JEE (ADVANCED) 2019 PAPER II

MATHS

SECTION-1 (Maximum Marks : 32)

This section contains EIGHT (08) questions.

* Each question has FOUR options ONE OR MORE THAN ONE of these four option(s) is(are) correct answer(s).

* For each question, choose the option(s) corresponding to (all) the correct answer(s).

* Answer to each question will be evaluated according to the following marking scheme.

Full Marks : +4 If only (all) the correct option(s) is (are) chosen.

Partial Marks : +3 If all the four options are correct but ONLY three options are chosen.

Partial Marks : +2 If three or more options are correct but ONLY two options are chosen and both of which are correct.

Partial Marks : +1 If two or more options are correct but ONLY one option is chosen and it is a correct option.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : –1 In all other cases

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1. Let $f: R \to R$ be a function. We say that f has **Question ID: 337911202**

PROPERTY 1 if $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{\sqrt{|h|}}$ exists and is finite and

PROPERTY 2 if $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h^2}$ exists and is finite and

Then which of the following options is/are correct ?

(1) $f(x) = |x|$ has PROPERTY 1 (2) $f(x) = x|x|$ has PROPERTY 2

(3) $f(x) = \sin x$ has PROPERTY 2 (4) $f(x) = x^{2/3}$ has PROPERTY 1

माना कि $f: R \to R$ एक फलन है। हम कहते हैं कि f में

गुण 1 (PROPERTY 1) है यदि $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{\sqrt{|h|}}$ का अस्तित्व (exists) है और वह परिमित (finite) है,

गुण 2 (PROPERTY 2) है यदि $\lim\limits_{h\to 0}\dfrac{f(h)-f(0)}{h^2}$ का अस्तित्व (exists) है और वह परिमित (finite) है।

तब निम्न में से कौन सा (से) विकल्प सही है (हैं) ?

(1) $f(x) = |x|$ में गुण 1 है (2) $f(x) = x|x|$ में गुण 2 है

(3) $f(x) = \sin x$ में गुण 2 है (4) $f(x) = x^{2/3}$ में गुण 1 है

Ans. 1,4

Option (1) $f(x) = |x| \quad f(0) = 0$

Property -1

$$\lim_{h\to0}\frac{|h|}{\sqrt{|h|}} = \lim_{h\to0}|h|^{1/2} = 0 \text{ limit exists}$$

Option (2) $f(x) = x|x| \quad f(0) = 0$

Property -2

$$\lim_{h\to0}\frac{h|h|-0}{h^2} = \lim_{h\to0}\frac{|h|}{h}$$

LHL = –1

RHL = 1

LHL ≠ RHL

⇒ Does not exist

Option (3) $f(x) = \sin x \quad f(0) = 0$

Property -2

$$\lim_{h\to0}\frac{\sin h}{h^2} = \lim_{h\to0}\left(\frac{\sin h}{h}.\frac{1}{h}\right)$$

⇒ Does not exist

Option (4) $f(x) = x^{2/3}$

$= (x^2)^{1/3} = |x|^{2/3} \qquad f(0) = 0$

Property -1

$$\lim_{h\to0}\frac{|h|^{2/3}-0}{\sqrt{|h|}} = \lim_{h\to0}|h|^{2/3-1/2} = \lim_{h\to0}|h|^{1/6} = 0 \text{ limit exists}$$

2. Let $x \in R$ and let **Question ID: 337911200**

$$P = \begin{bmatrix}1 & 1 & 1\\0 & 2 & 2\\0 & 0 & 3\end{bmatrix}, Q = \begin{bmatrix}2 & x & x\\0 & 4 & 0\\x & x & 6\end{bmatrix}$$

Then which of the following options is are/correct

(1) For x = 1 there exists a unit vector $\alpha\hat{i}+\beta\hat{j}+\gamma\hat{k}$ for which $R\begin{bmatrix}\alpha\\\beta\\\gamma\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

(2) $\det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$, for all $x \in R$

(3) For $x = 0$, if $R \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6 \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$ then $a + b = 5$

(4) There exists a real number x such that PQ = QP

माना कि $x \in R$ और माना कि

$P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}, Q = \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 6 \end{bmatrix}$

तब निम्न में से कौन सा (से) विकल्प सही है (हैं) ?

(1) $x = 1$ के लिए, एक ऐसा मात्रक सदिश (unit vector) $\alpha\hat{i} + \beta\hat{j} + \gamma\hat{k}$ सम्भव है, जिसके लिए $R \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

(2) सभी $x \in R$ के लिए, $\det R = \det \begin{bmatrix} 2 & x & x \\ 0 & 4 & 0 \\ x & x & 5 \end{bmatrix} + 8$

(3) $x = 0$ के लिए, यदि $R \begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6 \begin{bmatrix} 1 \\ a \\ b \end{bmatrix}$, तब $a + b = 5$

(4) एक ऐसी वास्तविक संख्या x सम्भव है जिसके लिए PQ = QP

Ans. **2,3**

S. Option (2) $PQP^{-1} = R$

$PQ = RP$

$\det P \det Q = \det R \det P$

$\det R = \det Q$

$\det Q = 48 - 4x^2$

2^{nd} option $\Rightarrow 40 - x^2 + 8$

$R = PQP^{-1}$

Option (3) $R=\frac{1}{6}\begin{bmatrix}6x+12 & 3x+6 & 4-10x\\ 12 & 24 & 8-4x\\ 18x & 0 & 36-6x\end{bmatrix}$

if $x=0$

$$R=\frac{1}{6}\begin{bmatrix}2 & 1 & \frac{2}{3}\\ 0 & 4 & \frac{4}{3}\\ 0 & 0 & 6\end{bmatrix}$$

$$R\begin{bmatrix}1\\ a\\ b\end{bmatrix}=6\begin{bmatrix}1\\ a\\ b\end{bmatrix}=\begin{bmatrix}2+a+\frac{2}{3}b\\ 4a+\frac{4}{3}b\\ 6b\end{bmatrix}\Rightarrow a=2 \text{ and } b=3$$

$a+b=5$

Option (4) $PQ=QP$ not possible

Option (1) if we put $x=1$ then

$$R=\frac{1}{6}\begin{bmatrix}18 & 9 & -6\\ 12 & 24 & 4\\ 18 & 0 & 30\end{bmatrix}$$

$|R|\neq 0$

$\Rightarrow$ system will have unique solution

$\Rightarrow$ only $\alpha=\beta=\gamma=0$ will satisfy the system

3. Let $f(x)=\frac{\sin \pi x}{x^2}, x>0$ **Question ID : 337911203**

Let $x_1<x_2<x_3<......<x_n<....$ be all the points of local maxmimum of f and $y_1<y_2<y_3<....<y_n<.........$ be all the points of local minimum of f then which of the following options is are correct

(1) $x_{n+1}-x_n>2$ for every n

(2) $x_1<y_1$

(3) $x_n\in\left(2n, 2n+\frac{1}{2}\right)$ for every n

(4) $|x_n-y_n|>1$ for every n

माना कि $f(x)=\frac{\sin \pi x}{x^2}, x>0$

माना कि f के सभी स्थानीय उच्चतम (local maximum) बिन्दु $x_1 < x_2 < x_3 < ......< x_n < ....$ है और f के सभी स्थानीय न्यूनतम (local minimum) बिन्दु $y_1 < y_2 < y_3 < .... < y_n < ....$ हैं। तब निम्न में से कौन सा(से) विकल्प सही है (हैं) ?

