JEE Main July 2021

Question Paper With Text Solution

27 July. | Shift-2

PHYSICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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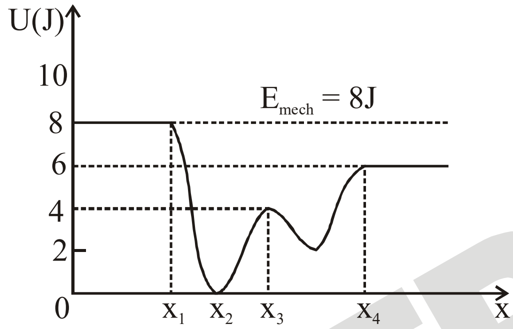
JEE MAIN JULY 2021 | 27TH JULY SHIFT-2

SECTION – A

1. Given below is the plot of a potential energy function U(x) for a system, in which a particle is in one dimensionl motion, while a conservative force F(x) acts on it. suppose that E_{mech} = 8 J, the incorrect statement for this system is :



[where K.E. = kinetic energy]

(1) at x = x_2, K.E. is greatest and the particle is moving at the fastest speed.

(2) at x < x_1, K.E. is smallest and the particle is moving at the slowest speed.

(3) at x > x_4, K.E. is constant throughout the region.

(4) at x = x_3, K.E. = 4 J.

Ans. Official Answer NTA (2)

Sol. By energy conservation theorem ⇒

ME = U + K.E. = Constant

Option – 1, at $x_1 = x_2$ ⇒ U is minimum ....(given in graph)

K.E = ME – U ⇒ as, U is minimum ⇒ K.E is maximum ⇒ speed is maximum

option – 2, for x < x_1 ⇒ U = 8 J ....(given in graph)

K.E = ME – U

⇒ K.E = 8 J – 8 J = O

⇒ $\frac{1}{2}MV^2$ = O ⇒ V = O, Particle remain at rest

option – 3, for x > x_4 ⇒ U = 6J = constant .... (from graph)

K.E = ME – U
K.E = ME – U
K.E = 8 J – 6 J = 2 J = constant
option – 4, at $x = x_3 \Rightarrow U = 4$ J .... (from graph)
K.E = ME – U
K.E = 8 J – 4 J = 4 J

2. Figure A and B show two long straight wires of circular cross–section (a and b with a < b), carrying current I which is uniformly distributed across the cross–section. The magnitude of magnetic field B varies with radius r and can be represented as :

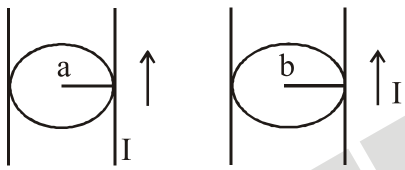
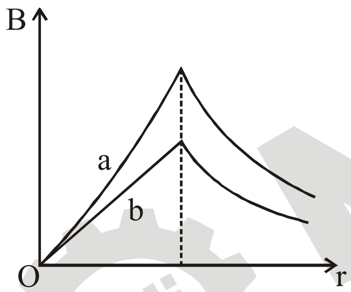


Fig. A Fig. B

(1)


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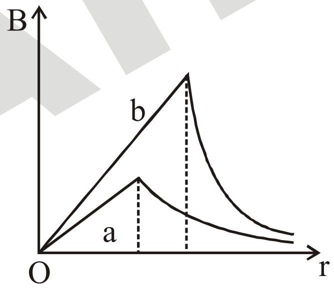

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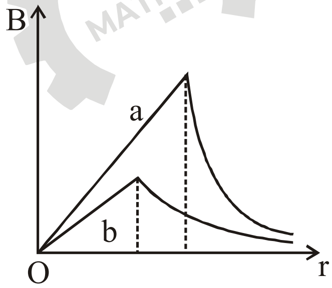
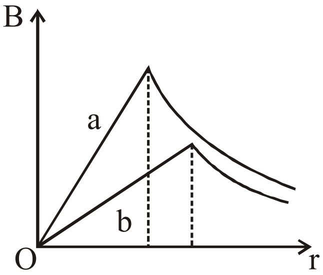


(4)


Ans. Official Answer NTA (4)

Sol. Current density in wire – (a) $\Rightarrow J_a = \dfrac{I}{\pi a^2}$

Current density in wire – (b) $\Rightarrow J_b = \dfrac{I}{\pi b^2}$

as b > a .... (given)

then $J_a > J_b$ ....(i)

magnetic field inside wire $\Rightarrow B = \frac{\mu_0 J}{2}(r)$

$B \propto r \Rightarrow$ curve b/w 'B' and 'r' is straight line

slope of straight line (m) = $\frac{\mu_0 J}{2}$

$m \propto J$

as $J_a > J_b$ $m_a > m_b$

Slope for wire – a > slope for wire – b (straight line)

for wire – a straight line will be till r = a

for wire – b straight line will be till r = b

and a > b

3. A raindrop with radius R = 0.2 mm falls from a cloud at a height h = 2000 m above the ground. Assume that the drop is spherical throughout its fall and the force of buoyance may be neglected, then the terminal speed attained by the raindrop is :

[Density of water $f_w = 1000\ kg\ m^{-3}$

and Density of air $f_a = 1.2\ kg\ m^{-3}$, $g = 10\ m/s^2$

Coefficient of viscosity of air = $1.8 \times 10^{-5}\ Nsm^{-2}$]

