ASSOCIATION OF MATHEMATICS TEACHRS OF INDIA

Screening Test – Bhaskara Contest

(NMTC-- JUNIOR LEVEL—IX and X Grades)

Saturday, the 15th October 2022

Note:

1. **Fill in the Response Sheet with your Name, Class and the Institution through which you appear, in the specified places.**
2. **Diagrams given are only Visual aids; they are not drawn to scale.**
3. **You may use separate sheets to do rough work.**
4. **Use of Electronic gadgets such as Calculator, Mobile Phone or Computer is not permitted.**
5. **Duration of the Test: 2 pm to 4 pm (2 hours).**

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01. ABCD is a trapezium in which AB is parallel to CD.

If AB = 30 cm, CD = 15 cm, AD = 13 cm and BC = 14 cm, then the area of the trapezium (in square cm) is

a) 263 **b**) 248 **c**) 252 **d**) 293

02. If a + b = 2, where a, b are real and $4^a + 4^b = 6$, then the numerical value of $2^{2(2a-1)} + 2^{2(2b-1)}$ is

a) 8 **b**) 12 **c**) 36 **d**) 1

03. If $\left(x+\frac{1}{x}\right)^2 = 3$, then the value of $x^{33} + x^{23} + x^{27} + x^{17} + 2$ is

a) 1 **b**) 2 **c**) 0 **d**) 4

04. The solution x of the equation $(5x)^x = 5^{5^5}$ is of the form a^b, then $a+b$ is

a) 5 **b**) 10 **c**) 20 **d**) 9

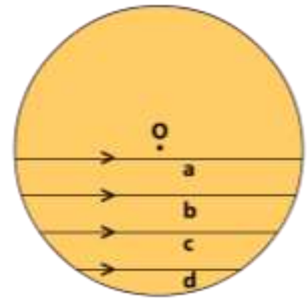
05. ABC is a right-angled isosceles triangle in which $\angle A = 90^{o}$.

D is a point on BC. Then $\frac{BD^2+CD^2}{AD^2}$ is equal to

a) 1 **b**) 2 **c**) 3 **d**) 4

06. There are four equidistant parallel chords of a circle whose lengths are a,b,c,d as shown in the figure.

The numerical value of $\frac{a^2 - d^2}{b^2 - c^2}$ is



a) 1 **b**) 2 **c**) 3 **d**) 4

07. In an arithmetical progression, the entries are integers. The sum of the first two terms is −2. The product of the second and third term is 5. The least number of the term which exceeds 2022 is

a) 508 **b**) 507 **c**) 510 **d**) 515

08. The maximum number of equal pieces that can be cut from the two lengths of wire of 74 cm and 92 cm, with a piece of 2 cm left out of each (in cm) is

a) 16 **b**) 18 **c**) 20 **d**) 14

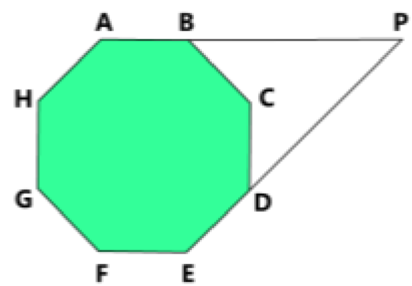
09. The value of $\sqrt{\frac{343^4 + 49^8}{343^6 + 49^7}}$ is

a) 7 **b**) $\frac{1}{7}$ **c**) 49 **d**) $\frac{1}{49}$

10. In the adjoining figure, ABCDEFGH is a regular octagon.

AB, ED are produced to meet at P.

Then the measure of $\angle BPD$ is equal to



a) 45^o **b**) 40^o **c**) 30^o **d**) 35^o

11. If $4x^2 + \frac{1}{x^2} = 2$, then the value of $8x^3 + \frac{1}{x^3}$ is

a) 1 **b**) -1 **c**) 8 **d**) 0

12. For permissible real values of *x, y, z,* the value of the expression

$$\frac{(2x+5y-3z)^3+(2x-5y+3z)^3+2x(2x+5y-3z)(2x-5y+3z)}{x^3} \text{ is}$$

a) 16 **b**) 32 **c**) 64 **d**) 128

13. When $\theta \neq 0^o, 90^o$ the value of the expression

$$\frac{(1+\sec\theta+\tan\theta)(1+\operatorname{cosec}\theta+\cot\theta)}{1+\tan\theta+\cot\theta+\sec\theta+\operatorname{cosec}\theta} \text{ is equal to}$$

a) 1 **b**) 2 **c**) -1 **d**) $\frac{1}{2}$

14. The number of real ordered pairs (*x, y*) which satisfy

$$4^{\frac{x}{y}+\frac{y}{x}} = 32, \textit{ and } \log_3(x-y) = 1-\log_3(x+y) \text{ is}$$

a) 0 **b**) 1 **c**) 2 **d**) 3

15. *a, b* are natural numbers such that $\frac{a}{b}+\frac{b}{a} = a+b$; then

a) *a* is odd and *b* is even.

b) *a, b* are both even.