(1) प्रत्येक n के लिए $x_{n+1} - x_n > 2$ है

(2) $x_1 < y_1$

(3) प्रत्येक n के लिए $x_n \in \left(2n, 2n + \frac{1}{2}\right)$ है

(4) प्रत्येक n के लिए $|x_n - y_n| > 1$ है

Ans. **1,3,4**

S. $f(x) = \dfrac{\sin \pi x}{x^2}, x > 0$

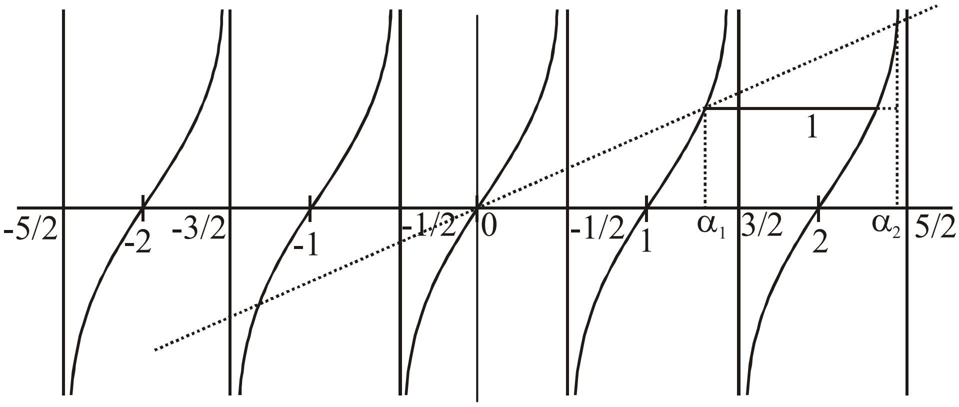
$$f'(x) = \frac{x^2 \cos(\pi x)\pi - \sin(\pi x).2x}{x^4}$$

$$= \frac{[\pi x \cos(\pi x) - 2\sin(\pi x)]}{x^3}$$

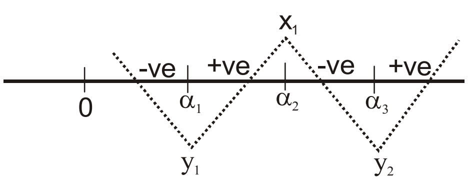
$f'(x) = 0$

$\Rightarrow \pi x \cos \pi x = 2\sin \pi x$

$\tan\pi x = \dfrac{\pi x}{2}$



At 0^+ $\quad f'(x) = -ve$



$\alpha_1 = x_1$

$\alpha_2 = y_1$ (and so on)

$\alpha_3 = x_2$

$x_1 > y_1$

$x_1 \in (2, 5/2)$

using periodicity

$x_n \in (2n, 2n+1/2)$

$x_{n+1} - x_n > 2 \;\; \forall$ n correct

$|x_n - y_n| > 1 \; \forall$ n correct

4. Let **Question ID: 337911199**

$$P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

and $X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$

where P_k^T denotes the transpose of the matrix P_k then which of the following options is/are correct

(1) The sum of diagonal entries of X is 18

(2) If $X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ then $\alpha = 30$

(3) X is a symmetric matrix

(4) X–30I is an invertible matrix

माना कि

$$P_1 = I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}, P_3 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, P_4 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}, P_5 = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, P_6 = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

और $X = \sum_{k=1}^{6} P_k \begin{bmatrix} 2 & 1 & 3 \\ 1 & 0 & 2 \\ 3 & 2 & 1 \end{bmatrix} P_k^T$

जहाँ आव्यूह (matrix) P_k के परिवर्त (transpose) को P_k^T से दर्शाया गया है। तब निम्न में से कौन सा (से) विकल्प सही है (हैं) ?

(1) X के विकर्ण (diagonal) की प्रविष्टियों (entries) का योग 18 है

(2) यदि $X \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \alpha \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$, तब $\alpha = 30$

(3) X एक सममित (symmetric) आव्यूह है

(4) $X-30I$ एक व्युत्क्रमणीय (invertible) आव्यूह है

Ans. 1,2,3

S. $P_1 = P_1^T = P_1^{-1}$

$P_2 = P_2^T = P_2^{-1}$

$P_6 = P_6^T = P_6^{-1}$

and $A^T = A$

$(A+B)^T = A^T + B^T$

$X^T = (P_1AP_1T + ...P_6AP_6^T)^T$

$= X$ so $\times$ is symmetry

trace $\Rightarrow (2+0+1)+(2+0+1).........6$ time

$= 18$

Let $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$XB = P_1AP_1^TB + ...P_6A_6^TB$

$XB = (P_1 + P_2 + P_3.......P_6)\begin{bmatrix} 6 \\ 3 \\ 6 \end{bmatrix}$

$= \begin{bmatrix} 6\times2+3\times2+6\times2 \\ 6\times2+3\times2+6\times2 \\ 6\times2+3\times2+6\times2 \end{bmatrix} = \begin{bmatrix} 30 \\ 30 \\ 30 \end{bmatrix} = 300 \Rightarrow \alpha = 30$

Since x $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 30\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow$ (x-301)B = 0 has a non trivial solution $B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\Rightarrow |x-30| = 0$

5. Three lines **Question ID : 337911206**

$L_1 : \vec{r} = \lambda\hat{i}, \lambda \in R$, $L_2 : \vec{r} = \hat{k} + \mu\hat{j}, \mu \in R$ and, $L_3 : \vec{r} = \hat{i} + \hat{j} + v\hat{k}, v \in R$

are given. For which point (s) Q on L_2 can we find a point P on L_1 and a point R on L_3 so that P,Q and R are collinear

तीन रेखाऐं

$L_1 : \vec{r} = \lambda\hat{i}, \lambda \in R$, $L_2 : \vec{r} = \hat{k} + \mu\hat{j}, \mu \in R$ और $L_3 : \vec{r} = \hat{i} + \hat{j} + v\hat{k}, v \in R$

दी गयी है। L_2 के किस बिन्दु (किन बिन्दुओं) Q के लिए हम L_1 पर एक बिन्दु P और L_3 पर एक बिन्दु R प्राप्त कर सकते हैं ताकि P, Q और R सरेख (collinear) हों जाएँ ?