(1) $14.4\ ms^{-1}$ (2) $4.94\ ms^{-1}$ (3) $250.6\ ms^{-1}$ (4) $43.56\ ms^{-1}$

Ans. Official Answer NTA (2)

Sol. $F_{net} = 0$

$6\pi\eta rv = mg$

$$6\pi\eta rv = \left(\frac{4}{3}\pi r^3\right)P_w g$$

$$V = \frac{2P_w r^2 g}{9\eta} = \frac{2(1000)(2\times10^{-4})^2(10)}{9\times1.8\times10^{-5}}$$

$V = 4.94\ ms^{-1}$

4. An electron and proton are separated by a large distance. The electron starts approaching the proton with energy 3eV. The proton captures the electron and forms a hydrogen atom in second excited state. The resulting photon is incident on a photosensitive metal of threshold wavelength 4000 Å. What is the maximum kinetic energy of the emitted photoelectron ?

(1) 7.61 eV (2) 3.3 eV

(3) No photoelectron would be emitted (4) 1.41 eV

Ans. Official Answer NTA (4)

Sol. Initial energy of electron (E_i) = 3ev

final Total energy of electron in 2nd excited state (n = 3)

$$E_f = -13.6\left(\frac{1^2}{3^2}\right)\text{ev} = -1.51 \text{ ev}$$

Energy of emitted photon (E_p) = $-(\Delta E)_{electron} = -(E_f - E_i)$

$E_P = -(-1.51 - 3)\text{ev} = 4.51\text{ev}$

work function of metal (ϕ) = $\dfrac{hc}{\lambda} = \dfrac{12400}{4000} = 3.1$ ev

$E_P = \phi + KE_{max}$,

$\Rightarrow 4.51 = 3.1 + K.E_{max}$

$\Rightarrow K.E_{max} = 1.41$ ev

5. One mole of an ideal gas is taken through an adiabatic process where the temperature rises from 27ºC to 37ºC. If the ideal gas is composed of polyatomic molecule that has 4 vibrational modes, which of the following is true ?

[R = 8.314 J mol^{-1} K^{-1}]

(1) work done on the gas is close to 582 J

(2) work done on the gas is close to 332 J

(3) work done by the gas is close to 332 J

(4) work done by the gas is close to 52 J

Ans. Official Answer NTA (1)

Sol. For polyatomic gas of 'f' vibrational mode $\Rightarrow$

$C_P = (4 + f)R$ and $C_v = (3 + f)R$

given, f = 4

$\gamma = \frac{C_P}{C_V} = \frac{(4+4)R}{(4+3)R} = \frac{8}{7}$

Work done on gas = $\frac{nR(\Delta T)}{\gamma - 1} = \frac{(1)(8.314)(37-27)}{\frac{8}{7}-1}$

W = 582 J

Work done is positive does mean work is done on gas.

6. A physical quantity 'y' is represented by the formula $y = m^2 r^{-4} g^x l^{-\frac{3}{2}}$

If the percentage errors found in y,m,r,l and g are 18, 1, 0.5, 4 and p respectively, then find the value of x and p.

(1) 4 and ± 3 (2) $\frac{16}{3}$ and $\pm\frac{3}{2}$ (3) 5 and ± 2 (4) 8 and ± 2

Ans. Official Answer NTA (2)

Sol. $y = M^2 r^{-4} g^x l^{-3/2}$

take log both sides

Log (y) = 2 log (m) – 4 log (r) + x log (g) – $\frac{3}{2}$ log (l)

differentiate both sides ⇒

$$\frac{\Delta y}{y} = 2\left(\frac{\Delta m}{m}\right) - 4\left(\frac{\Delta r}{r}\right) + x\left(\frac{\Delta g}{g}\right) - \frac{3}{2}\left(\frac{\Delta l}{l}\right)$$

Multiply by 100% both sides and errors always add, so take positive signs →

$$\frac{\Delta y}{y}\times 100\% = 2\left(\frac{\Delta m}{m}\times 100\%\right) + 4\left(\frac{\Delta r}{r}\times 100\%\right) + x\left(\frac{\Delta g}{g}\times 100\%\right) + \frac{3}{2}\left(\frac{\Delta l}{l}\times 100\%\right)$$

18 = 2 (1) + 4 (0.5) + x (P) + $\frac{3}{2}$ (4)

18 = 10 + xP

xP = 8, check options, product of x and P should be ± 8, because P will be '±'

7. A particle of mass M originally at rest is subjected to a force whose direction is constant but magnitude varies with time according to the relation

$$F = F_0\left[1-\left(\frac{t-T}{T}\right)^2\right]$$

Where F_0 and T are constants. The force acts only for the time interval 2T. The velocity v of the particle after time 2T is :

(1) $4F_0T/3M$ (2) $F_0T/3M$ (3) $2F_0T/M$ (4) $F_0T/2M$

Ans. Official Answer NTA (1)

