda_2=-4$

If Equation $K^2x^2+(K^2-5K+\lambda_1)xy+\left(3K+\frac{\lambda_2}{2}\right)y^2-8x+12y+\lambda_2=0$ represents a circle

coeffceint of x^2 = coefficient y^2 and coefficient $xy=0$

$K^2=3K-2$ $\quad$ $K^2-5K+4=0$

$K=1$ or 2 $\quad$ $K=1$ or 4

$\Rightarrow K=1$

Question ID : 8606541129

11. If $x^2+x+1=0$, then the value of $\left(x+\frac{1}{x}\right)^4+\left(x^2+\frac{1}{x^2}\right)^4+\left(x^3+\frac{1}{x^3}\right)^4+\ldots+\left(x^{25}+\frac{1}{x^{25}}\right)^4$ is :

यदि $x^2+x+1=0$ है, तो $\left(x+\frac{1}{x}\right)^4+\left(x^2+\frac{1}{x^2}\right)^4+\left(x^3+\frac{1}{x^3}\right)^4+\ldots+\left(x^{25}+\frac{1}{x^{25}}\right)^4$ का मान है :

(1) 175 (2) 128 (3) 145 (4) 162

Ans. Official answer NTA (3)

Sol. $x=\omega$ or ω^2

$$x^2+\frac{1}{x^n}\begin{bmatrix}-1 & x\neq 3k\\ 2 & x=3k\end{bmatrix}$$

$$\sum_{x=1}^{25}\left(x^n+\frac{1}{x^n}\right)^4=17(-1)^4+8(2)^4=17+128=145$$

Question ID : 8606541132

12. If the coefficient of x in the expansion of $(ax^2+bx+c)(1-2x)^{26}$ is -56 and the coefficients of x^2 and x^3 are both zero, then $a+b+c$ is equal to :

यदि $(ax^2 + bx + c)(1 - 2x)^{26}$ के प्रसार में x का गुणांक –56 और x^2 तथा x^3 दोनों के गुणांक शून्य हैं, तो a + b + c बराबर है:

(1) 1483 (2) 1300 (3) 1500 (4) 1403

Ans. Official answer NTA (4)

Sol. $(ax^2 + bx + c)(1 - 2x)^{26}$

(i) coefficient of x = –56

$b - 52c = -56$ ______(1)

(ii) coefficient of $x^2 = 0$

$a - 52b + 1300c = 0$ ______(2)

(iii) coefficient of $x^3 = 0$

$-52a + 1300b - 20800c = 0$ ______(3)

Solve (1), (2) and (3)

$a = 1300,\ b = 100,\ c = 3$

$\Rightarrow a + b + c = 1403$

Question ID : 8606541134

13. Let the foci of a hyperbola coincide with the foci of the ellipse $\frac{x^2}{36} + \frac{y^2}{16} = 1$. If the eccentricity of the hyperbola is 5, then the length of its latus rectum is :

यदि एक अतिपरवलय की नाभियाँ, दीर्घवृत्त $\frac{x^2}{36} + \frac{y^2}{16} = 1$ की नाभियों के सम्पाती हैं। यदि अतिपरवलय की उत्केन्द्रता 5 है, तो इसकी नाभिलंब जीवा की लंबाई है :

(1) 16 (2) $24\sqrt{5}$ (3) 12 (4) $\frac{96}{\sqrt{5}}$

Ans. Official answer NTA (4)

Sol. $E : \frac{x^2}{36} + \frac{y^2}{16} = 1$

$H : \frac{x^2}{\alpha} - \frac{y^2}{20 - \alpha} = 1$ (Foci are same for ellipse and hyperbola)