(1) $\hat{k} + \hat{j}$ (2) $\hat{k} - \frac{1}{2}\hat{j}$ (3) $\hat{k} + \frac{1}{2}\hat{j}$ (4) $\hat{k}$

S. P(λ,0,0) Q(0,μ,1) R(1,1,ν)

P,Q,R are collinear

$\overrightarrow{PQ} = t\overrightarrow{QR} \quad t \in R$

$\overrightarrow{PQ} = (-\lambda, \mu, 1)$

$\overrightarrow{QR} = (-1, \mu - 1, 1 - \nu)$

$\frac{-\lambda}{-1} = \frac{\mu}{\mu - 1} = \frac{1}{1 - \nu}$

$\mu \neq 0, 1$

So Q cannot be (0, 0, 1) and (0, 1, 1)

Ans. **2,3**

6. For a ∈ R, |a| > 1, Let **Question ID : 337911204**

$$\lim_{n\to\infty}\left(\frac{1+\sqrt[3]{2}+....+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+....+\frac{1}{(an+n)^2}\right)}\right) = 54$$

Then the possible value (s) of a is/are

माना कि a ∈ R, |a| > 1 के लिए

$$\lim_{n\to\infty}\left(\frac{1+\sqrt[3]{2}+....+\sqrt[3]{n}}{n^{7/3}\left(\frac{1}{(an+1)^2}+\frac{1}{(an+2)^2}+....+\frac{1}{(an+n)^2}\right)}\right) = 54$$

तब a का (के) सम्भावित मान है (हैं)

(1) –6 (2) 8 (3) 7 (4) –9

Ans. 2,4

S.
$$\lim_{x\to\infty}\left(\frac{1+(2)^{1/3}+(3)^{1/3}....(n)^{1/3}}{n^{1/3}.\frac{n^2}{n^2}\left(\frac{1}{\left(a+\frac{1}{n}\right)^2}+\frac{1}{\left(a+\frac{2}{n}\right)^2}....\frac{1}{\left(a+\frac{n}{n}\right)^2}\right)}\right)$$

$$\lim_{x\to\infty}\left(\frac{\frac{1}{n}\sum_{r=1}^{n}\left(\frac{r}{n}\right)^{1/3}}{\frac{1}{n}\sum_{r=1}^{n}\frac{1}{\left(a+\frac{r}{n}\right)^2}}\right)$$

$$\frac{r}{n}=x,\frac{1}{x}=dx$$

$$\frac{\int_0^1 (x)^{1/3}dx}{\int_0^1 \frac{1}{(a+x)^2}dx}=54$$

$$\frac{\left(\frac{x^{4/3}}{4/3}\right)_0^1}{-\left(\frac{1}{a+x}\right)_0^1}=54$$

$$\frac{\frac{3}{4}}{-\left[\frac{1}{a+1}-\frac{1}{a}\right]}=54$$

$$\frac{a(a+1)}{-(a-a-1)}=72$$

$$a^2+a-72=0$$

$$a^2+9a-8a-72=0$$

$$a=-9,\ a=8$$

7. Ler $f : R \rightarrow R$ be given by $f(x) = (x-1)(x-2)(x-5)$ define **Question ID : 337911205**

$$F(x) = \int_0^x f(t)dt, x > 0$$

Then which of the following options is/are correct

(1) $F(x) \neq 0$ for all $x \in (0,5)$

(2) F has a local maximum at $x = 2$

(3) F has a local minimum at $x = 1$

(4) F has two local maxima and one local minimum in $(0,\infty)$

माना कि $f : R \rightarrow R, f(x) = (x-1)(x-2)(x-5)$ द्वारा दिया गया है। परिभाषित करें।

$$F(x) = \int_0^x f(t)dt, x > 0$$

तब निम्न में से कौन सा (से) विकल्प सही है (हैं) ?

(1) सभी $x \in (0,5)$ के लिए $F(x) \neq 0$

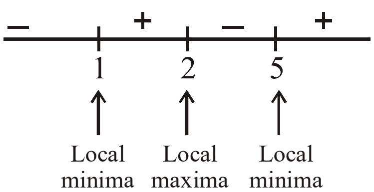
(2) F का एक स्थानीय उच्चतम (local maximum) $x = 2$ है

(3) F का एक स्थानीय निम्नतम (local minimum) $x = 1$ पर है

(4) F के दो स्थानीय उच्चतम और एक स्थानीय निम्नतम $(0,\infty)$ में हैं

Ans. 1,2,3

S. $$F'(x) = f(x) = (x-1)(x-1)(x-5)$$



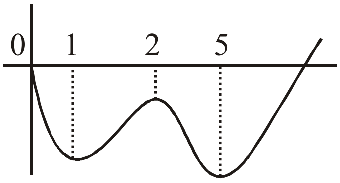
$$F(x) = \int_0^x (x^3 - 8x^2 + 17x - 10)dx$$

$$F(x) = \frac{x^4}{4} - \frac{8x^3}{3} + \frac{17}{2}x^2 - 10x$$

$$F(1) = \frac{1}{4} - \frac{8}{3} + \frac{17}{2} - 10 < 0$$

$$F(2) = \frac{16}{4} - \frac{64}{3} + \frac{68}{2} - 20 < 0$$

$$F(5) = \frac{625}{4} - \frac{1000}{3} + \frac{17 \times 25}{2} - 50 < 0$$



8. For non negative integers n let **Question ID : 337911201**

$$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^{2}\left(\frac{k+1}{n+2}\pi\right)}$$

Assuming $\cos^{-1}x$ takes values in $[0,\pi]$ which of the following options is/are correct

(1) $f(4)=\frac{\sqrt{3}}{2}$

(2) If $\alpha=\tan(\cos^{-1}f(6))$ then $\alpha^2+2\alpha-1=0$

(3) $\lim_{n\to\infty} f(n)=\frac{1}{2}$

(4) $\sin(7\cos^{-1}f(5))=0$

अऋणात्मक पूर्णांकों (non negative integers) n के लिए माना कि

$$f(n)=\frac{\sum_{k=0}^{n}\sin\left(\frac{k+1}{n+2}\pi\right)\sin\left(\frac{k+2}{n+2}\pi\right)}{\sum_{k=0}^{n}\sin^{2}\left(\frac{k+1}{n+2}\pi\right)}$$

माना कि $\cos^{-1}x$ का मान $[0,\pi]$ में है, तब निम्न में से कौन सा (से) विकल्प सही है (हैं) ?