Sol. $F = ma = F_o\left[1-\left(\frac{t-T}{T}\right)^2\right]$

$$\Rightarrow a = \frac{F_o}{M}\left[1-\left(\frac{t-T}{T}\right)^2\right]$$

$$\Rightarrow a = \frac{dv}{dt} = \frac{F_o}{M}\left[1-\left(\frac{t-T}{T}\right)^2\right]$$

$$\Rightarrow \int_o^v dv = \frac{F_o}{M}\int_o^{2T}\left[1-\left(\frac{t-T}{T}\right)^2\right]dt$$

$$\Rightarrow v-0 = \frac{F_o}{M}\left[t-\frac{T}{3}\left(\frac{t-T}{T}\right)^3\right]_o^{2T} = \frac{F_o}{M}\left[2T-\frac{T}{3}-\left(0+\frac{T}{3}\right)\right]$$

$$\Rightarrow v = \frac{4F_oT}{3M}$$

8. The planet Mars has two moons, if one of them has a period 7 hours, 30 minutes and an orbital radius of 9.0×10^3 km. Find the mass of Mars.

$$\left\{\text{Given } \frac{4\pi^2}{G} = 6\times10^{11}\,N^{-1}m^{-2}kg^2\right\}$$

(1) 3.25×10^{21} kg (2) 5.96×10^{19} kg (3) 7.02×10^{25} kg (4) 6.00×10^{23} kg

Ans. Official Answer NTA (4)

Sol. Time period (T) = 7 hour 30 minutes = (7.5 × 3600) seconds

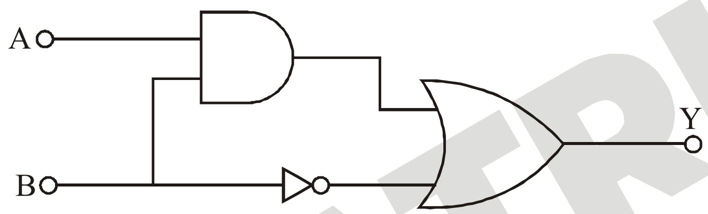
radius (r) = 9×10^3 Km = 9×10^6 meter

$$T^2 = \frac{4\pi^2}{G}\left(\frac{r^3}{M}\right)$$

$$\Rightarrow (7.5 \times 3600)^2 = (6 \times 10^{11})\left(\frac{(9\times10^6)^3}{M}\right)$$

$$\Rightarrow M = 6 \times 10^{23} \text{ kg}$$

9. Find the truth table for the function Y of A and B represented in the following figure.

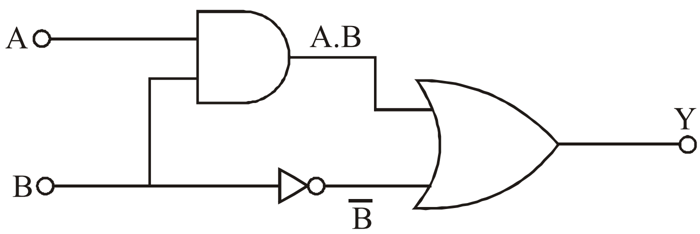


(1) (2)

(3) (4)

Ans. Official Answer NTA (3)

Sol.



$y = A.B + \overline{B}$

10. A 100 Ω resistance, a 0.1 μF capacitor and an inductor are connected in series across a 250 V supply at variable frequency. Calculate the value of inductance of inductor at which resonance will occur. Given that the resonant frequency is 60 Hz.

(1) 70.3 mH (2) 0.70 H (3) 70.3 H (4) 7.03×10^{-5} H

Ans. Official Answer NTA (3)

Sol. At resonance $\Rightarrow X_C = X_L$

$$\Rightarrow \frac{1}{wc} = wL \Rightarrow L = \frac{1}{w^2 c}$$

and $w = 2\pi f = 2\pi(6o) = 120\pi$

$$\Rightarrow L = \frac{1}{(120\pi)^2 (0.1 \times 10^{-6})}$$

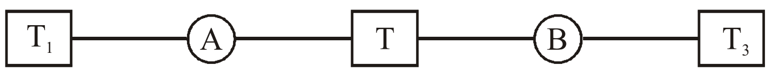
$\Rightarrow L = 70.3$ H

11. Two Carnot engines A and B operate in series such that engine A absorbs heat at T_1 and rejects heat to a sink at temperature T. Engine B absorbs half of the heat rejected by Engine A and rejects heat to the sink at T_3. When workdone in both the cases is equal, the value of T is :

(1) $\frac{1}{3}T_1 + \frac{2}{3}T_3$ (2) $\frac{3}{2}T_1 + \frac{1}{3}T_3$ (3) $\frac{2}{3}T_1 + \frac{1}{3}T_3$ (4) $\frac{2}{3}T_1 + \frac{3}{2}T_3$

Ans. Official Answer NTA (3)

Sol.



Heat absorbs by 'A' = Q_A

Work done by 'A' = $W_A = Q_A\left(1 - \frac{T}{T_1}\right)$

heat rejected by 'A' = $E_A = Q_A\left(\frac{T}{T_1}\right)$

Given, heat obsorbs by 'B' = $Q_B = \frac{1}{2}E_A$

Work done by 'B' = $\Rightarrow W_B = Q_B\left(1-\frac{T_3}{T}\right) = \frac{1}{2}E_A\left(1-\frac{T_3}{T}\right)$

$W_B = \frac{1}{2}\left(Q_A\frac{T}{T_1}\right)\left(1-\frac{T_3}{T}\right)$

given is $W_A = W_B$

$Q_A\left(1-\frac{T}{T_1}\right) = \frac{1}{2}\left(Q_A\frac{T}{T_1}\right)\left(1-\frac{T_3}{T}\right)$

