JEE (ADVANCED) 2019 PAPER I

MATHS

SECTION-1 (Maximum Marks : 12)

- * This section contains FOUR (04) questions.
- * Each question has FOUR options ONLY ONE of these four options is the correct answer.
- * For each question, choose the correct option corresponding to the correct answer.
- * Answer to each question will be evaluated according to the following marking scheme :

  Full Marks : +3 If ONLY the correct option is chosen.

  Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

  Negative Marks : –1 In all other cases.

1 A line $y = mx + 1$ intersects the circle $(x-3)^2 + (y+2)^2 = 25$ at the points P and Q. If the midpoint of the line segment PQ has x–coordinate $-\frac{3}{5}$ then which one of the following options is correct ?

एक रेखा $y = mx + 1$ वृत्त $(x-3)^2 + (y+2)^2 = 25$ को बिन्दुओं P और Q पर प्रतिच्छेद करती है अगर रेखाखण्ड (line segment) PQ के मध्य बिन्दु का x–निर्देशांक (coordinate) $-\frac{3}{5}$ है तब निम्नलिखित में से कौन सा एक विकल्प सही है।

(1) $6 \le m < 8$ (2) $4 \le m < 6$ (3) $2 \le m < 4$ (4) $-3 \le m < -1$

Question ID-337911147

Ans. 3

S. Circle $(x-3)^2 + (y+2)^2 = 25$

$y = mx + 1$

Solve

$(x-3)^2 + (mx + 1 + 2)^2 = 25$

$(1 + m^2)x^2 + x(6m - 6) - 7 = 0 \left\langle \begin{matrix} x_1 \\ x_2 \end{matrix} \right.$

$\frac{x_1 + x_2}{2} = \frac{-3}{5}$

$-\frac{(6m-6)}{2(m^2+1)} = \frac{-3}{5}$

$5(m-1) = m^2 + 1$

$m^2 - 5m + 6 = 0$

$(m-2)(m-3) = 0$

$m = 2,3$

2 The area of the region $\{(x,y): xy \le 8, 1 \le y \le x^2\}$ is

क्षेत्र $\{(x,y): xy \le 8, 1 \le y \le x^2\}$ का क्षेत्रफल है

(1) $16\log_e 2 - 6$ (2) $8\log_e 2 - \frac{14}{3}$ (3) $16\log_e 2 - \frac{14}{3}$ (4) $8\log_e 2 - \frac{7}{3}$

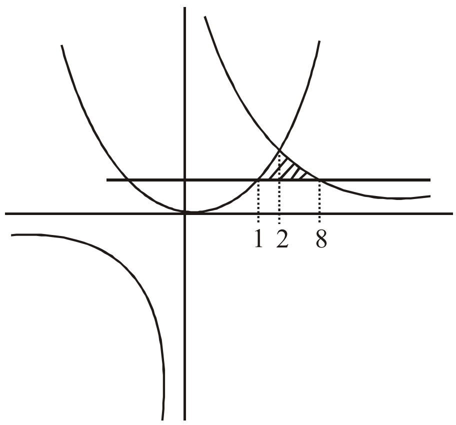
Question ID-337911148

Ans. 3

S0. $x^3 = 8$

$x = 2$

$\text{Area} = \int_1^2 (x^2 - 1)dx + \int_2^8 \left(\frac{8}{x} - 1\right)dx$



$= \left(\frac{x^3}{3} - x\right)_1^2 + (8\ln x - x)_2^8$

$= \left(\frac{8}{3} - 2 - \frac{1}{3} + 1\right) + [8\ln 8 - 8\ln 2 - 8 + 2]$

$= \left(\frac{7}{3} - 1\right) + 8\ln 4 - 6$

$= 16\ln 2 + \frac{7}{3} - 7$

$= 16\ln 2 - \frac{14}{3}$

3 Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

where $\alpha = \alpha(\theta)$ and $\beta = \beta(\theta)$ are real numbers and I is the 2×2 identity matrix. If α^* is the minimum of the set $\{\alpha(\theta): \theta \in [0,2\pi)\}$ and β^* is the minimum of the set $\{\beta(\theta): \theta \in [0,2\pi)\}$ then the value of $\alpha^* + \beta^*$

माना कि $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

जहाँ $\alpha = \alpha(\theta)$ और $\beta = \beta(\theta)$ वास्तविक संख्याएँ है और I एक 2×2 तत्समक आव्यूह है, यदि समुच्चय $\{\alpha(\theta): \theta \in [0,2\pi)\}$ का निम्नतम α^* है और समुच्चय $\{\beta(\theta): \theta \in [0,2\pi)\}$ का निम्नतम β^* है।

तो $\alpha^* + \beta^*$ का मान है

(1) $-\frac{31}{16}$ (2) $-\frac{29}{16}$ (3) $-\frac{37}{16}$ (4) $-\frac{17}{16}$

Question ID-337911146

Ans. 2

S. $M = \alpha I + \beta M^{-1}$

$M^2 = \alpha M + \beta I$

$M^2 - \alpha M - \beta I = 0$

$\alpha = tr(M) = \sin^4\theta + \cos^4\theta$

$\alpha = 1 - \frac{1}{2}\sin^2 2\theta$

$(\alpha)_{min} = 1 - \frac{1}{2} = \frac{1}{2}$ when $\sin^2 2\theta = 1$

$-\beta = |M|$

$\beta = -|M|$

$= -\left(\sin^4\theta\cos^4\theta + \left(1+\sin^2\theta\right)\left(1+\cos^2\theta\right)\right) = -\left(\sin^4\theta\cos^4\theta + \sin^2\theta\cos^2\theta + 2\right)$