(1) $f(4)=\frac{\sqrt{3}}{2}$

(2) यदि a $=\tan(\cos^{-1}f(6))$ तब $\alpha^2+2\alpha-1=0$

(3) $\lim_{n\to\infty} f(n)=\frac{1}{2}$

(4) $\sin(7\cos^{-1}f(5))=0$

Ans. **1,2,4**

S. $$f(n)=\frac{\sum_{k=0}^{n}2\sin\left((k+1)\frac{\pi}{n+2}\right)\sin\left((k+2)\frac{\pi}{n+2}\right)}{\sum_{k=0}^{n}2\sin^{2}\left((k+1)\frac{\pi}{n+2}\right)}$$

$$\frac{\sum_{k=0}^{n}\left(\cos\frac{\pi}{n+2}-\cos\left((2k+3)\frac{\pi}{n+2}\right)\right)}{\sum_{k=0}^{n}1-\cos\left((2x+2)\frac{\pi}{n+2}\right)}$$

$$f(n)=\frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)-\sum_{k=0}^{n}\cos\left((2k+3)\frac{\pi}{n+2}\right)}{(n+1)-\sum_{k=0}^{n}\cos\left((2k+2)\frac{\pi}{n+2}\right)} \quad ....(1)$$

$$\sum_{k=0}^{n}\cos\left((2k+3)\frac{\pi}{n+2}\right)=\cos\left(\frac{3\pi}{n+2}\right)+\cos\frac{5\pi}{n+2}+\cos\frac{7\pi}{n+2}+.....+(2n+3)\frac{\pi}{n+2}$$

$$=\frac{\sin\left(\frac{(n+1)\frac{2\pi}{n+2}}{2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}.\cos\left(\frac{\frac{3\pi}{n+2}+(2n+3)\frac{\pi}{n+2}}{2}\right)$$

$$=\frac{\sin\left((n+1)\frac{\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}.\cos\left((n+3)\frac{\pi}{n+2}\right) \quad [\sin(\pi-\theta)=\sin\theta]$$

$$=\cos\left((n+3)\frac{\pi}{n+2}\right)=\cos\left(\pi+\frac{\pi}{n+2}\right)=-\cos\left(\frac{\pi}{n+2}\right)$$

$$\sum_{k=0}^{h}\cos\left((2k+2)\frac{\pi}{n+2}\right)=\cos\left(\frac{2\pi}{n+2}\right)+\cos\frac{4\pi}{n+2}+.....+\cos\left((2n+2)\frac{\pi}{n+2}\right)$$

$$=\frac{\sin\left((n+1)\frac{2\pi}{n+2}\right)}{\sin\left(\frac{\pi}{n+2}\right)}\cos\left((n+2)\frac{\pi}{n+2}\right)$$

$$=\cos\pi=-1$$

$$f(n)=\frac{(n+1)\cos\left(\frac{\pi}{n+2}\right)-\left(-\cos\frac{\pi}{n+2}\right)}{n+1-(-1)}$$

$$f(n) = \cos\frac{(n+2)\cos\left(\frac{\pi}{n+2}\right)}{n+2}$$

$$f(n) = \cos\left(\frac{\pi}{n+2}\right)$$

Option: (1) $f(4) = \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

(2) $f(6) = \cos^{-1}(f(6) = \frac{\pi}{8}$

$\alpha = \tan\frac{\pi}{8}$

$\alpha^2 + 2\alpha - 1 = 0$

$2\alpha = 1 - \alpha^2$

$\frac{2\alpha}{1-\alpha^2} = 1$

$\tan\left(\frac{\pi}{8} \times 2\right) = 1$

$\tan 45° = 1$ true

(3) $n \to \infty$

$f(n) \to 1$

(4) $f(5) = \cos\left(\frac{\pi}{7}\right)$

$\cos^{-1}(f(5)) = \frac{\pi}{7}$

$\sin\left(7 \times \frac{\pi}{7}\right) = \sin\pi = 0$

SECTION-II (Maximum Marks : 18)

This section contains SIX (06) questions. The answer to each question is a NUMERICAL VALUE.

- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places, truncate/round-off the value to TWO decimal places.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the correct numerical value is entered.

Zero Marks : 0 In all other cases.

1. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ and $\hat{b} = \hat{i} + 2\hat{j} + \hat{k}$ be two vectors. Consider a vector $\vec{c} = \alpha\vec{a} + \beta\vec{b}$, $\alpha, \beta \in R$. If the projection of $\vec{c}$ on the vector $(\vec{a} + \vec{b})$ is $3\sqrt{2}$ then the minimum value of $\left(\vec{c} - \left(\vec{a} \times \vec{b}\right)\right) \cdot \vec{c}$ equals________

Question ID : 337911212

माना कि $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$ और $\hat{b} = \hat{i} + 2\hat{j} + \hat{k}$ दो सदिश (vector) हैं। माना कि एक सदिश $\vec{c} = \alpha\vec{a} + \beta\vec{b}$, $\alpha, \beta \in R$ है। यदि सदिश $(\vec{a} + \vec{b})$ पर $\vec{c}$ का प्राक्षेप (projection) $3\sqrt{2}$ है, तब $\left(\vec{c} - \left(\vec{a} \times \vec{b}\right)\right) \cdot \vec{c}$ का निम्नतम (minimum) मान बराबर ________

Ans. 18

S. $|\vec{a}|^2 = |\vec{b}|^2 = 6$ $\quad\quad$ $\vec{a} + \vec{b} = 3\hat{i} + 3\hat{j}$

$\vec{a}.\vec{b} = 2 + 2 - 1 = 3$

Projection of $\vec{c}$ on $\vec{a} + \vec{b}$ is

$$\frac{\vec{c}.\left(\vec{a} + \vec{b}\right)}{\left|\vec{a} + \vec{b}\right|} = 3\sqrt{2}$$

$$\frac{\left(\alpha\vec{a} + \beta\vec{b}\right).\left(\vec{a} + \vec{b}\right)}{3\sqrt{2}} = 3\sqrt{2}$$

$9(\alpha + \beta) = 18$

$\alpha + \beta = 2$

Let $y = \left(\vec{c} - \left(\vec{a} \times \vec{b}\right)\right).\vec{c}$

$= \vec{c}.\vec{c} - \left(\alpha\vec{a} + \beta\vec{b}\right).\left(\vec{a} \times \vec{b}\right)$