$T = \frac{2}{3}T_1 + \frac{1}{3}T_3$

12. Consider the following statements :

A. Atoms of each element emit characteristics spectrum.

B. According to Bohr's Postulate, an electron in a hydrogen atom, revolves in a certain stationary orbit.

C. The density of nuclear matter depends on the size of the nucleus.

D. A free neutron is stable but a free proton decay is possible.

E. Radioactivity is an indication of the instability of nuclei. T

(1) B and D only (2) A, B and E only

(3) A, B, C, D and E (4) A, C and E only

Ans. Official Answer NTA (2)

Sol. Part of theory

13. Two identical particles of mass 1 kg each go round a circle of radius R, under the action of their mutual gravitational attraction. the angular speed of each particle is :

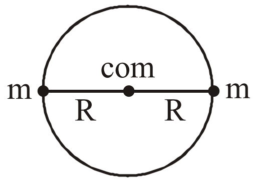
(1) $\sqrt{\frac{2G}{R^3}}$ (2) $\frac{1}{2}\sqrt{\frac{G}{R^3}}$ (3) $\frac{1}{2R}\sqrt{\frac{1}{G}}$ (4) $\sqrt{\frac{G}{2R^3}}$

Ans. Official Answer NTA (2)

Sol. Two particle system revolve around their center of mass.

Mass of both particles are same, so $R_1 = R_2 = R$

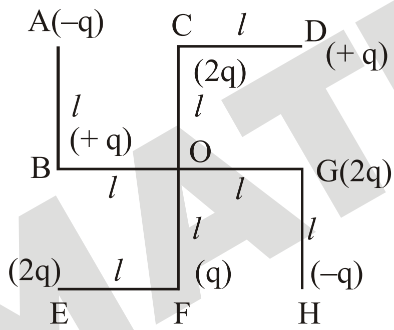
Centripetal force = $M\omega^2(R)$



$$\frac{GMM}{(2R)^2} = m\omega^2(R)$$

$$\omega = \frac{1}{2}\sqrt{\frac{G}{R^3}}$$

14. What will be the magnitude of electric field at point O as shown in figure ? Each side of the figure is l and perpendicular to each other ?



(1) $\frac{q}{4\pi\omega_0(2l)^2}$

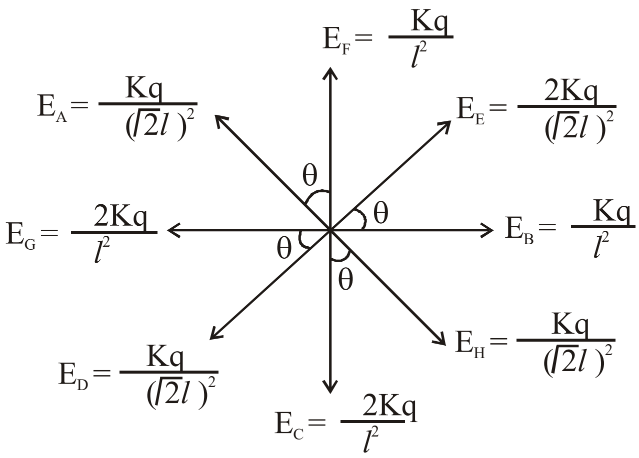
(2) $\frac{1}{4\pi\varepsilon_0}\frac{2q}{2l^2}\left(\sqrt{2}\right)$

(3) $\frac{1}{4\pi\varepsilon_0}\frac{q}{l^2}$

(4) $\frac{1}{4\pi\varepsilon_0}\frac{q}{\left(2l^2\right)}\left(2\sqrt{2}-1\right)$

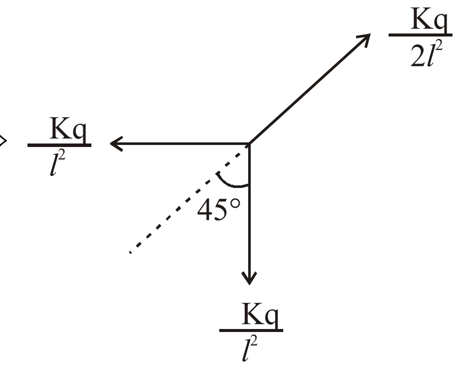
Ans. Official Answer NTA (4)

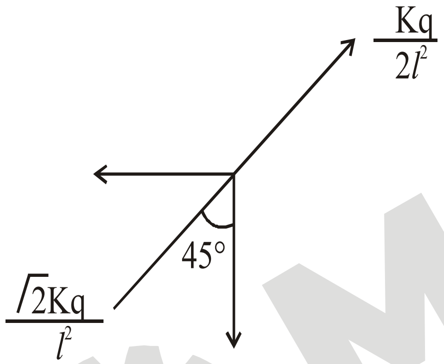
Sol.