$= -\left(\left(\sin^2\theta\cos^2\theta + \frac{1}{2}\right)^2 + \frac{7}{4}\right) = -\left(\left(\frac{\sin^2 2\theta}{4} + \frac{1}{2}\right)^2 + \frac{7}{4}\right)$

$(\beta)_{min} = \frac{-37}{16}$ when $\sin^2 2\theta = 1$

$(\alpha)_{min} + (\beta)_{min} = \frac{-29}{16}$

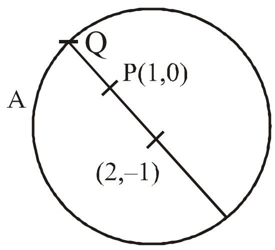
4 Let S be the set of all complex numbers z satisfying $|z-2+i| \geq \sqrt{5}$ if the complex number z_0 is such that $\frac{1}{|z_0-1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$ then the principal argument of $\frac{4 - z_0 - \overline{z}_0}{z_0 - \overline{z}_0 + 2i}$ is

माना कि S उन सभी सम्मिश्र संख्याओं z का समूच्चय है जो $|z-2+i| \geq \sqrt{5}$ को संतुष्ट करती है। यदि एक सम्मिश्र संख्या z_0 ऐसी है जिससे $\frac{1}{|z_0-1|}$ समुच्चय $\left\{\frac{1}{|z-1|} : z \in S\right\}$ का उच्चतम है, तब $\frac{4 - z_0 - \overline{z}_0}{z_0 - \overline{z}_0 + 2i}$ का मुख्य कोणांक है

(1) $\frac{\pi}{4}$ (2) $-\frac{\pi}{2}$ (3) $\frac{\pi}{2}$ (4) $\frac{3\pi}{4}$

Question ID-337911145

Ans. 2

S. 

Set S will be set of all points being outside (on) circle A whose center is (2,–1) & radius is $\sqrt{5}$.

$|z-1|$ = distance of z from P(1,0)

Q is the required point when $\dfrac{1}{|z-1|}$ is maximum.

$Q(x_0, y_0) \Rightarrow z_0 = x_0 + y_0$

$$\frac{4-z_0-\overline{z}_0}{z_0-\overline{z}_0+z_i} = \frac{4-2x}{2iy_0+2i} = -\frac{(2-x)i}{(y+1)}$$

$$\therefore \frac{2-x}{y+1} > 0$$

$\Rightarrow$ principal argument $= -\dfrac{\pi}{2}$

SECTION-II (Maximum Marks : 32)

* This section contains EIGHT (08) questions.
* Each question has FOUR options. ONE OR MORE THAN ONE of these four option(s) is (are) correct answer(s).
* For each question, choose the option(s) corresponding to (all) the correct answer(s).
* Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : –1 In all other cases.

1 In a non right angled triangle ΔPQR let p,q,r denote the length of the sides opposite to the angles at P,Q,R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O, If $p = \sqrt{3}$, $q = 1$ and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct

एक असमकोणीय त्रिभुज (non right angled triangle) ΔPQR के लिए माना कि p,q,r क्रमशः कोण P,Q,R के सामने वाली भुजाओं की लम्बाईयॉ दर्शाती है। R से खीचीं गयी माध्यिका (median) भुजा PQ से S पर मिलती है P से खीचा गया अभिलम्ब (perpendicular) भुजा QR से E पर मिलता है तथा RS और PE एक दुसरे को O पर काटती है यदि $p = \sqrt{3}$, $q = 1$ और ΔPQR के परिवृत्त (circumcircle) की त्रिज्या 1 है, तब निम्न में से कौनसा(से) विकल्प सही है(हैं)

(1) Length of OE = $\frac{1}{6}$ / OE की लम्बाई = $\frac{1}{6}$

(2) Length of RS = $\frac{\sqrt{7}}{2}$ / RS की लम्बाई = $\frac{\sqrt{7}}{2}$

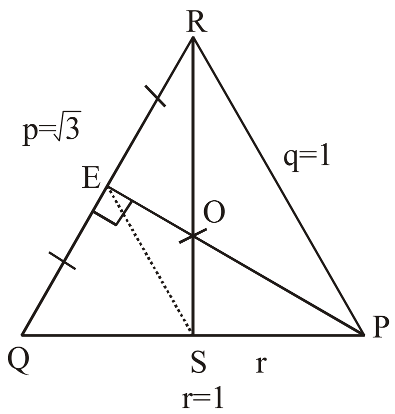
(3) Radius of incircle of ΔPQR = $\frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$/ΔPQR के अंतवृर्त की त्रिज्या = $\frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

(4) Area of ΔSOE = $\frac{\sqrt{3}}{12}$ / ΔSOE का क्षेत्रफल = $\frac{\sqrt{3}}{12}$

Question ID = 337911152

Ans. 1,2,3

S.



$$\frac{p}{\sin P} = \frac{q}{\sin Q} = \frac{r}{\sin R} = 2R$$

$$\frac{\sqrt{3}}{2} = \sin P, \frac{1}{2} = \sin Q$$

$$P = \frac{\pi}{3}, \frac{2\pi}{3}; \qquad Q = \frac{\pi}{6}, \frac{5\pi}{6}$$

$\therefore p > q$ then $P > Q$

if $P = \frac{\pi}{3}$, $Q = \frac{\pi}{6}$ then $R = \frac{\pi}{2}$ (not possible)

$$P = \frac{2\pi}{3}, Q = \frac{\pi}{6} \qquad R = \frac{\pi}{6}$$

$\Delta = \frac{1}{2}1 \times 1 \times \sin\frac{2\pi}{3} = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$

$s = \frac{1+1+\sqrt{3}}{2} = \frac{2+\sqrt{3}}{2}$

$r = \frac{\sqrt{3}}{4} \times \frac{2}{\left(2+\sqrt{3}\right)} = \frac{\sqrt{3}}{2} \times \frac{1}{\left(2+\sqrt{3}\right)}$