$= |\vec{c}|^2 - 0$

$= \alpha^2|\vec{a}|^2 + \beta^2|\vec{b}|^2 + 2\alpha\beta\vec{a}.\vec{b}$

$= 6\alpha^2 + 6\beta^2 + 6\alpha\beta$

$= 6\left(\alpha^2 + \beta^2 + \alpha\beta\right)$ $\quad\quad$ $(\alpha = 2 - \beta)$

$= 6\left((2-\beta)^2 + \beta^2 + (2-\beta)\beta\right)$

$= 6\left(4 - 4\beta + \beta^2 + \beta^2 + 2\beta - \beta^2\right)$

$= 6\left(\beta^2 - 2\beta + 1 + 3\right)$

$y = 6\left((\beta - 1)^2 + 3\right)$

$y_{min} = 6(0 + 3) = 18$

2. The value of

Question ID : 337911210

$$\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$$

in the interval $\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$ equals

अन्तराल (interval) $\left[-\frac{\pi}{4},\frac{3\pi}{4}\right]$ में

$$\sec^{-1}\left(\frac{1}{4}\sum_{k=0}^{10}\sec\left(\frac{7\pi}{12}+\frac{k\pi}{2}\right)\sec\left(\frac{7\pi}{12}+\frac{(k+1)\pi}{2}\right)\right)$$

का मान बराबर है –

Ans. 0

S. $$\frac{1}{4}\sum_{k=10}^{10}\frac{1}{\cos\left(\frac{k\pi}{2}+\frac{7\pi}{12}\right)\cos\left((k+1)\frac{\pi}{2}+\frac{7\pi}{12}\right)}$$

$$=\frac{1}{4}\sum_{k=0}^{10}\frac{\sin\left(\left((k+1)\frac{\pi}{2}+\frac{7\pi}{12}\right)-\left(\frac{k\pi}{2}+\frac{7\pi}{12}\right)\right)}{\cos\left(\frac{k\pi}{2}+\frac{7\pi}{12}\right)\cos\left((k+1)\frac{\pi}{2}+\frac{7\pi}{12}\right)}$$

$$=\frac{1}{4}\sum_{k=0}^{10}\left(\tan\left((k+1)\frac{\pi}{2}+\frac{7\pi}{12}\right)-\tan\left(\frac{k\pi}{2}+\frac{7\pi}{12}\right)\right)$$

$$=\frac{1}{4}\left(\tan\left(\frac{11\pi}{2}+\frac{7\pi}{12}\right)-\tan\frac{7\pi}{12}\right)$$

$$=\frac{1}{4}\left(\tan\left(\frac{\pi}{12}\right)-\tan\frac{7\pi}{12}\right)$$

$$=\frac{1}{4}\left(\tan\frac{\pi}{12}+\cot\frac{\pi}{12}\right)$$

$$=\frac{1}{4}2\cos ec\left(2\times\frac{\pi}{12}\right)$$

$$=\frac{1}{4}\times 2\cos ec\left(\frac{\pi}{6}\right)$$

$$=\frac{1}{4}\times 2\times 2=1$$

$\sec^{-1}(1)=0$

3. Suppose **Question ID : 337911207**

$$\det\begin{bmatrix}\sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^{n}C_k k^2 \\ \sum_{k=0}^{n} {}^{n}C_k k & \sum_{k=0}^{n} {}^{n}C_k 3^k\end{bmatrix} = 0$$

holds for some positive integer n then $\sum_{k=0}^{n}\frac{{}^{n}C_k}{k+1}$ equals

माना कि किसी धनात्मक पूर्णांक (positive integer) n के लिए

$$\det\begin{bmatrix}\sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^{n}C_k k^2 \\ \sum_{k=0}^{n} {}^{n}C_k k & \sum_{k=0}^{n} {}^{n}C_k 3^k\end{bmatrix} = 0$$

तब $\sum_{k=0}^{n}\frac{{}^{n}C_k}{k+1}$ बराबर

Ans. 6.2

S. $$\begin{vmatrix}\frac{n(n+1)}{2} & n(n+1)2^{n-2} \\ n2^{n-1} & 4^n\end{vmatrix} = 0$$

$n = 4$

put $n = 4$ In $\sum_{k=0}^{n}\frac{{}^{n}C_k}{k+1}$

$$\sum_{k=0}^{4}\frac{{}^{4}C_k}{k+1} = \frac{2^5-1}{5} = \frac{31}{5}$$

$= 6.2$

4. Five persons A,B,C,D and E are seated in a circular arrangment.If each of them is given a hat of one of the three colours red, blue and green then the number of ways of distributing the hats such that the persons seated in adjacent seats get different coloured hats is________ **Question ID : 337911208**

पॉच व्यक्ति A,B,C,D और E वृत्तीय क्रम (circular arrangment) में बैठे हैं। यदि प्रत्येक को तीन रंगो लाल, नीले और हरे

रंग की टोपियों में से एक रंग की टोपी दी जाती है, तब टोपियो को कितने प्रकार से बाँट सकते हैं जिससे संलग्न (adjacent) बैठे व्यक्तियों की टोपियों के रंग भिन्न हों ________

Ans. 30

S. Maximum no. of hats used the same color are 2

They cannot be 3 otherwise at least two hats of same color are consecutive.
That can be choose by 3 ways
RRGGB, RGGBB, RRGBB
Let RRGGB is present
Now number of ways disturbing blue hat in 5 person equal to 5
Let blue goes to person A
Now either B & D are filled by green and C & E filled by red.
$\Rightarrow$ 2 way possible
Total $\Rightarrow 3 \times 5 \times 2 = 30$ ways

5. Let |X| denote the number of elements in a set X. Let S = {1,2,3,4,5,6} be a sample where each element is equally likely to occur. If A and B are independent events associated with S, then the number of ordered pairs (A,B) such that $1 \le |B| < |A|$ equals______ **Question ID : 337911209**

माना |X| समुच्चय (set) X के तत्वों (elements) की संख्या दर्शाता है। माना कि S = {1,2,3,4,5,6} एक प्रतिदर्श समिष्ट (sample space) है जिसमें प्रत्येक तत्व के आने की संभावना समान है। यदि A और B, प्रतिदर्श समिष्ट S से सम्बद्ध स्वतंत्र घटनाएँ (independent events) हैं तब उन क्रमित–युग्मों (ordered pairs) (A,B) की संख्या, जिसमें $1 \le |B| < |A|$ हो, बराबर ______