$\tan\theta = \frac{l}{l} = 1$

$\theta = 45^o$

find resultant of these vectors ⇒ 

⇒ 

$\Rightarrow E_{net} = \frac{\sqrt{2}Kq}{l^2} - \frac{Kq}{2l^2} = \frac{Kq}{(2l^2)}(2\sqrt{2}-1)$

15. An automobile of mass 'm' accelerates starting from origin and initially at rest, while the engine supplies constant power P. The position is given as a function of time by :

(1) $\left(\frac{9m}{8P}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$ (2) $\left(\frac{8p}{9m}\right)^{\frac{1}{2}} t^{\frac{2}{3}}$ (3) $\left(\frac{9p}{8m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$ (4) $\left(\frac{8p}{9m}\right)^{\frac{1}{2}} t^{\frac{3}{2}}$

Ans. Official Answer NTA (4)

Sol. P = F.V = constant

$\Rightarrow m(a)V = P$

$\Rightarrow m\left(\frac{dv}{dt}\right)V = P$

$$\Rightarrow \int_{o}^{v} Vdv \int_{o}^{t} \frac{P}{m} dt$$

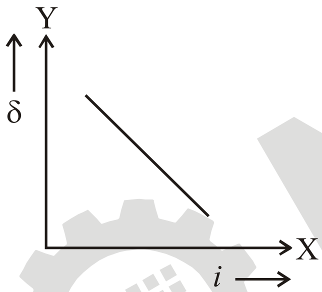
$$\Rightarrow \frac{V^2}{2} = \frac{P}{m}(t)$$

$$\Rightarrow V = \left(\frac{2P}{m}\right)^{1/2} t^{1/2}$$

$$\Rightarrow V = \frac{dx}{dt} \Rightarrow \int dx = \int vdt$$

$$\Rightarrow \int_{o}^{x} dx = \int_{o}^{t} \left(\frac{2P}{M}\right)^{1/2} (t^{1/2})\, dt \Rightarrow \times = \left(\frac{8P}{9M}\right)^{1/2} t^{3/2}$$

16. The expected graphical representation of the variation of angle of deviation 'δ' with angle of incidence '*i*' in a prism is :T

(1)


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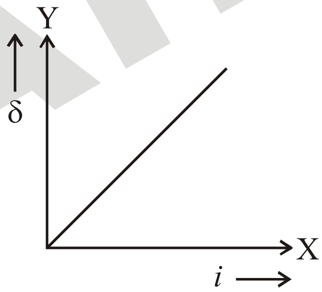

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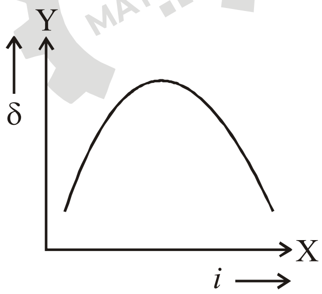

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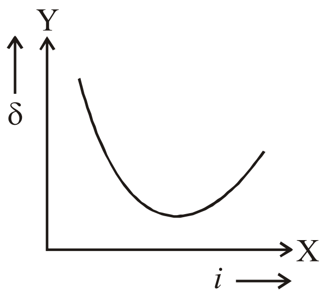


Ans. Official Answer NTA (4)

Sol. Part of theory $\Rightarrow \delta = i + e - A$

17. An object of mass 0.5 kg is executing simple harmonic motion. Its amplitude is 5 cm and time period (T) is 0.2 s. What will be the potential energy of the object at an instant $t = \frac{T}{4}$ s starting from mean postion. Assume that the initial phase of the oscillation is zero.

(1) 6.2×10^{-3} J (2) 1.2×10^{3} J (3) 0.62 J (4) 6.2×10^{3}J

Ans. Official Answer NTA (3)

Sol. $\omega = \frac{2\pi}{T} = \frac{2\pi}{0.2} = 10\pi$

Amplitude (A) = 5 c.m = 5×10^{-2} m

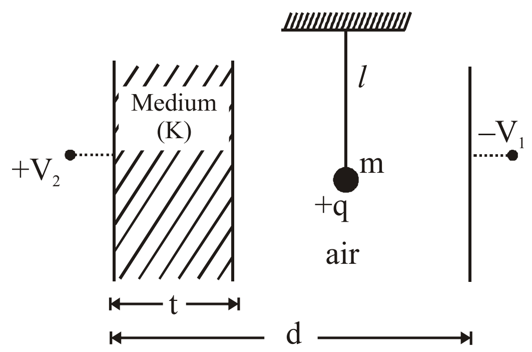
as particle starts from mean position, so at time $t = \frac{T}{4}$, Particle will be at Extreme position

P.E. = $\frac{1}{2}$ $(m\omega^2A^2)$

= $\frac{1}{2}$ $(0.5 \times (10\pi)^2 \times (5 \times 10^{-2}))$

= 0.625 J

18. A simple pendulum of mass 'm', length 'l' and charge '+q' suspended in the electric field produced by two conducting parallel plates as shown. The value of deflection of pendulum in equilibrium position will be :



(1) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_2 - V_1)}{(C_1 + C_2)(d - t)}\right]$

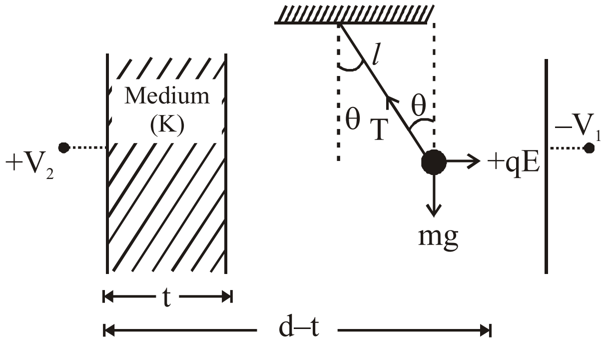
(2) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_1(V_2 - V_1)}{(C_1 + C_2)(d - t)}\right]$

(3) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_1(V_1 + V_2)}{(C_1 + C_2)(d - t)}\right]$

(4) $\tan^{-1}\left[\frac{q}{mg} \times \frac{C_2(V_1 + V_2)}{(C_1 + C_2)(d - t)}\right]$

Ans. Official Answer NTA (4)

Sol.