$r = \frac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

$RS = \frac{1}{2}\sqrt{2(3)+2(1)-1} = \frac{\sqrt{7}}{2}$

$OE = \frac{1}{3}PE \Rightarrow OE = \frac{1}{3}.\frac{1}{2}\sqrt{2(1)+2(1)-3} = \frac{1}{3}.\frac{1}{2} = \frac{1}{6}$

Area of $\Delta SOE = \frac{1}{12}\left(\text{area of } \Delta PQR\right) = \frac{\sqrt{3}}{48}$

2 Define the collections $\{E_1, E_2 E_3.......\}$ of ellipses and $\{R_1, R_2 R_3.......\}$ of rectangles as follows

$E_1 = \frac{x^2}{9} + \frac{y^2}{4} = 1;$

R_1 : rectangles of largest area, with sides parallel to the axes, inscribed in E_1

E_n : ellipse $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ of largest area inscribed in R_{n-1}, $n > 1$;

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n, $n > 1$
Then which of the following options is/are correct

(1) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

(2) The length of latus rectum of E_9 is $\frac{1}{6}$

(3) The eccentricities of E_{18} and E_{19} are not equal

(4) $\sum_{n=1}^{N}\left(\text{area of } R_n\right) < 24$ for each positive integer N

दीर्घवृतो (ellipses) $\{E_1, E_2 E_3.......\}$ और आयतों (rectangles) $\{R_1, R_2 R_3.......\}$ के संग्रहो को निम्न प्रकार से परिभाषित करें

$E_1 = \frac{x^2}{9} + \frac{y^2}{4} = 1;$

R_1 : अधिकतम क्षेत्र (largest area) का आयत जिसकी भुजाएं अक्षों (axes) के समान्तर है और जो E_1 में अंतर्स्थित (inscribed) है

E_n : अधिकतम क्षेत्र वाला दीर्घवृत $\frac{x^2}{a_n^2} + \frac{y^2}{b_n^2} = 1$ जो R_{n-1}, $n>1$ में अंतर्स्थित है

R_n : अधिकतम क्षेत्र का आयत जिसकी भुजाएं अक्षों के समान्तर है और जो E_n, $n > 1$ में अंतर्स्थित है।

तब निम्न में से कौनसा (से) विकल्प सहीं है (हैं)

(1) E_9 में केंन्द्र से एक नाभि (focus) की दूरी $\frac{\sqrt{5}}{32}$ है।

(2) E_9 के नाभिलम्ब (latus rectum) की लम्बाई $\frac{1}{6}$ है।

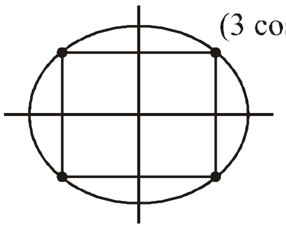
(3) E_{18} और E_{19} की उत्केंन्द्रतायें (eccentricities) समान नहीं है।

(4) प्रत्येक पूर्णांक N के लिए $\sum_{n=1}^{N}(R_n \text{ का क्षेत्रफल}) < 24$ है।

Question ID-337911153

Ans. 2,4

S. E_1 $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$



Area of rectangle = $(2a\cos\theta)(2b\sin\theta)$
$= 2ab\sin 2\theta$

area of rectangle will be maximum when $\theta = \frac{\pi}{4}$

$$E_2 = \frac{x^2}{(a\cos\theta)^2}+\frac{y^2}{(b\sin\theta)^2}=1$$

$$\Rightarrow a_n = \frac{a_{n-1}}{\sqrt{2}} \ \& \ b_n = \frac{b_{n-1}}{\sqrt{2}}$$

$\Rightarrow$ Forming G.P.

for E_9 : $a_9 = 3\left(\frac{1}{\sqrt{2}}\right)^8 = \frac{3}{16}$

$$b_9 = 2\left(\frac{1}{\sqrt{2}}\right)^8 = \frac{2}{16}$$

eccentricity of all ellipses will remain same as $\frac{b_n}{a_n}$ is same for all ellipse

(1) distance of focus from centre in $E_9 = a_9 e = \frac{\sqrt{5}}{3}\times\frac{3}{16}=\frac{\sqrt{5}}{16}$

(2) $l_{LR} = \frac{2b_9}{a_9} = \frac{2\left(\frac{1}{8}\right)^2}{3/16} = \frac{2\times 16}{3\times 64} = \frac{1}{6}$

(4) area of $R_n = \frac{1}{2}\left(\text{area of } R_{n-1}\right)$

area of $R_1 = 12 = A_1$

$$\sum_{n=1}^{N} A_n = A_1 + A_2 + ... + A_n < \lim_{n\to\infty}\sum_{r=1}^{N} A_n$$

$$< \left(12 + \frac{12}{2} + ...\infty\right)$$

$$< \frac{12}{1-\frac{1}{2}} = 24$$

$$\sum_{n=1}^{N}\left(\text{area of } R_n\right) < 24$$

3 Let L_1 and L_2 denote the lines $\vec{r} = \hat{i} + \lambda\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)\lambda \in \mathbb{R}$, $\vec{r} = \mu\left(2\hat{i} - \hat{j} + 2\hat{k}\right), \mu \in \mathbb{R}$ respectively. If L_3 is line which is perepndiclar to both L_1 and L_2 and cuts both of them then which of the following options describe (s) L_3

माना कि L_1 और L_2 क्रमशः निम्न रेखाएं है $\vec{r} = \hat{i} + \lambda\left(-\hat{i} + 2\hat{j} + 2\hat{k}\right)\lambda \in \mathbb{R}$ और $\vec{r} = \mu\left(2\hat{i} - \hat{j} + 2\hat{k}\right), \mu \in \mathbb{R}$ यदि L_3 एक रेखा है जो L_1 और L_2 दोनों के लम्बवत् है और दोनों को काटती है, तब निम्नलिखित विकल्पों में से कौन सा (से)L_3 को निरूपित करता (करतें) है (हैं) ?