Ans. 422

S. 5. S {1, 2, 3, 4, 5, 6}

$1 \le |B| < |A|$
$P(B/A) = P(B)$

$$\frac{n(A \cap B)}{n(A)} = \frac{n(B)}{n(S)}$$

n(A) should have 2 or 3 as prime factor
n(A) can be 2, 3, 4 or 6 as n(A) > 1
n(A) = 2 doesnot satisfy the constraint
n(A) = 3 n(B) = 2
$n(A \cap B) = 1$

No. of ordered pair = $^6C_4 \frac{4!}{2!} = 180$

$n(A) = 4, n(B) = 3\ n(A \cap B) = 2$

No. of ordered pair = $^6C_5 \frac{5!}{2!2!} = 180$

n(A) = 6 n(B) = 1, 2, 3, 4, 5
No. of ordered pair = $2^6 - 2 = 62$
Total = 180+180+62 = 422

6. The value of the integral $\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5} d\theta$ equals______ **Question ID : 337911211**

समाकल (integral) $\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5} d\theta$ का मान बराबर ______

Ans. 0.5

S. 6.

$$\int_0^{\pi/2} \frac{3\sqrt{\cos\theta}}{\left(\sqrt{\cos\theta}+\sqrt{\sin\theta}\right)^5} d\theta$$

$$\int_0^{\pi/2} \frac{3\sec^2\theta}{\left(1+\sqrt{\tan\theta}\right)^5} d\theta$$

Let $\tan\theta = t^2$

$$\int_0^{\infty} \frac{6t\,dt}{(1+t)^5}$$

$\sec^2\theta\, d\theta = 2t\,dt$

$$=6\left[\int_0^{\infty}\left(\frac{1}{(1+t)^4}-\frac{1}{(1+t)^5}\right)dt\right]$$

$$=6\left(-\frac{1}{3}\frac{1}{(1+t)^3}+\frac{1}{4(1+t)^4}\right)_0^{\infty}$$

$$=6\left(\frac{1}{3}-\frac{1}{4}\right)$$

$$=\frac{6}{12}=0.5$$

SECTION-III (Maximum Marks : 12)

This section contains TWO (02) List-Match sets.

- Each List-Match set has TWO (02) Multiple Choice Questions.
- Each List-Match set has two lists : List-I and List-II.

• List-I has Four entries (I),(II), (III) and (IV) List-II has Six entries (P),(Q), (R), (S), (T) and (U).

• FOUR options are given in each Multiple Choice Question based on List-I and List-II and ONLY ONE of these four options satisfies the condition asked in the Multiple Choice Question.

• Answer to each question will be evaluated according to the following marking scheme :

Full Marks : +3 If ONLY the option corresponding to the correct combination is chosen.

Zero Marks : 0 If none of the options is chosen (i.e. the question is unanswered).

Negative Marks : –1 In all other cases.

Answer the following by appropriately matching the lists based on the information given in the paragraph.

Let $f(x) = \sin(\pi \cos x)$ and $g(x) = \cos(2\pi \sin x)$ be two functions defined for $x > 0$. Define the following sets whose elements are written in the increasing order :

$X = \{x : f(x) = 0\}$, $Y = \{x : f'(x) = 0\}$

$Z = \{x : g(x) = 0\}$, $W = \{x : g'(x) = 0\}$

List-I contains the sets X, Y, Z and W. List II contains some information regarding these sets.

List I	**List II**
(I) X	(P) $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II) Y	(Q) an arithmetic progression
(III) Z	(R) NOT an arithmetic progression
(IV) W	(S) $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(T) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(U) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

1. Which of the following is the only CORRECT combination : **Question ID : 337911213**

Options

(1) (I), (Q), (U) (2) (II), (R), (S) (3) (I), (P), (R) (4) (II), (Q), (T)

2. Which of the following is the only CORRECT combination **Question ID : 337911214**

(1) (III), (P), (Q), (U) (2) (III), (R), (U)

(3) (IV), (P), (R), (S) (4) (IV), (Q), (T)

अनुच्छेद में दी गई जानकारी के आधार पर सूचियों का उचित मिलान करके प्रश्न का उत्तर दें।

माना कि $f(x) = \sin(\pi \cos x)$ और $g(x) = \cos(2\pi \sin x)$ दो फलन (function) है जो $x > 0$ में परिभाषित हैं।

$X = \{x : f(x) = 0\}$, $Y = \{x : f'(x) = 0\}$

$Z = \{x : g(x) = 0\}$, $W = \{x : g'(x) = 0\}$

सूची-I में X, Y, Z और W समुच्चय है। सूची-II में इन समुच्चयों के बारे में कुछ सूचनाएं हैं।

List I	**List II**
(I) X	(P) $\supseteq \left\{\frac{\pi}{2}, \frac{3\pi}{2}, 4\pi, 7\pi\right\}$
(II) Y	(Q) समान्तर श्रेणी (an arithmetic progression)
(III) Z	(R) समान्तर श्रेणी नहीं है (NOT an arithmetic progression)
(IV) W	(S) $\supseteq \left\{\frac{\pi}{6}, \frac{7\pi}{6}, \frac{13\pi}{6}\right\}$
	(T) $\supseteq \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi\right\}$
	(U) $\supseteq \left\{\frac{\pi}{6}, \frac{3\pi}{4}\right\}$

1. निम्न में से कौन सा एकमात्र संयोजन सही है :

विकल्प

(1) (I), (Q), (U) (2) (II), (R), (S) (3) (I), (P), (R) (4) (II), (Q), (T)

Ans. **4**

2. निम्न में से कौन सा एकमात्र संयोजन सही है ?

विकल्प

(1) (III), (P), (Q), (U) (2) (III), (R), (U)

(3) (IV), (P), (R), (S) (4) (IV), (Q), (T)