$e = 5$

$$\sqrt{1+\frac{20-\alpha}{\alpha}}=5$$

$$\frac{20-\alpha}{\alpha}=24$$

$$\Rightarrow \alpha=\frac{4}{5}$$

$$l_{LR}=\frac{2(20-\alpha)}{\sqrt{\alpha}}=\frac{2\left(20-\frac{4}{5}\right)}{2/\sqrt{5}}$$

$$=\frac{96}{\sqrt{5}}$$

Question ID : 8606541127

14. If the domain of the function $f(x)=\cos^{-1}\left(\frac{2x-5}{11-3x}\right)+\sin^{-1}\left(2x^2-3x+1\right)$ is the interval $[\alpha, \beta]$, then $\alpha+2\beta$ is equal to :

यदि फलन $f(x)=\cos^{-1}\left(\frac{2x-5}{11-3x}\right)+\sin^{-1}\left(2x^2-3x+1\right)$ का प्रांत अंतराल $[\alpha, \beta]$ है, तो $\alpha + 2\beta$ बराबर है :

(1) 2 (2) 1 (3) 5 (4) 3

Ans. Official answer NTA (4)

Sol. $-1\le\frac{2x-5}{11-3x}\le 1$

$x\in\left(-\infty,\frac{16}{5}\right]\cup[6\infty)$

$-1 \le 2x^2 - 3x + 1 \le 1$

$x \in [0, \frac{3}{2}]$

$$\Rightarrow x\in\left[0,\frac{3}{2}\right]$$

$$\Rightarrow \alpha = 0,\ \beta = \frac{3}{2}$$

Question ID : 8606541139

15. Let (α, β, γ) be the co-ordinates of the foot of the perpendicular drawn from the point (5, 4, 2) on the line

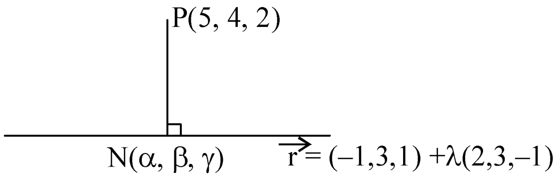
$\vec{r}=\left(-\hat{i}+3\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-\hat{k}\right)$. Then the length of the projection of the vector $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ on the vector $6\hat{i}+2\hat{j}+3\hat{k}$ is :

माना बिन्दु (5, 4, 2) से रेखा $\vec{r}=\left(-\hat{i}+3\hat{j}+\hat{k}\right)+\lambda\left(2\hat{i}+3\hat{j}-\hat{k}\right)$ पर डाले गए लंब के पाद के निर्देशांक (α, β, γ) हैं, तो सदिश $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ के सदिश $6\hat{i}+2\hat{j}+3\hat{k}$ पर प्रक्षेप की लंबाई है :

(1) $\frac{15}{7}$ (2) 4 (3) 3 (4) $\frac{18}{7}$

Ans. Official answer NTA (4)

Sol.



Let $N(-1+2\lambda, 3+3\lambda, 1-\lambda)$

$\overline{PN}\cdot\left(2\hat{i}+3\hat{j}-\hat{k}\right)=0 \quad \Rightarrow \lambda = 1$

$\Rightarrow N(1, 6, 0) \Rightarrow \alpha = 1, \beta = 6, \gamma = 0$

$$\text{projection} = \frac{\left(\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}\right)\cdot\left(6\hat{i}+2\hat{j}+3\hat{k}\right)}{\sqrt{36+4+9}}$$

$$=\frac{6+12+0}{7}=\frac{18}{7}$$

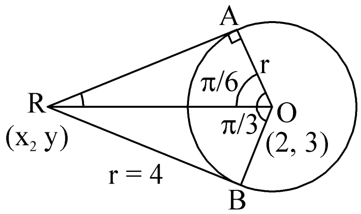
Question ID : 8606541136

16. Let PQ and MN be two straight lines touching the circle $x^2 + y^2 - 4x - 6y - 3 = 0$ at the points A and B respectively. Let O be the centre of the circle and $\angle AOB = \frac{\pi}{3}$. Then the locus of the point of intersection of the lines PQ and MN is :