Let E be electric field in air

T $\sin\theta = qE$

T $\cos\theta = mg$

$$\tan\theta = \frac{qE}{mg}$$

$$Q = \left[\frac{C_1C_2}{C_1+C_2}\right][V_1+V_2]$$

$$E\frac{Q}{A\in_0} = \left[\frac{C_1C_2}{C_1+C_2}\right]\frac{[V_1+V_2]}{A\in_0}$$

$$C_1\frac{\in_0 A}{d-1} \Rightarrow E = \frac{C_2[V_1+V_2]}{(C_1+C_2)(d-t)}$$

$$\text{Now } \theta = \tan^{-1}\left[\frac{q.E}{mg}\right]$$

$$\theta = \tan^{-1}\left[\frac{q}{mg}\times\frac{C_2(V_1+V_2)}{(C_1+C_2)(d-t)}\right]$$

19. Match Lisht I with List II.

List I	List II
(a) Capacitance, C	(i) $M^1 L^1 T^{-3} A^{-1}$
(b) Permittivity of free space, ε_0	(ii) $M^{-1} L^{-3} T^4 A^2$
(c) Permeability of free space, μ_0	(iii) $M^{-1} L^{-2} T^4 A^2$

(d) Electric field, E (iv) $M^1 L^1 T^{-2} A^{-2}$

Choose the correct answer from the options given below :

(1) (a) → (iv), (b) → (ii), (c) → (iii), (d) → (i)

(2) (a) → (iv), (b) → (iii), (c) → (ii), (d) → (i)

(3) (a) → (iii), (b) → (iv), (c) → (ii), (d) → (i)

(4) (a) → (iii), (b) → (ii), (c) → (iv), (d) → (i)

Ans. Official Answer NTA (4)

Sol. $F = \frac{1}{4\pi\varepsilon_o}\left(\frac{q^2}{r^2}\right)$

$$\varepsilon_o = \frac{q^2}{F(r^2)} = \frac{(It)^2}{(ma)(r^2)}$$

$$\varepsilon_o = \frac{(AT)^2}{\left(M\frac{L}{T^2}\right)(L^2)} = M^{-1}L^{-3}T^4A^2$$

(c) velocity of light (c) = $\frac{1}{\sqrt{\mu_o\varepsilon_o}}$

$$\mu_o = \frac{1}{c^2\varepsilon_o} = \frac{1}{(L^1T^{-1})^2(M^{-1}L^{-3}T^4A^2)} = M^1L^1T^{-2}A^{-2}$$

(d) $E = \frac{F}{q} = \frac{F}{It} = \frac{M^1L^1T^{-2}}{AT} = M^1L^1T^{-3}A^{-1}$

(a) $C = \frac{A\varepsilon_o}{d} = \frac{(L^2)M^{-1}L^{-3}T^4A^2}{L^1} = M^{-1}L^{-2}T^4A^2$

20. The resistance of a conductor at 15ºC is 16Ω and at 100ºC is 20Ω. What will be the temperature coefficient of resistance of the conductor ?

(1) 0.042ºC^{-1} (2) 0.033ºC^{-1} (3) 0.003ºC^{-1} (4) 0.010ºC^{-1}

Ans. Official Answer NTA (3)

Sol. $R = R_o(1 + \alpha\Delta T)$

$20 = 16\,(1 + \alpha\,(100 - 15))$

$$\frac{20}{16} = 1 + 85(\alpha)$$

$\alpha = \frac{1}{4 \times 85} = 0.003°C^{-1}$

SECTION - B

1. The K_{α} X-ray of molybdenum has wavelength 0.071 nm. If the energy of a molybdenum atom with a K electron knocked out is 27.5 keV, the energy of this atom when an L electron is knocked out will be ____keV.(Round off to the nearest integer)

[$h = 4.14 \times 10^{-15}$eVs, $c = 3 \times 10^{8}ms^{-1}$]

Ans. Official Answer NTA (10)

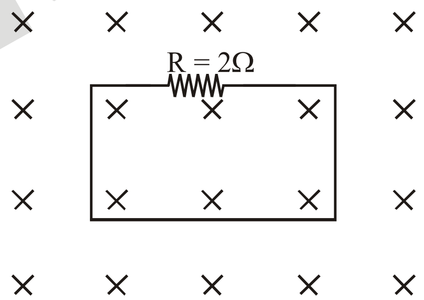
Sol. $E_{k\alpha} = E_K - E_L$

$\Rightarrow \frac{hc}{\lambda_{k\alpha}} = E_K - E_L$

$\Rightarrow \frac{4.14 \times 10^{-15} \times 3 \times 10^{8}}{0.071 \times 10^{-9}} = 27.5 \times 10^{3} - E_L$

$\Rightarrow E_L = 10 \times 10^{3}$ ev = 10 Kev

2. In the given figure the magnetic flux through the loop increases according to the relation $\phi_{\beta}(t) = 10t^2 + 20t$, where ϕ_{β} is in milliwebers and t is in seconds.
The magnitude of current through R = 2Ω resistor at t = 5s is ______mA.