(1) $\vec{r} = \frac{1}{3}\left(2\hat{i} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

(2) $\vec{r} = \frac{2}{9}\left(2\hat{i} - \hat{j} + 2\hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

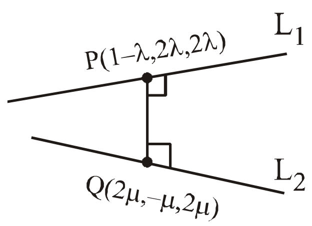
(3) $\vec{r} = \frac{2}{9}\left(4\hat{i} + \hat{j} + \hat{k}\right) + t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

(4) $\vec{r} = t\left(2\hat{i} + 2\hat{j} - \hat{k}\right), t \in \mathbb{R}$

Question ID : 337911156

Ans. 1,2,3

S.



$\overrightarrow{PQ} = (1-\lambda-2\mu,\ 2\lambda+\mu,\ 2\lambda-2\mu)$

$PQ \perp L_1$ and

$-(1-\lambda-2\mu) + 2(2\lambda+\mu) + 2(2\lambda-\mu) = 0$

$-1 + \lambda + 2\mu + 4\lambda + 2\mu + 4\lambda - 4\mu = 0$

$9\lambda = 1, \quad \lambda = \frac{1}{9}$

$PQ \perp L_2$

$2(1-\lambda-2\mu) - (2\lambda+\mu) + 2(2\lambda-2\mu) = 0$

$2 - 2\lambda - 4\mu - 2\lambda - \mu + 4\lambda - 4\mu = 0$

$\mu = \frac{2}{9}$

$P\left(\frac{8}{9},\frac{2}{9},\frac{2}{9}\right)$; $Q\left(\frac{4}{9},\frac{-2}{9},\frac{4}{9}\right)$

mid point of $PQ\left(\frac{2}{3},0,\frac{1}{3}\right)$

4 Let α and β be the roots of $x^2 - x - 1 = 0$ with $\alpha > \beta$. For all positive integers n, define

$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \ge 1$, $b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}, n \ge 2$ Then which of the following options is/are correct

माना कि $x^2 - x - 1 = 0$ के मूल α और β हैं जहा है। सभी धनात्मक पूर्णाकों n के लिए निम्न को परिभाषित किया गया है।

$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $n \ge 1$, $b_1 = 1$ and $b_n = a_{n-1} + a_{n+1}, n \ge 2$

तब निम्न मे से कौन सा (से) विकल्प सही है

(1) $b_n = \alpha^n + \beta^n$ for all $n \ge 1$/प्रत्येक $n \ge 1$ के लिए $b_n = \alpha^n + \beta^n$

(2) $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$ for all $n \ge 1$/ प्रत्येक $n \ge 1$ के लिए $a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$

(3) $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$

(4) $\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$

Question ID-337911149

Ans. **1,2,3**

S. $x^2 - x - 1 = 0$ roots are α and β $a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}$, $\alpha = \frac{1+\sqrt{5}}{2}$, $\beta = \frac{1-\sqrt{5}}{2}$

$\alpha^2 - \alpha - 1 = 0$ and $\beta^2 - \beta - 1 = 0$

Option (1) $b_n = a_{n-1} + a_{n+1}$

$$= \frac{\left(\alpha^{n-1} - \beta^{n-1}\right) + \left(\alpha^{n+1} - \beta^{n-1}\right)}{\alpha - \beta}$$

$$= \frac{\alpha^{n-1}\left(1+\alpha^2\right) - \beta^{n-1}\left(1+\beta^2\right)}{\alpha - \beta}$$

$$= \frac{\alpha^{n-1}\left(\alpha+2\right) - \beta^{n-1}\left(\beta+2\right)}{\alpha - \beta}$$

$$= \frac{\alpha^{n-1}\left(\frac{5+\sqrt{5}}{2}\right) - \beta^{n-1}\left(\frac{5-\sqrt{5}}{2}\right)}{\alpha - \beta}$$

$$= \frac{\sqrt{5}\left(\alpha^{n-1}\left(\frac{\sqrt{5}+1}{2}\right) - \beta^{n-1}\left(\frac{\sqrt{5}-1}{2}\right)\right)}{\alpha - \beta}$$

$$=\sqrt{5}\left(\frac{\alpha^{n-1}(\alpha)+\beta^{n-1}(\beta)}{\alpha-\beta}\right)$$

$$b_n=\frac{(\alpha^n+\beta^n)\sqrt{5}}{\sqrt{5}}=\alpha^n+\beta^n$$

Option (2) $a_n=\dfrac{\alpha^n-\beta^n}{\alpha-\beta}$

$$=\frac{\alpha^{n-2}(\alpha^2)-\beta^{n-2}(\beta^2)}{\alpha-\beta}$$

$$=\frac{\alpha^{n-2}(\alpha+1)-\beta^{n-2}(\beta+1)}{\alpha-\beta}$$

$$=\frac{\alpha^{n-1}-\beta^{n-1}}{\alpha-1}+\frac{\alpha^{n-2}-\beta^{n-2}}{\alpha-\beta}$$