माना दो सरल रेखाएँ PQ और MN, वृत्त $x^2 + y^2 - 4x - 6y - 3 = 0$ को क्रमशः बिन्दुओं A और B पर स्पर्श करती हैं। माना वृत्त का केन्द्र O है और $\angle AOB = \frac{\pi}{3}$ है, तो रेखाओं PQ और MN के प्रतिच्छेदन बिन्दु का बिन्दुपथ है :

(1) $x^2 + y^2 - 18x - 12y - 25 = 0$
(2) $3(x^2 + y^2) - 12x - 18y - 25 = 0$
(3) $3(x^2 + y^2) - 18x - 12y + 25 = 0$
(4) $x^2 + y^2 - 12x - 18y - 25 = 0$

Ans. Official answer NTA (2)

Sol. 

(R is point of intersection of tangents)

$$\cos\frac{\pi}{6} = \frac{r}{OR}$$

$$OR = \frac{2r}{\sqrt{3}}$$

$3(OR)^2 = 4r^2$

$3(x-2)^2 + 3(y-3)^2 = 64$

$3x^2 + 3y^2 - 12x - 18y - 25 = 0$

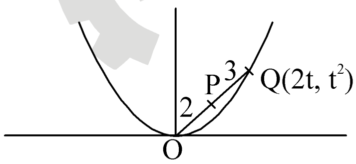
Question ID : 8606541137

17. Let O be the vertex of the parabola $x^2 = 4y$ and Q be any point on it. Let the locus of the point P , which divides the line segment OQ internally in the ratio 2 : 3 be the conic C . Then the equation of the chord of C, which is bisected at the point (1, 2), is :

माना परवलय $x^2 = 4y$ का शीर्ष O है और इस पर कोई बिन्दु Q है। माना बिन्दु P, जो रेखाखण्ड OQ को अंत: 2 : 3 के अनुपात में विभाजित करता है, का बिन्दुपथ शांकव C है। तो C की जीवा, जो बिन्दु (1, 2) पर समद्विभाजित होती है, का समीकरण है :

(1) $x - 2y + 3 = 0$ (2) $4x - 5y + 6 = 0$ (3) $5x - y - 3 = 0$ (4) $5x - 4y + 3 = 0$

Ans. Official answer NTA (4)

Sol. 

$$P\left(\frac{4t}{5}, \frac{2t^2}{5}\right)$$

$$h = \frac{4t}{5} \ \& \ k = \frac{2t^2}{5}$$

Eliminate t

$$\left(\frac{5h}{4}\right)^2 = \frac{5k}{2}$$

$5h^2 = 8k$

$5x^2 = 8y$

Equation of chord which is bisected at the point (1, 2)
$T = S_1$
$5x(1) + 4(y + 2) = 5 - 16$
$5x - 4y + 3 = 0$

Question ID : 8606541130

18. Let $a_1, a_2, a_3, ...$ be a G.P. of increasing positive terms such that $a_2.a_3.a_4 = 64$ and $a_1 + a_3 + a_5 = \frac{813}{7}$. Then $a_3 + a_5 + a_7$ is equal to :

माना वर्धमान धनात्मक पदों की एक G.P. $a_1, a_2, a_3, ...$ के लिए $a_2.a_3.a_4 = 64$ और $a_1 + a_3 + a_5 = \frac{813}{7}$ है। तो $a_3 + a_5 + a_7$ बराबर है :

(1) 3248 (2) 3252 (3) 3256 (4) 3244

Ans. Official answer NTA (2)

Sol. $a_2a_3a_4 = 64 \quad \Rightarrow a_3 = 4$

$= a_1 + a_3 + a_5 = \frac{813}{7}$

$\frac{4}{r^2} + 4 + 4r^2 = \frac{813}{7}$

$\Rightarrow r^2 = 28$

$a_3 + a_5 + a_7 = 4 + 4r^2 + 4r^4 = 4(1 + 28 + 784) = 3252$

Question ID : 8606541141

19. Let $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b} = 8\hat{i} + 7\hat{j} - 3\hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} \times \vec{c} = \vec{b}$. If $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$, then $|\vec{a} + \vec{c}|^2$ is equal to :