Ans. Official Answer NTA (60)

Sol. Induced *e*mf (V) = $\frac{d\phi}{dt} = \frac{d}{dt}(10t^2 + 20t)$

V = 20 t + 20

at, t = 5 sec $\Rightarrow$ V = 20(5) + 20 = 120 Volt

$I = \frac{V}{R} = \frac{120}{2} = 60$ Ampere

3. For the circuit shown, the value of current at time t = 3.2 s will be _____A.

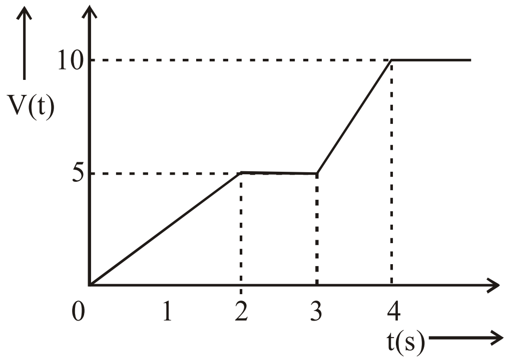


Figure 1

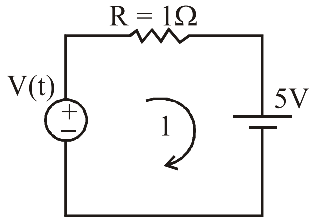
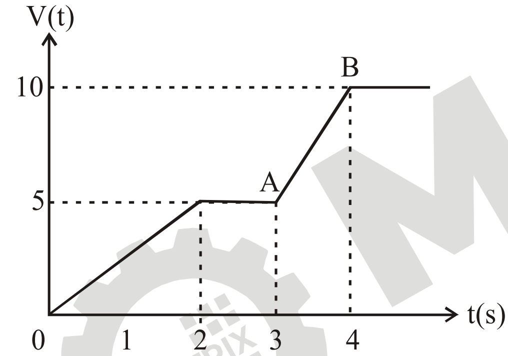


Figure 2

[Voltage distribution V(t) is shown by Fig. (1) and the circuit is shown in Fig.(2)]

Ans. Official Answer NTA (1)

Sol.



Co-ordinates of points A & B

$A \equiv (3, 5)$, $B \equiv (4, 10)$

equation of line AB $\Rightarrow$

$$V - 5 = \frac{10-5}{4-3}(t-3)$$

$V = 5t - 10$

at t = 3.2 second, $V(t) = 5\,(3.2) - 10$

$V(t) = 6$ Volt

net potential of circuit (V) = V(t) – 5

$V = 6 - 5$

$V = 1$ volt

current through Resistor (I) = $\frac{V}{R}$

$I = \frac{1}{1} = 1$ Ampere

4. The difference in the number of waves when yellow light propagates through air and vacuum columns of the same thickness is one. The thickness of the air column is _____mm.

[Refractive index of air = 1.0003, wavelength of yellow light in vacuum = 6000Å]

Ans. Official Answer NTA (2)

Sol. $\lambda \propto \frac{1}{\mu} \Rightarrow \frac{\lambda_{vaccum}}{\lambda_{air}} = \frac{\mu_{air}}{\mu_{vaccum}} = \frac{1.0003}{1}$

$\lambda_{air} = \frac{\lambda_{vaccum}}{1.003} = \frac{6000}{1.0003}$ Å

as $\lambda_{air} < \lambda_{vaccum}$, so number of waves in air will be greater than number of waves in vaccum
suppose number of waves in vaccume = n

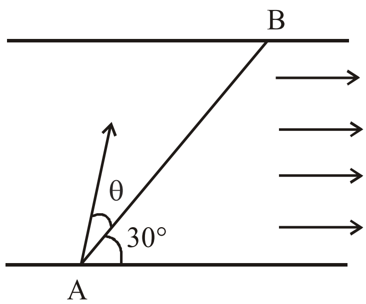
then number of waves in air will be = n + 1

thickness of column = $t = n\ \lambda_{vaccume} = (n+1)\lambda_{air}$

$\Rightarrow n(6000) = (n+1)\left(\frac{6000}{1.0003}\right) \Rightarrow n = \frac{10^4}{3}$

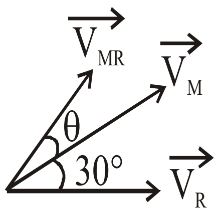
$t = n\,\lambda_{vaccum} = \frac{10^4}{3}(6000 \times 10^{-9}) = 2 \times 10^{-3}\,m = 2mm$

5. A swimmer wants to cross a river from point A to point B. Line AB makes an angle of 30° with the flow of river. Magnitude of velocity of the swimmer is same as that of the river. The angle θ with the line AB should be____°, so that the swimmer reaches point B.



Ans. Official Answer NTA (30)

Sol.



$\overrightarrow{V_{MR}} = \overrightarrow{V_M} - \overrightarrow{V_R}$

$\overrightarrow{V_M} = \overrightarrow{V_{MR}} + \overrightarrow{V_R}$

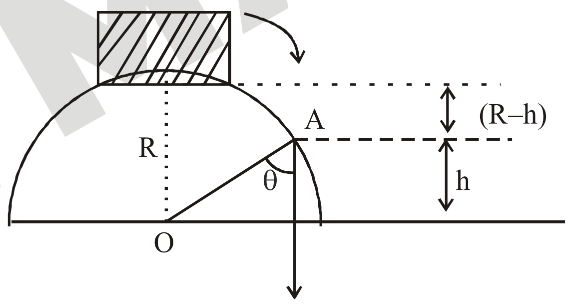
given, $\left|\overrightarrow{V_{MR}}\right| = \left|\overrightarrow{V_R}\right|$

So , $\overrightarrow{V_M}$ will be along the angle bisector of $\overrightarrow{V_{MR}}$ and $\overrightarrow{V_R}$

so, $\theta = 30°$

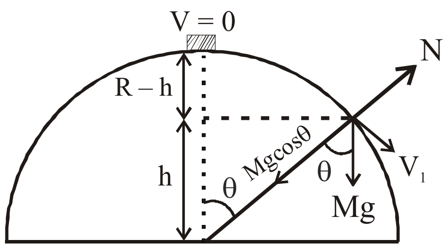
6. A small block slides down from the top of hemisphere of radius R = 3 m as shown in the figure. The height 'h' at which the block will lose contact with the surface of the sphere is ______m.