$a_n=a_{n-1}+a_{n-2}$

$a_{n+2}=a_{n+1}+a_n$

$a_{n-1}=a_n+a_{n-1}$

$a_n=a_{n-1}+a_{n-2}$

|

|

|

$a_3=a_2+a_1$

$$a_2=\frac{\alpha^2-\beta^2}{\alpha-\beta}=\alpha+\beta=1$$

Add

$a_{n+2}=(a_n+a_{n-1}+a_{n-2}+.......+a_2+a_3)+a_2$

$a_{n+2}-a_2=a_n+a_{n-1}+.......+a_1$

$a_{n+2}-1=a_n+a_{n-1}+.......+a_1$

Option (3) $\sum\dfrac{\alpha^n-\beta^n}{(\alpha-\beta)10^n}=\dfrac{1}{\alpha-\beta}\left(\sum\dfrac{\alpha^n}{10^n}-\sum\dfrac{\beta^n}{10^n}\right)$

$$=\frac{1}{\alpha-\beta}\left(\frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}}-\frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}\right)$$

$$=\frac{1}{\alpha-\beta}\left(\frac{\alpha}{10-\alpha}-\frac{\beta}{10-\beta}\right)$$

$$=\frac{1}{\alpha-\beta}\left(\frac{(10-\beta)\alpha-\beta(10-\alpha)}{(10-\alpha)(10-\beta)}\right)$$

$$=\frac{(10\alpha-10\beta)}{(\alpha-\beta)(100-(\alpha+\beta)10+\alpha\beta)}$$

$$=\frac{10}{100-10+1}=\frac{10}{89}$$

Option (4) $\sum\left(\frac{\alpha^n+\beta^n}{10^n}\right)=\sum\frac{\alpha^n}{10^n}+\sum\frac{\beta^n}{10^n}$

$$=\frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}}+\frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}$$

$$=\frac{\alpha}{10-\alpha}+\frac{\beta}{10-\beta}$$

$$=\frac{\alpha(10-\beta)+\beta(10-\alpha)}{(10-\alpha)(10-\beta)}$$

$$=\frac{10(\alpha-\beta)-2\alpha\beta}{100-(\alpha+\beta)+\alpha\beta}$$

$$=\frac{10+2}{100-10-1}=\frac{12}{89}$$

5 There are three bags B_1, B_2 and B_3.The bag B_1 contains 5red and 5 green balls , B_2 contains 3red and 5 green balls and B_3 contains 5 red and 3 green balls.Bags B_1, B_2 and B_3 have probabilities $\frac{3}{10},\frac{3}{10}$ and $\frac{4}{10}$ respectively of being chosen. A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct

(1) Probability that the selected bag is B_3 and the chosen ball is green equals $\frac{3}{10}$

(2) Probability that the selected bag is B_3 given that the chosen ball is green equals $\frac{5}{13}$

(3) Probability that the chosen ball is green given that the selected bag is B_3 equals $\frac{3}{8}$

(4) Probability that the chosen ball is green equals $\frac{39}{80}$

तीन थैले B_1, B_2 और B_3 है। B_1 थैले में 5 लाल और 5 हरी गेंदे है B_2 में 3 लाल और 5 हरी गेंदें है और B_3 में 5 लाल और 3 हरी गेंदे है थैलें B_1, B_2 तथा B_3 के चुने जाने की प्रायिकताये क्रमशः $\frac{3}{10},\frac{3}{10}$ and $\frac{4}{10}$ है। एक थैला यादृिच्छक लिया जाता है और एक गेंद उस थैले में से यादृिच्छया चुनी जाती है तब निम्न में से कौन सा (से) विकल्प सही है

(1) चुने हुए थैले के B_3 होने के साथ साथ गेंद के हरे होने की प्रायिकता $\frac{3}{10}$ है।

(2) चुने हुए थैले के B_3 होने की प्रायिकता $\frac{5}{13}$ है जब यह ज्ञात है कि चुनी गयी गेंद हरी है।

(3) चुनीगयी गेंद के हरे होने की प्रायिकता 3/8 है, जब ज्ञात है कि चुना हुआ थैला B_3 है।

(4) चुनी गयी गेंद के हरे होने की प्रायिकता $\frac{39}{80}$ है।

Question ID-337911151

Ans. 3,4

S. $B_i \equiv$ Bag B_i is selected
$R \equiv$ Red ball is selected
$G \equiv$ Green ball is selected

(1) $P(B_3 \cap G) = P(B_3)P\left(\frac{G}{B_3}\right)$

$$= \frac{4}{10} \times \frac{3}{8} = \frac{3}{20}$$

(2) $P\left(\frac{B_3}{G}\right) = \frac{P(B_3 \cap G)}{P(G)}$

$$= \frac{\frac{4}{10} \times \frac{3}{8}}{\left(\frac{3}{10} \times \frac{5}{10}\right) + \left(\frac{3}{10} \times \frac{5}{8}\right) + \left(\frac{4}{10} \times \frac{3}{8}\right)} = \frac{12}{12+15+12} = \frac{12}{39} = \frac{4}{13}$$

(3) $P\left(\frac{G}{B_3}\right) = \frac{P(B_3 \cap G)}{P(B_3)} = \frac{\frac{3}{20}}{\frac{4}{10}} = \frac{3}{8}$

(4) $P(G) = \left(\frac{3}{10} \times \frac{5}{10}\right) + \left(\frac{3}{10} \times \frac{5}{8}\right) + \left(\frac{4}{10} \times \frac{3}{8}\right) = \frac{39}{80}$

6 Let f: R $\to$R be given by

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \ge 3 \end{cases}$$

Then which of the following options is/are correct

(1) f is increasing on $(-\infty, 0)$

(2) f' has a local maximum at x = 1

(3) f' is NOT differentiable at x = 1

(4) f is onto

माना कि $f: R \to R$ निम्न प्रकार से दिया है।

$$f(x) = \begin{cases} x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1, & x < 0; \\ x^2 - x + 1, & 0 \le x < 1; \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3}, & 1 \le x < 3; \\ (x-2)\log_e(x-2) - x + \frac{10}{3}, & x \ge 3 \end{cases}$$