माना $\vec{a} = -\hat{i} + 2\hat{j} + 2\hat{k}$, $\vec{b} = 8\hat{i} + 7\hat{j} - 3\hat{k}$ और एक सदिश $\vec{c}$ के लिए $\vec{a} \times \vec{c} = \vec{b}$ हैं। यदि $\vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = 4$ है, तो $|\vec{a} + \vec{c}|^2$ बराबर है :

(1) 30 (2) 27 (3) 35 (4) 33

Ans. Official answer NTA (2)

Sol. $\vec{a}\times\vec{c}=\vec{b}\Rightarrow \vec{a}\times(\vec{a}\times\vec{c})=\vec{a}\times\vec{b}$

$(\vec{a}\cdot\vec{c})\vec{a}-(\vec{a}\cdot\vec{a})\vec{c}=\vec{a}\times\vec{b}$ let $(\vec{a}.\vec{c})=\lambda$

$\frac{\lambda\vec{a}-\vec{a}\times\vec{b}}{9}=\vec{c}$

$\vec{c}\cdot(\hat{i}+\hat{j}+\hat{k})=4$

$\lambda\vec{a}\cdot(\hat{i}+\hat{j}+\hat{k})-(\vec{a}\times\vec{b})\cdot(\hat{i}+\hat{j}+\hat{k})=36$

$\lambda(3)+30=36\Rightarrow\lambda=2$

$\vec{c}=\frac{2\vec{a}-\vec{a}\times\vec{b}}{9}$

$\Rightarrow\vec{c}=2\hat{i}-\hat{j}+3\hat{k}$

$\Rightarrow|\vec{c}+\vec{a}|=|\hat{i}+\hat{j}+5\hat{k}|=\sqrt{27}$

Question ID : 8606541135

20. Let a point A lie between the parallel lines L_1 and L_2 such that its distances from L_1 and L_2 are 6 and 3 units, respectively. Then the area (in sq. units) of the equilateral triangle ABC, where the points B and C lie on the lines L_1 and L_2, respectively, is :

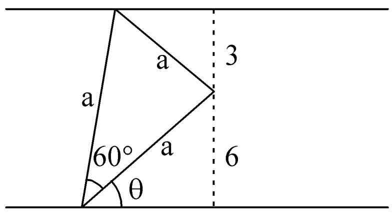
दो समान्तर रेखाओं L_1 और L_2 के मध्य एक बिन्दु A इस प्रकार है कि इसकी L_1 और L_2 से दूरियाँ क्रमशः 6 और 3 इकाई हैं। तो समबाहु त्रिभुज ABC, जहाँ बिन्दु B और C क्रमशः रेखाओं L_1 और L_2 पर हैं, का क्षेत्रफल (वर्ग इकाई में) है :

(1) $21\sqrt{3}$ (2) 27 (3) $12\sqrt{2}$ (4) $15\sqrt{6}$

Ans. Official answer NTA (1)

Sol. $\sin\theta=\frac{6}{a}$

$\sin(60°+\theta)=\frac{9}{a}$



$\sin 60°\cos\theta+\cos 60°\sin\theta=\frac{9}{a}$

$\frac{\sqrt{3}}{2}\sqrt{1-\frac{36}{a^2}}+\frac{1}{2}\left(\frac{6}{a}\right)=\frac{9}{a}$

$\Rightarrow a=\sqrt{84}$

$\text{Area} = \frac{\sqrt{3}}{4}a^2 = 21\sqrt{3}$

SECTION – B

Question ID : 8606541149

21. Let $f: R \to R$ be a twice differentiable function such that the quadratic equation $f(x)m^2 - 2f'(x)m + f''(x) = 0$ in m, has two equal roots for every $x \in R$. If $f(0) = 1, f'(0) = 2$, and (α, β) is the largest interval in which the function $f(\log_e x - x)$ is increasing, then $\alpha + \beta$ is equal to ________.