(Assume there is no friction between the block and the hemisphere)



Ans. Official Answer NTA (2)

Sol.



$$\text{Cos}\theta = \frac{h}{R} \quad ....(1)$$

Apply ECT for $V_1 \Rightarrow Mg(R-h) = \frac{1}{2}M(V_1^2 - 0^2)$

$$\Rightarrow V_1^2 = 2g(R-h) \text{ ....(2)}$$

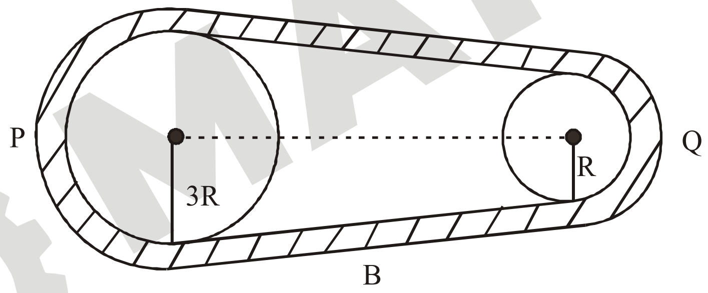
Apply equation of centripetal force $\Rightarrow$

$$Mg\cos\theta = \frac{MV_1^2}{R}$$

From equation (1) and (2) $\Rightarrow Mg\left(\frac{h}{R}\right) = \frac{M}{R}(2g(R-h))$

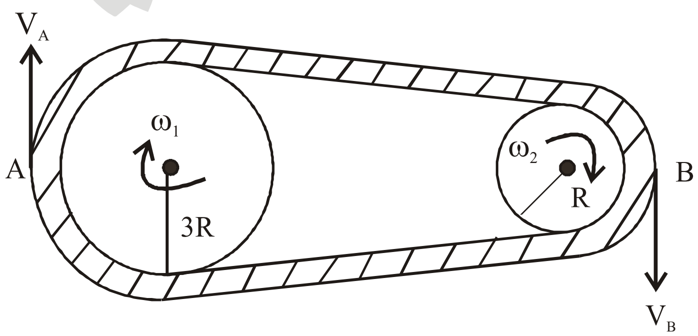
$$\Rightarrow h = \frac{2R}{3} = \frac{2}{3}(3) = 2\,\text{meter}$$

7. In the given figure, two wheels P and Q are connected by a belt B. The radius of P is three times as that of Q. In case of same rotational kinetic energy, the ratio of rotational inertias $\left(\frac{I_1}{I_2}\right)$ will be x : 1. The value of x will be ________.



Ans. Official Answer NTA (9)

Sol.



Speed of point A and Point B will be equal to speed of belt

$V_A = V_B$

$\omega_1(3R) = \omega_2(R)$

$\left(\frac{\omega_2}{\omega_1}\right) = 3$ ....(i)

given, $K.E_1 = K.E_2$ ....Rotational kinetic energy

$\frac{1}{2}(I_1)(\omega_1^2) = \frac{1}{2}(I_2)(\omega_2^2)$

$\frac{I_1}{I_2} = \left(\frac{\omega_2}{\omega_1}\right)^2$

$= (3)^2$ .... from equation (1)

$= 9$

8. The water is filled upto height of 12 m in a tank having vertical sidewalls. A hole is made in one of the walls at a depth 'h' below the water level. The value of 'h' for which the emerging stream of water strikes the ground at the maximum range is ______m.

Ans. Official Answer NTA (6)

Sol. For maximum Range, hole should be at midpoint of height of tank

$h = \frac{12}{2} = 6$ meter

9. A particle executes simple harmonic motion represented by displacement function as

$x(t) = A\sin(\omega t + \phi)$

If the position and velocity of the particle at t = 0 s are 2cm and 2ω cms^{-1} respectively, then its amplitude is $x\sqrt{2}$ cm where the value of x is ______.

Ans. Official Answer NTA (2)

Sol. Velocity at displacement 'x' from mean position.

$V = \omega\sqrt{A^2 - X^2}$

$(2\omega)^2 = \omega\left(\sqrt{A^2 - (2)^2}\right.$

$A^2 = 8 \Rightarrow A = 2\sqrt{2} = x\sqrt{2}$

$x = 2$

10. The maximum amplitude for an amplitude modulated wave is found to be 12 V while the minimum amplitude is found to be 3V. The modulation index is 0.6x where x is _____.

Ans. Official Answer NTA (1)

Sol. $A_{max} = 12, A_{min} = 3$

Modulation index (m) = $\frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

$m = \frac{12-3}{12+3} = \frac{9}{15}$

$m = 0.6 \equiv 0.6(x)$

$x = 1$