तब निम्न में से कौन सा (से) विकल्प सही है।

(1) f अंतराल $(-\infty, 0)$ में वर्धमान है।

(2) f' का एक स्थानीय उच्चतम x = 1 पर है।

(3) $x = 1$ पर f' अवकलनीय नहीं है।

(4) f आच्छादक है।

SOL. $$f(x) = \begin{cases} (x+1)^5 - 2x & x < 0 \\ x^2 - x + 1 & 0 \le x < 1 \\ \frac{2}{3}x^3 - 4x^2 + 7x - \frac{8}{3} & 1 \le x < 3 \\ (x-2)\ln(x-2) - x + \frac{10}{3} & x \ge 3 \end{cases}$$

$f(x)$ is continuous at x = 0, x = 1, x = 3

$\Rightarrow$ continuous for all $x \in R$

$$f'(x) = \begin{cases} 5(x+1)^4 - 2 & x < 0 \\ 2x - 1 & 0 \le x < 1 \\ 2x^3 - 8x + 7 & 1 < x < 3 \\ \ln(x-2) & x > 3 \end{cases}$$

$f'(x)$ is continuous at x = 1,

Discountinuous at x = 0,3

Option (1) for $x < 0$ $f'(x)$ is changing its sign

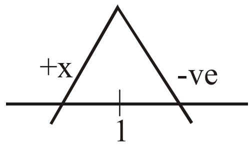
$\Rightarrow$ not increasing in $(0, \infty)$

Option (2) at x = 1

$$f''(x) = \begin{cases} 2 & 0 < x < 1 \\ 4x - 8 & 1 < x < 3 \end{cases}$$

$f''(1^+) = -ve, f''(1^-) = +ve$

f' has a local max^m at x = 1



Option (3) $f''(1^+) = -4 \neq f''(1^-) = 2$

f' is not differentiable at x = 1

Option (4) for $x \in R$

for $x < 0$, $(x+1)^5 - 2x \in (-\infty, 1)$

for $x \geq 3$, $(x-2)\ln(x-2) - x + \frac{10}{3} \in \left[\frac{1}{3}, \infty\right)$

Range of $f(x) = (-\infty, \infty)$

Question ID-337911154

Ans. 2,3,4

S.

7 Let Γ denote a curve $y = y(x)$ which is in the first quadrant and let the point (1,0) on it.Let the tangent to Γ at a point P intersect the y-axis at Y_p. If PY_P has length 1 for each point P on Γ, then which of the following options is/are correct

माना कि Γ एक वक्र $y = y(x)$ है जो प्रथम चतुर्थांश (first quadrant) में है और माना कि बिंन्दु (1,0) उस पर स्थित है। माना कि Γ कें बिन्दु P पर खिचीं गयी स्पर्श रेखा (tangent) y-अक्ष को Y_p पर प्रतिच्छेद (intersect) करती है, यदि Γ के प्रत्येक बिन्दु P के लिए PY_P की लम्बाई 1 है। तब निम्न में से कौन सा (से) कथन सही है

(1) $xy' + \sqrt{1-x^2} = 0$

(2) $y = -\log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2}$

(3) $y = \log_e\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2}$

(4) $xy' - \sqrt{1-x^2} = 0$

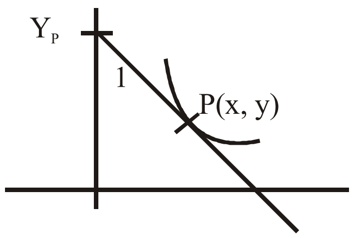
Question ID-337911155

Ans. 1,3

7. Equation of tangent at P

$Y - y = m(X - x)$

$Y_p(0, y - mx)$



$PY_p = 1$

$\sqrt{x^2 + m^2x^2} = 1$

$x^2 + m^2x^2 = 1$

$m^2 = \frac{1-x^2}{x^2}$

$$m = \pm\sqrt{\frac{1-x^2}{x^2}}$$

$$\frac{dy}{dx} = \sqrt{\frac{1-x^2}{x}}$$

$$\int dy = \int \frac{\sqrt{1-x^2}}{x} dx$$

Put $x = \sin\theta$

$$y = \int \frac{\cos^2\theta}{\sin\theta} d\theta$$

$$= \int \frac{1-\sin^2\theta}{\sin\theta} d\theta$$

$$= \int (\operatorname{cosec}\theta - \sin\theta) d\theta$$

$$= -\ell n(\operatorname{cosec}\theta + \cot\theta) + \cos\theta + c$$

$$= -\ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) + \sqrt{1-x^2} + c$$

$\Rightarrow c = 0$

(Rejected because curve is lying in IV Quadrant)

$$\frac{dy}{dx} = -\frac{\sqrt{1-x^2}}{x}$$

$$\int dy = -\int \frac{\sqrt{1-x^2}}{x} dx$$

Put $x = \sin\theta$

$$y = \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right) - \sqrt{1-x^2} + c$$

$\Rightarrow c = 0$

8 Let

माना कि

$$M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix} \text{ and adj } M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$$

Where a and b are real numbers, which of the following options is/are correct

जहॉं a और b वास्तविक संख्याएँ है। निम्न में से कौन सा (से) विकल्प सही है (है)

(1) $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$

(2) If M $\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ then $\alpha - \beta + \gamma = 3$/ यदि M $\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ तब $\alpha - \beta + \gamma = 3$

(3) $a + b = 3$

(4) $\det(\text{adj}M^2) = 81$

Question ID-337911150

Ans. 1,2,3

S. $M=\begin{bmatrix}0&1&a\\1&2&3\\3&b&1\end{bmatrix}$

$\text{adj}(M)=\begin{bmatrix}2-3b&...&...\\...&-3a&...\\...&...&...\end{bmatrix}$