माना दो बार अवकलनीय एक फलन $f : R \to R$ इस प्रकार है कि प्रत्येक $x \in R$ के लिए m में द्विघात समीकरण $f(x)m^2 - 2f'(x)m + f''(x) = 0$ के दो समान मूल हैं। यदि $f(0) = 1$, $f'(0) = 2$ और सबसे बड़ा अंतराल, जिसमें फलन $f(\log_e x - x)$ वर्धमान है, (α, β) है, तो $\alpha + \beta$ बराबर है ________

Ans. Official answer NTA (1)

Sol. $D = 0 \Rightarrow (f'(x))^2 = f(x)f''(x)$

$$\frac{f'(x)}{f(x)} = \frac{f''(x)}{f'(x)}$$

$\ln c + \ln f(x) = \ln f'(x)$

$f'(x) = cf(x)$

$\because f(0) = 1, f'(0) = 2 \quad \Rightarrow c = 2$

$f'(x) = 2f(x)$

$$\frac{dy}{dx} = 2y$$

$$\int \frac{dy}{y} = \int 2dx$$

$\ln y = 2x + \ln c_1$

$f(x) = y = c_1 e^{2x}$

$\because f(0) = 1 \quad \Rightarrow c_1 = 1$

$\Rightarrow f(x) = e^{2x}$

Let $p(x) = f(\ln x - x)$

$p(x) = e^{2(\ln x - x)}$

$$p'(x) = 2e^{2(\ell n x - x)}\left(\frac{1}{x} - x\right)$$

$= 2e^{2(\ell n x - x)}\left(\frac{1-x^2}{x}\right)$

$\underset{0}{\vert} \; + \; \underset{1}{\vert} \; -$

$\Rightarrow$p is increasing in $x \in (0, 1)$
$\Rightarrow \alpha = 0, \beta = 1$

Question ID : 8606541147

22. Let $a_1 = 1$ and for $n \geq 1$, $a_{n+1} = \frac{1}{2}a_n + \frac{n^2-2n-1}{n^2(n+1)^2}$. Then $\left|\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right|$ is equal to ________.

माना $a_1 = 1$ है और $n \geq 1$ के लिए, $a_{n+1} = \frac{1}{2}a_n + \frac{n^2-2n-1}{n^2(n+1)^2}$ है। तो $\left|\sum_{n=1}^{\infty}\left(a_n - \frac{2}{n^2}\right)\right|$ बराबर है ________

Ans. Official answer NTA (2)

Sol. $a_{n+1} = \frac{a_n}{2} + \frac{n^2-2n-1}{n^2(n+1)^2}$

$\Rightarrow$ divide by $\left(\frac{1}{2}\right)^n$

$2^n a_{n+1} = 2^{n-1}a_n + 2^n\left(\frac{n^2-2n-1}{n^2(n+1)^2}\right)$

$= 2^{n-1}a_n + 2^n\left(\frac{2n^2-(n+1)^2}{n^2(n+1)^2}\right)$

$2^n a_{n+1} = 2^{n-1}a_n + \frac{2^{n+1}}{(n+1)^2} - \frac{2^n}{n^2}$

$2^n a_{n+1} - 2^{n-1}a_n = \frac{2^{n+1}}{(n+1)^2} - \frac{2^n}{n^2}$

This has been converted in telescopic sum

$2^{n-1}a_n - a_1 = \frac{2^n}{n^2} - 2$

$a_n = \frac{2}{n^2} - \frac{1}{2^{n-1}}$

$$a_n - \frac{2}{n^2} = -\frac{1}{2^{n-1}}$$

$$\left|\sum_{n=1}^{\infty} -\frac{1}{2^{n-1}}\right| = \frac{1}{1-\frac{1}{2}} = 2$$

Question ID : 8606541148

23. Let S = {(m, n) : m, n ∈ {1, 2, 3, ..., 50}}. If the number of elements (m, n) in S such that $6^m + 9^n$ is a multiple of 5 is p and the number of elements (m, n) in S such that m + n is a square of a prime number is q, then p + q is equal to _________.