Ans. **3**

S. X: $f(x) = 0$

$\sin(\pi \cos x) = 0$

$\pi \cos x = n\pi \quad n \in I$

$\cos x = n \quad n \in I$

$\therefore \cos x = 0$ or $\cos x = -1$ or $\cos x = 1$

$x = (2n+1)\frac{\pi}{2} \qquad x = n\pi$

$\therefore \in \left\{n\pi, \left(n + \frac{1}{2}\right)\pi\right\}$

$\therefore x = \left\{\frac{\pi}{2}, \pi, \frac{3\pi}{2}, ...\right\}$

Y: $f'(x) = 0$

$\cos(\pi\cos x).\ \pi(-\sin x) = 0$

$\cos(\pi \cos x) = 0$ or $\sin x = 0$

$\pi \cos x = (2n+1)\frac{\pi}{2}$ $\quad\quad x = n\pi$

$$\cos x = \frac{(2n+1)}{2}$$

$$\therefore \cos x = \frac{1}{2}, \frac{-1}{2}$$

$$x = 2n\pi \pm \frac{\pi}{3} = \left(2n + \frac{1}{3}\right)\pi, \left(2n \frac{-1}{3}\right)\pi$$

$$x = 2n\pi \pm \frac{2\pi}{3} = \left(2n + \frac{2}{3}\right)\pi, \left(2n \frac{-2}{3}\right)\pi$$

$$y = \left\{\frac{\pi}{3}, \frac{2\pi}{3}, \pi, \frac{4\pi}{3}, ...\right\}$$

Z: $g(x) = 0$

$\cos(2\pi \sin x) = 0$

$2\pi \sin x = (2n+1)\pi/2 \quad\quad n \in I$

$$\sin x = \frac{(2n+1)}{4}$$

$$\sin x = \frac{1}{4}, \frac{-1}{4}, \frac{3}{4}, \frac{-3}{4}$$

$x = n\pi \pm (-1)^n \sin^{-1}(1/4)$

or $x = n\pi \pm (-1)^n \sin^{-1}(3/4)$

W: $g'(x) = 0$

$-\sin(2\pi \sin x).\ 2\pi \cos x = 0$

$\sin(2\pi \sin x) = 0$ or $\cos x = 0$

$2\pi \sin x = n\pi$ $\quad\quad\quad\quad x = (2n+1)\frac{\pi}{2}$

$\sin x = n/2$

$\therefore \sin x = 0, ½, \frac{-1}{2}, 1, -1$

$\therefore x = n\pi$

or $x = n\pi \pm (-1)^n \frac{\pi}{6}$

or $x = 2n\pi \pm \frac{\pi}{2}$

Ans. Q.1 : (II) Q, (T)

Q.2 : (IV), P,R,S

Let the circle $C_1: x^2 + y^2 = 9$ and $C_2: (x-3)^2 + (y-4)^2 = 16$ intersect at the points X and Y. Suppose that another circle $C_3: (x-h)^2 + (y-k)^2 = r^2$ satisfies the following conditions.

(i) Centre of C_3 is collinear with the centres of C_1 & C_2,

(ii) C_1 & C_2 both lie inside C_3 and

(iii) C_3 touches C_1 at M and C_2 at N

Let the line through X and Y intersect C_3 at Z and W and let a common tangent of C_1 & C_3 be a tangent to the parabola $x^2 = 8\alpha y$

There are some expressions given in the list-I, whose values are given in list-II below :

List I	**List II**
(I) $2h + k$	(P) 6
(II) $\dfrac{\text{Length of ZW}}{\text{Length of XY}}$	(Q) $\sqrt{6}$
(III) $\dfrac{\text{Area of triange MZN}}{\text{Area of triangle ZMW}}$	(R) $\dfrac{5}{4}$
(IV) α	(S) $\dfrac{21}{5}$
	(T) $2\sqrt{6}$
	(U) $\dfrac{10}{3}$

माना कि वृत्त (circle) $C_1: x^2 + y^2 = 9$ और वृत्त $C_2: (x-3)^2 + (y-4)^2 = 16$, एक दूसरे को बिन्दुओं X और Y पर काटते हैं। मान लीजिये एक और वृत्त $C_3: (x-h)^2 + (y-k)^2 = r^2$ निम्नलिखित शर्तों को सन्तुष्ट करता है :

(i) C_3 का केन्द्र (centre), C_1 और C_2 के केन्द्रों के संरेख (collinear) है।

(ii) C_1 और C_2 दोनों C_3 के अन्दर है और

(iii) C_3, C_1 को M और C_2 को N पर स्पर्श करता है।

माना कि X और Y से होकर जाने वाली रेखा C_3 को Z और W पर काटती है तथा C_1 और C_3 की एक उभयनिष्ठ स्पर्श–रेखा (common tangent), परवलय $x^2 = 8\alpha y$ की स्पर्श–रेखा है।

सूची-I (List-I) में कुछ व्यंजक (expression) हैं जिनका मान नीचे दी गयी सूची-II (List-II) में है।

सूची-I	**सूची-II**
(I) $2h + k$	(P) 6
(II) $\dfrac{\text{ZW की लम्बाई}}{\text{XY की लम्बाई}}$	(Q) $\sqrt{6}$

(III) $\dfrac{\text{त्रिभुज MZN का क्षेत्रफल}}{\text{त्रिभुज ZMW का क्षेत्रफल}}$ (R) $\dfrac{5}{4}$

(IV) α (S) $\dfrac{21}{5}$

(T) $2\sqrt{6}$

(U) $\dfrac{10}{3}$

3. Which of the following is the only incorrect combination : **Question ID : 337911216**

निम्न में से कौन सा एकमात्र संयोजन गलत है ?

Options

(1) (III), (R) (2) (IV), (S) (3) (I), (P) (4) (IV), (U)

Ans. 2

S.

4. Which of the following is the only correct combination : **Question ID : 337911215**

निम्न में से कौन सा एकमात्र संयोजन सही है ?

Options

(1) (I), (U) (2) (II), (Q) (3) (I), (S) (4) (II), (T)

Ans. 2

S. $C_1 : x^2 + y^2 = 9$ $C_1\ (0, 0)$ $r_1 = 3$

$C_2 : (x-3)^2 + (y-4)^2 = 16$ $C_2 : (3, 4)$ $r_2 = 4$

$C_3 : (x-h)^2 + (y-k)^2 = r^2$ $C_3 : (h, k)$ $r_3 = r$

$MN = MC_1 + C_1C_2 + C_2N$

$MN = r_1 + \sqrt{9+16} + r_2$

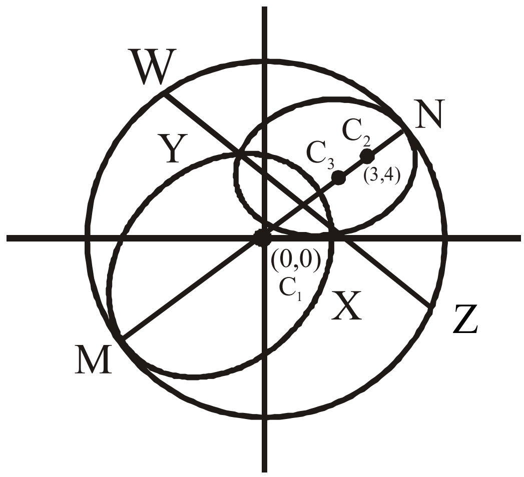
$MN = 3 + 5 + 4 = 12 = 2r \quad \Rightarrow r = 6$