$2-3b=-1$, $3b=3$, $b=1$ | $-3a=-6$, $a=2$

$M=\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}\Rightarrow |M|=-2$

(1) $\text{adj}(M^{-1})+(\text{adj } M)^{-1}=-M$

$(\text{adj } M)^{-1}+(\text{adj } M)^{-1}=-M$

$2(\text{adj }(M))^{-1}=-M$

$2(\text{adj } M)^{-1}\text{ adj } M=-M\text{ adj } M$

$2I=-|M|\, I$

$2I=2I$

(2) $\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}1\\2\\3\end{bmatrix}$

$\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}0&1&2\\1&2&3\\3&1&1\end{bmatrix}^{-1}\begin{bmatrix}1\\2\\3\end{bmatrix}$

$M^{-1}=\dfrac{\text{adj}(M)}{(M)}=-\dfrac{1}{2}\begin{bmatrix}-1&1&-1\\8&-6&2\\-5&3&-1\end{bmatrix}$

$\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=-\dfrac{1}{2}\begin{bmatrix}-1&1&-1\\8&-6&2\\-5&3&-1\end{bmatrix}\begin{bmatrix}1\\2\\3\end{bmatrix}$

$=-\dfrac{1}{2}\begin{bmatrix}-2\\2\\-2\end{bmatrix}$ $\qquad \begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix}=\begin{bmatrix}1\\-1\\1\end{bmatrix}\Rightarrow \alpha=1,\ \beta=-1,\ 4=1$

(3) $a+b=1+2=3$

(4) $|\text{adj}(M^2)|=|M^2|^2=|M^4|=16$

SECTION-3 (Maximum Marks : 18)

* This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.

* For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places truncate/round-off the value to TWO decimal placed.

* Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the correct numerical value is entered.

Zero Marks : 0 In all other cases.

1 Let the point B be the reflection of the point A(2, 3) with respect to the line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is.....

माना कि बिन्दु B रेखा $8x - 6y - 23 = 0$ के सापेक्ष बिन्दु A(2,3) का प्रतिबिम्ब (reflection) है माना कि Γ_A तथा Γ_B क्रमशः त्रिज्याएँ 2 और 1 वाले वृत्त है। जिनकें केन्द्र क्रमशः A और B है माना कि वृतों Γ_A और Γ_B की एक ऐसी उभयनिष्ठ स्पर्श (common tangent) रेखा T है दोनो वृत्त जिसके एक ही तरफ है। यदि C बिन्दुओं A और B से जाने वाली रेखा T का प्रतिच्छेद बिंदु है तब रेखाखण्ड (line segment) AC की लम्बाई है।

Question ID-337911160

Ans. 10

S. $$\frac{x-2}{8} = \frac{y-3}{-6} = -2\left(\frac{16-18-23}{100}\right)$$

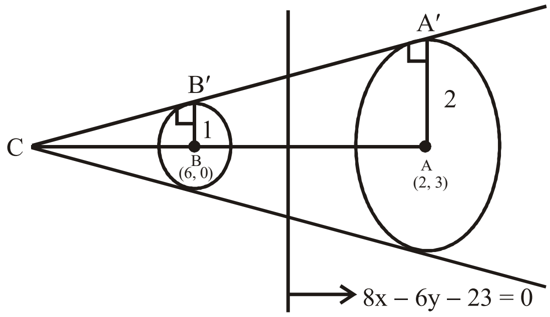
$$\frac{x-2}{8} = \frac{y-3}{-6} = \frac{-2\times -25}{100} = \frac{1}{2}$$

$$\frac{x-2}{4} = \frac{y-3}{-3} = 1$$

$x - 2 = 4$ $\quad$ $y - 3 = -3$ $\quad \Rightarrow y = 0$

$x = 6$

B (6, 0)



$\frac{CB}{CA} = \frac{1}{2}$ (Δ'S CBB' and CAA' are congruent)

$$\frac{CB}{CB + AB} = \frac{1}{2}$$

$2CB = CB + AB$

$CB = AB = \sqrt{16+9} = 5$

$CA = 2(CB) = 10$

2 Let $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\{|\alpha + b\omega + c\omega^2|^2 : a,b,c \text{ distinct no-zero integers}\}$ equals

माना कि $\omega \neq 1$ एकक का एक घनमूल है। तब समुच्चय $\{|\alpha + b\omega + c\omega^2|^2 : a,b,c$ भिन्न अशून्य पूर्णांक है।$\}$

का निम्नतम बराबर _____

Question ID-337911157

Ans. 3

S. $|a+b\omega+c\omega^2|^2$

$= (a+b\omega+c\omega^2)\ (a+b\omega^2+c\omega)$

$= a^2 + b^2 + c^2 - ab - bc - ca$

$= \frac{1}{2}\ ((a-b)^2 + (b-c)^2 + (c-a)^2)$

for minimum put a = 1, b = 2 c = 3

minimum = 3

3 Let AP (a;d) denote the set of all the terms of an infinite arithmeitc progression with first term a and common difference d > 0. If $AP(1;3) \cap AP(2;5) \cap AP(3;7) = AP(a;d)$ then a + d equals

माना कि AP (a;d)एक अनंत समान्तर श्रेण के पदों का समूच्चय है जिसका प्रथम पद a तथा सार्वन्तर d > 0 है।

यदि $AP(1;3) \cap AP(2;5) \cap AP(3;7) = AP(a;d)$ है तब a + d बराबर

Question ID-337911158

Ans. 157

S. $AP(1,3) \cap AP(2,5) \cap AP(3,7)$

I AP 1,4,7,10,...........

II AP 2,7,12,17,.........