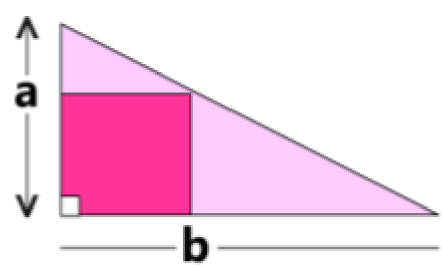
c) Such natural numbers *a* and *b* do not exist.

d) There is exactly one value of '*a*' and '*b*' which satisfy the equation.

Fill in the blanks:

16. The sum of all the roots of the equation $3^{\frac{x+2}{3x-4}} - 7 = 2\left(3^{\frac{5x-10}{3x-4}}\right)$ is ______.

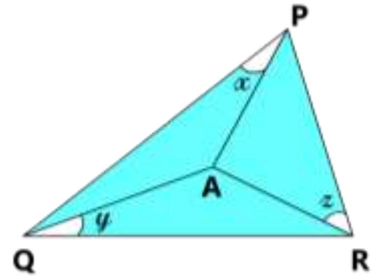
17. A square is inscribed in a right angled Triangle as shown in the figure. One leg of the triangle is twice the other. If the perimeter of the square is 64 cm, then the length of longer leg of the triangle (in cm) is ___________.



18. If $\cos\theta(\tan\theta+2)(2\tan\theta+1) = a\sec\theta + b\sin\theta$, then $a+b$ is equal to _________.

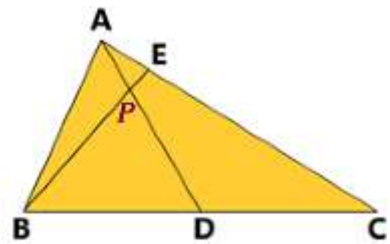
19. The number of real roots of the equation $x^4+x^3+x^2+2=0$ is __________.

20. In the adjoining figure, A is a point inside the triangle PQR, such that AP = AQ = AR. Given $x + 2y = 109^\circ$ and $3x - y = 54^\circ$. Then z (in degrees) is ________.



21. a, b, c are non-zero reals. Given $a + b + c = abc$ and $a^2 = bc$. Then the minimum value of a^2 is _________.

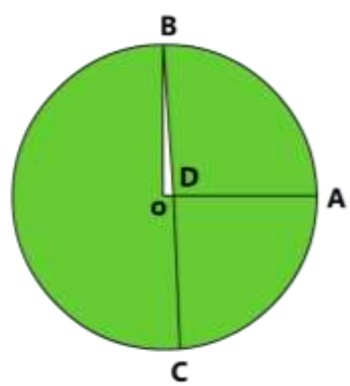
22. In the given figure (not drawn to scale) AC = 2AB. D, E are respectively points on BC and AC such that ∠ABE = ∠CAD. If the triangle PBD is equilateral, and the measure of ∠ABC is x°, then x = _______.



23. The number of solutions x of the equation $(3|x|-3)^2 = |x| + 7$ such that $\sqrt{x(x-3)}$ exists is __________.

24. The difference between the fourth and first terms of a G.P. is 52. The sum of the first three terms is half of this difference. The n^{th} term of this G.P. just exceeds 2022. Then the value of n is __________.

25. In the adjoining figure, OA and OB are two perpendicular radii. With A as centre and AO as radius, an arc is drawn to cut the circle at C. BC cuts OA at D. If ∠ADC = x°, then x = _____.



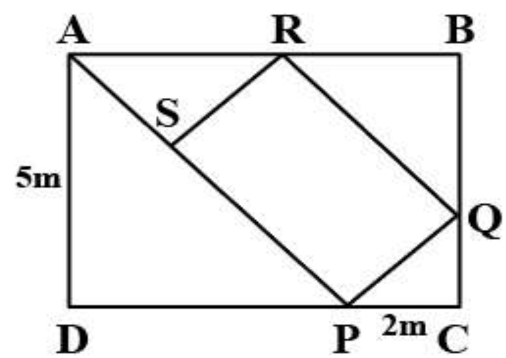
26. Three pipes p_1, p_2 and p_3 can fill a tank in 10 hours. After working at it together for 2 hours, p_1 is closed and p_2 and p_3 can fill it in 16 hours. The time required by p_1 to fill the tank alone is ____________hours.

27. The least number which when divided by 8, 9, 12 and 15 leaves 1 as remainder *each* time is __________.

28. The sum of the digits of a two digit number is 15. If the digits are interchanged, the number of reverse digits is increased by 9. The original two digit number is ________.

29. The number of numbers divisible by 17 between 300 and 500 is _______.

30. ABCD is a non-standard billiards table. AD = 5*m*.
A ball is projected from A along a line which makes 45^{o} with AD.
It bounces at P on DC, again bounces respectively at Q and R as shown and reaches the line AP at S.
The total distance covered by the ball is __________*m*



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