C_3C_1 and C_2 are colinear

$C_1(0, 0)\ C_2(3, 4)$

$\dfrac{4}{3} = \dfrac{y}{x} \quad \Rightarrow y = \dfrac{4x}{3}$

$C_3\ (h, k) \quad k = \dfrac{4h}{3}$



$C_3 : \left(h, \dfrac{4h}{3}\right)$

$C_1C_3 = MC_3 - C_1M$

$$\sqrt{h^2+\frac{16h^2}{9}} = 6-3=3$$

$$\frac{5|h|}{3}=3 \Rightarrow |h|=\frac{9}{5} \Rightarrow h=\frac{9}{5}$$

$$k=\frac{4h}{3}=\frac{4}{3}\times\frac{9}{5}=\frac{12}{5}$$

(I) $\quad 2h+k=\frac{18}{5}+\frac{12}{5}=\frac{30}{5}=6$

(II) Equation of line ZW

$C_1 - C_2 = 0$

$x^2 + y^2 - 9 - x^2 - 9 + 6x - y^2 + 8y - 16 + 16 = 0$

$6x + 8y = 18$

$3x + 4y = 9$

Distance of ZW from $C_1(0, 0)$

$$\left|\frac{-9}{\sqrt{25}}\right|=\frac{9}{5}$$

$$\text{Length XY} = 2\sqrt{3^2-\left(\frac{9}{5}\right)^2}$$

$$=2\sqrt{9-\frac{81}{25}}=\sqrt{\frac{225-81}{25}}$$

$$=2\sqrt{\frac{144}{25}}=\frac{12\times 2}{5}=\frac{24}{5}$$

$$C_3 : \left(\frac{9}{5},\frac{12}{5}\right)$$

Distance of C_3 from ZW

$$\left|\frac{3\times\frac{9}{5}+4\times\frac{12}{5}-9}{5}\right|$$

$$=\left|\frac{\frac{27}{5}+\frac{48}{5}}{5}-9\right|=\left|\frac{27+48-45}{25}\right|=\left|\frac{30}{25}\right|=\frac{6}{5}$$

$$\text{Length of WZ} = \sqrt[2]{36-\frac{36}{25}}$$

$$=12\sqrt{\frac{24}{25}}=\frac{12}{5}\times 2\sqrt{6}=\frac{24}{5}\sqrt{6}$$

$$\frac{\text{Length of ZW}}{\text{Length of XY}} = \frac{\frac{24\sqrt{6}}{5}}{\frac{12}{5}\times 2} = \sqrt{6}$$

(III) Area of $\Delta MZN = \frac{1}{2}(MN)\left(\frac{1}{2}ZW\right)$

$$= \frac{1}{4}\times 12\times\frac{24}{5}\times\sqrt{6}$$

$$= \frac{72\sqrt{6}}{5}$$

Area of $\Delta ZMW = \frac{1}{2}(ZW)$ (OM + Distance of C_1 from ZW)

$$= \frac{1}{2}\times\frac{24\sqrt{6}}{5}\times\left(3+\frac{9}{5}\right)$$

$$= \frac{12\sqrt{6}}{5}\times\left(\frac{24}{5}\right) = \frac{288\sqrt{6}}{25}$$

$$\frac{\text{Area of triangle MZN}}{\text{Area of triangle ZMW}} = \frac{\frac{72\sqrt{6}}{5}}{\frac{288\sqrt{6}}{25}}$$

$$= \frac{72\sqrt{6}}{5}\times\frac{25}{288\sqrt{6}} = \frac{5}{4}$$

(IV) Slope of tangent to C_1 at M $= \frac{-3}{4}$

Equation of tangent

$$y = mx - 3\sqrt{1+m^2}$$

$$y = \frac{-3x}{4} - 3\sqrt{1+\frac{9}{16}}$$

$$y = \frac{-3x}{4} - \frac{3\times 5}{4}$$

$4y = -3x - 15$

$4y + 3x + 15 = 0$ .... (i)

$T \Rightarrow x = my + \frac{2\alpha}{m}$

$3x + 4y + 15 = 0$

$mx - m^2y - 2\alpha = 0$

$$\frac{3}{m} = \frac{-4}{m^2} = \frac{-15}{2\alpha}$$

$3m^2 = 4m$

$3m^2 + 4m = 0$

1. $PQP^{-1} = R$

$PQ = RP$

det P det Q = det R det P

det R = detQ

$\det Q = 48 - 4x^2$

2^{nd} option $\Rightarrow 40 - x^2 + 8$

$R = PQP^{-1}$

$$R = \frac{1}{6}\begin{bmatrix} 6x+12 & 3x+6 & 4-10x \\ 12 & 24 & 8-4x \\ 18x & 0 & 36-6x \end{bmatrix}$$

if $x = 0$

$$R = \frac{1}{6}\begin{bmatrix} 2 & 1 & \frac{2}{3} \\ 0 & 4 & \frac{4}{3} \\ 0 & 0 & 6 \end{bmatrix}$$

$$R\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = 6\begin{bmatrix} 1 \\ a \\ b \end{bmatrix} = \begin{bmatrix} 2 + a + \frac{2}{3}b \\ 4a + \frac{4}{3}b \\ 6b \end{bmatrix}$$

$a \times b = 5$

PQ = QP not possible

if we put

$x = 1$

then

$$R = \frac{1}{6}\begin{bmatrix} 18 & 9 & -6 \\ 12 & 24 & 4 \\ 18 & 0 & 30 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & \frac{3}{2} & -1 \\ 2 & 4 & \frac{1}{3} \\ 3 & 0 & 5 \end{bmatrix}$$

$$R\begin{bmatrix}\alpha\\ \beta\\ 4\end{bmatrix}=\begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$3\alpha+\frac{3}{2}\beta-4=0$$

$$2\alpha+4\beta+\frac{2}{3}4=0$$

$3\alpha + 54 = 0$

$$\alpha_1-\frac{2\alpha}{S},\frac{-3a}{S}$$

not a unit vector

4. $P_1 = P_1^T = P_1^{-1}$
$P_2 = P_2^T = P_2^{-1}$
$P_6 = P_6^T = P_6^{-1}$
and $A^T = A$

$(A+B)^T = A^T + B^T$

$X^T = (P_1AP_1T + ...P_6AP_6^T)^T$

$= X$ so $\times$ is symmetri

trace $\Rightarrow (2 + 0 + 1) + (2 + 0 + 1)$.........6 time

$= 18$

Let $B=\begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$

$XB = P_1AP_1^TB + ...P_6A_6^TB$

$$XB=(P_1+P_2+P_3.......P_6)\begin{bmatrix}6\\ 3\\ 6\end{bmatrix}$$