III AP 3,10,17,24,.......

first time 52 will be present in all three AP

d = LCM (3,5,7) = 105

a = 52

d = 105

a+d = 157

4 Let S be the sample space of all 3 × 3 matrices with entries from the set {0, 1}. Let the events E_1 and E_2 be given by

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S : \text{Sum of entries of A is } 7\}$

If a matrix is chosen at random from S, then the conditional probability $P(E_1 | E_2)$ equals _____

माना कि S ऐसे 3 × 3 आव्यूहों का प्रतिदर्श समिष्ट है जिनकी प्रविष्टियॉ समूच्चय {0, 1} से है माना कि घटनाएं E_1 एवं E_2 निम्न है।

$E_1 = \{A \in S : \det A = 0\}$ और

$E_2 = \{A \in S :$ A की प्रविष्टियों का कुल योग 7 है ।$\}$

यदि एक आव्यूह S से यादृच्छिक चुना जाता है तब सप्रतिबंध प्रायिकता $P(E_1 \mid E_2)$ बराबर ______

Question ID-337911159

Ans. 0.5

S. $E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S : \text{Sum of entries of A is } 7\}$

$p(E_1/E_2)$ = probability that det A = 0 when entries of det have sum equal to 7

$p(E_2)$ = 7 one's and 2 zero's

$$= \frac{9!}{7!2!} = 36$$

det A will become zero if both zero's will come in same column or same row

$${}^3C_1 \bullet {}^3C_2 + {}^3C_1 \bullet {}^3C_2 = 18$$

$$p(E_1/E_2) = \frac{18}{36} = \frac{1}{2} = 0.5$$

5 If $I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{\left(1+e^{\sin x}\right)\left(2-\cos 2x\right)}$ then $27I^2$ equals

यदि $I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{\left(1+e^{\sin x}\right)\left(2-\cos 2x\right)}$ तब $27I^2$ बराबर

Question ID-337911161

Ans. 4

S. king and add

$$2E = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \frac{dx}{\left(2-\cos 2x\right)}$$

$$\Rightarrow E = \frac{2}{\pi}\int_{0}^{\frac{\pi}{4}} \frac{dx}{2-\left(\frac{1-\tan^2 x}{1+\tan^2 x}\right)}$$

let $\tan x = t$

$\sec^2 x \, dx = dt$

$$I = \frac{2}{\pi}\int_0^1 \frac{dt}{2+2t^2-1+t^2}$$

$$I = \frac{2}{\pi}\int_0^1 \frac{dt}{1+3t^2} = \frac{2}{3\pi}\int_0^1 \frac{dt}{t^2+\left(\frac{1}{\sqrt{3}}\right)^2}$$

$= \frac{2}{3\pi}.\frac{1}{\frac{1}{\sqrt{3}}}\left(\tan^{-1}\left(\sqrt{3}t\right)\right)_0^1$

$I = \frac{2}{\pi\sqrt{3}}.\frac{\pi}{3} = \frac{2}{3\sqrt{3}}$

$I^2 = \frac{4}{27}$

6. Three lines are given by

$\vec{r} = \lambda\hat{i}, \lambda \in R$, $\vec{r} = \mu\left(\hat{i}+\hat{j}\right), \mu \in R$ and $\vec{r} = \upsilon\left(\hat{i}+\hat{j}+\hat{k}\right), \upsilon \in R$

Let the lines cut the plane $x + y + z = 1$ at the points A,B and C respectively. If the area of the triagle ABC is Δ then the value of $(6\Delta)^2$equas

तीन रेखाएं क्रमशः

$\vec{r} = \lambda\hat{i}, \lambda \in R$, $\vec{r} = \mu\left(\hat{i}+\hat{j}\right), \mu \in R$ तथा $\vec{r} = \upsilon\left(\hat{i}+\hat{j}+\hat{k}\right), \upsilon \in R$

द्वारा दी गयी है। माना कि रेखाएं समतल (plane) $x + y + z = 1$ को क्रमशः बिन्दुओं A,B तथा C वा काटती है। यदि त्रिभुज ABC का क्षेत्रफल Δ है तब $(6\Delta)^2$ का मान बराबर_____

Question ID-337911162

Ans. 0.75

Sol Let $A(\lambda,0,0)$ $\beta(\mu,\mu,0)$ $C(\upsilon,\upsilon,\upsilon)$

A,B,C lie on the plane $x + y + z = 1$ so

$\lambda = 1,\ \mu = \frac{1}{2},\ \upsilon = \frac{1}{3}$

$A(1,0,0)\ B\left(\frac{1}{2},\frac{1}{2},0\right)\ \left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

$\overrightarrow{AB} = \left(-\frac{1}{2},\frac{1}{2},0\right)$

$\overrightarrow{AC} = \left(-\frac{2}{3},\frac{1}{3},\frac{1}{3}\right)$

$$AB \times AC = \begin{vmatrix} i & j & k \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ -\frac{2}{3} & \frac{1}{3} & \frac{1}{3} \end{vmatrix} = \frac{1}{6}\begin{vmatrix} i & j & k \\ -1 & 1 & 0 \\ -2 & 1 & 1 \end{vmatrix}$$

$= \frac{1}{6}\left(i(1-0)-\hat{j}\right)(-1-0)+k(-1+2)$

$\overrightarrow{AB} \times \overrightarrow{AC} = \frac{1}{6}\left(\hat{i}+\hat{j}+\hat{k}\right)$

$$\Delta AB = \frac{1}{2}\left|AB \times AC\right|$$

$$\Delta = \frac{1}{2}\cdot\frac{1}{6}.\sqrt{3} = \frac{\sqrt{3}}{12}$$

$$\left(6\Delta\right)^2 = \left(\frac{6}{12}\sqrt{3}\right)^2 = \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{3}{4} = 0.75$$