माना S = {(m, n) : m, n ∈ {1, 2, 3, ..., 50}} है। यदि S में अवयवों (m, n), जिनके लिए $6^m + 9^n$, 5 का एक गुणज है, की संख्या p है और S में अवयवों (m, n) जिनके लिए m + n एक अभाज्य संख्या का वर्ग है, की संख्या q है, तो p + q बराबर है _________.

Ans. Official answer NTA (1333)

Sol. (i) $6^m + 9^n = (5+1)^m + (10-1)^n$

$= 5k_1 + 1 + 10k_2 + (-1)^n$

m ∈ N, n = odd

p = 50 × 25 = 1250

(ii) m + n = 4 ⇒ ${}^3C_1 = 3$

m + n = 9 ⇒ ${}^8C_1 = 8$

m + n = 25 ⇒ ${}^{24}C_1 = 24$

m + n = 49 ⇒ ${}^{48}C_1 = 48$

Sum q = 83

Ans. = 1250 + 83 = 1333

Question ID : 8606541150

24. $6\int_0^{\pi}\left|(\sin 3x + \sin 2x + \sin x)\right|dx$ is equal to _________.

$6\int_0^{\pi}\left|(\sin 3x + \sin 2x + \sin x)\right|dx$ बराबर है_________

Ans. Official answer NTA (17)

Sol. $= 6\int_0^{\pi} |2\sin 2x \cos x + \sin 2x| dx$

$= 6\int_0^{\pi} |4\sin x \cos^2 x + 2\sin x \cos x| dx$

$= 12\int_0^{\pi} \sin x |2\cos^2 x + \cos x| dx$

$= 12\int_0^{1} |2t^2 + t| dt = 17$

Question ID : 8606541146

25. For some $\alpha, \beta \in R$, let $A = \begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 1 \\ 1 & \beta \end{bmatrix}$ be such that $A^2 - 4A + 2I = B^2 - 3B + I = O$. Then $\left(\det\left(\operatorname{adj}\left(A^3 - B^3\right)\right)\right)^2$ is equal to ________.

किसी $\alpha, \beta \in R$ के लिए, माना $A = \begin{bmatrix} \alpha & 2 \\ 1 & 2 \end{bmatrix}$ और $B = \begin{bmatrix} 1 & 1 \\ 1 & \beta \end{bmatrix}$ इस प्रकार हैं कि $A^2 - 4A + 2I = B^2 - 3B + I = O$ है।

तो $\left(\det\left(\operatorname{adj}\left(A^3 - B^3\right)\right)\right)^2$ बराबर है________

Ans. Official answer NTA (225)

Sol. $A^2 - (2+\alpha)A + (2\alpha - 2)I = 0 \quad \Rightarrow 2 + \alpha = 4 \;\Rightarrow \alpha = 2$

$B^2 - (1+\beta)B + (\beta - 1)I = 0 \Rightarrow 1 + \beta = 3 \Rightarrow \beta = 2$

$A^2 - 4A + 2I = 0$

$A^2 = 4A - 2I$

$A^3 = 4A^2 - 2A$

$= 4(4A - 2I) - 2A$

$A^3 = 14A - 8I$

$B^2 = 3B - I$

$B^3 = 3B^2 - B$

$= 3(3B - I) - B$

$B^3 = 8B - 3I$

$A^3 - B^3 = 14A - 8B - 5I$

$= \begin{bmatrix} 28 & 28 \\ 14 & 28 \end{bmatrix} - \begin{bmatrix} B & 8 \\ B & 16 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 15 & 20 \\ 6 & 7 \end{bmatrix}$

$\left|\operatorname{adj}\left(A^3 - B^3\right)\right|^2 = \left(\left|A^3 - B^3\right|\right)^2 = \left|A^3 - B^3\right|^2 \quad = (15)^2 = 225$