JEE Main July 2021

Question Paper With Text Solution

25 July. | Shift-2

PHYSICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JULY 2021 | 25TH JULY SHIFT-2

SECTION – A

1. Consider a planet in some solar system which has a mass double the mass of earth and density equal to the average density of earth. If the weight of an object on earth is W, the weight of the same object on that planet will be :

(1) 2 W

(2) $2^{\frac{1}{3}}$ W

(3) W

(4) $\sqrt{2}$ W

Ans. Official Answer NTA (2)

Sol. g(earth) = $\frac{GM}{R^2}$

For planet $M_1 = 2M$

$\Rightarrow \frac{4}{3}\pi R_1^3 \times \sigma = 2\left(\frac{4}{3}\pi R^3 \sigma\right)$

$\Rightarrow R_1 = (2)^{1/3} R$

$\therefore g' = \frac{GM_1}{R_1^2}$ So $g'/g = \frac{M_1 R^2}{MR_1^2} = 2^{1/3}$

$\because \frac{W^1}{W} = \frac{mg^1}{mg}$ $\Rightarrow \frac{W^1}{W} = 2^{1/3}$

2. An electron moving with speed *v* and a photon moving with speed c, have same D-Broglie wavelength. The ratio of kinetic energy of electron to that of photon is :

(1) $\frac{v}{3c}$

(2) $\frac{3c}{v}$

(3) $\frac{2c}{v}$

(4) $\frac{v}{2c}$

Ans. Official Answer NTA (4)

Sol. For electron $K_e = \frac{1}{2} m_e v_e^2 = \frac{Pv}{2}$

For photon $K_{ph} = Pc$

$\therefore \frac{K_e}{K_{Ph}} = \frac{Pv/2}{Pc} = \frac{v}{2c}$

3. When radiation of wavelength λ is incident on a metallic surface, the stopping potential of ejected photoelectrons is 4.8 V. If the same surface is illuminated by radiation of double the previous wavelength, then the stopping potential becomes 1.6 V. The threshold wavelength of the metal is :

(1) 4 λ

(2) 6 λ

(3) 2 λ

(4) 8 λ

Ans. Official Answer NTA (1)

Sol. When wavelength is λ

$4.8 \text{ eV} = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$ ....(1) [λ_0 is threshold wavelength]

When wavelength is doubled

$1.6 \text{ eV} = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$ ....(2)

From equation (1) and (2)

$\Rightarrow \lambda_0 = 4\lambda$

4. Two spherical soap bubbles of radii r_1 and r_2 in vacuum combine under isothermal conditions. The resulting bubble has a radius equal to :

(1) $\sqrt{r_1^2 + r_2^2}$

(2) $\frac{r_1 r_2}{r_1 + r_2}$

(3) $\frac{r_1 + r_2}{2}$

(4) $\sqrt{r_1 \ r_2}$

Ans. Official Answer NTA (1)

Sol. Ideal gas eqvation PV = nRT

excess pressure inside a soap bubble = $\frac{4T}{r}$

conserving number of moles

$$n_1 + n_2 = n$$

$$\Rightarrow \quad \frac{P_1V_1}{RT} + \frac{P_2V_2}{RT} = \frac{PV}{RT}$$

$$\Rightarrow \quad \left(\frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^3 + \left(\frac{4T}{r_2}\right)\frac{4}{3}\pi r_2^3 = \left(\frac{4T}{r}\right)\frac{4}{3}\pi r^3$$

$$\Rightarrow \quad r_1^2 + r_2^2 = r^2$$

$$\Rightarrow \quad r = \sqrt{r_1^2 + r_2^2}$$

5. A prism of refractive index μ and angle of prism A is placed in the position of minimum angle of deviation. If minimum angle of deviation is also A, then in terms of refractive index value of A is :

(1) $2\cos^{-1}\left(\frac{\mu}{2}\right)$

(2) $\sin^{-1}\left(\sqrt{\frac{\mu-1}{2}}\right)$

(3) $\cos^{-1}\left(\frac{\mu}{2}\right)$

(4) $\sin^{-1}\left(\frac{\mu}{2}\right)$

Ans. Official Answer NTA (1)

Sol. $$\mu = \frac{\sin\left(\frac{A+\delta_{min}}{2}\right)}{\sin\left(\frac{A}{2}\right)}$$

$$\Rightarrow \mu = \frac{\sin(A)}{\sin\left(\frac{A}{2}\right)}$$

$\Rightarrow \frac{\mu}{2} = \cos\left(\frac{A}{2}\right)$

$\Rightarrow \frac{\mu}{2} = \cos\left(\frac{A}{2}\right)$

$\Rightarrow A = 2\cos^{-1}\left(\frac{\mu}{2}\right)$

6. Two vectors $\vec{X}$ and $\vec{Y}$ have equal magnitude. The magnitude of $(\vec{X} - \vec{Y})$ is n times the magnitude of $(\vec{X} + \vec{Y})$. The angle between $\vec{X}$ and $\vec{Y}$ is :

(1) $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$

(2) $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$

(3) $\cos^{-1}\left(\frac{n^2+1}{-n^2-1}\right)$

(4) $\cos^{-1}\left(\frac{-n^2-1}{n^2-1}\right)$

Ans. Official Answer NTA (2)

Sol. $|\vec{x} - \vec{y}| = n|\vec{x} + \vec{y}|$

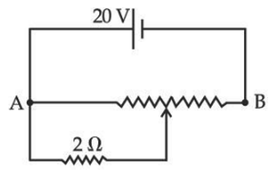
Squaring both sides

$\Rightarrow$ $x^2 + y^2 - 2\,x\,y\cos\theta = n^2(x^2 + y^2 + 2xy\cos\theta)$

$\Rightarrow$ $2 - 2\cos\theta = n^2(2 + 2\cos\theta)$

$\Rightarrow$ $\cos\theta = \left(\frac{1-n^2}{n^2+1}\right)$

7. The given potentiometer has its wire of resistance 10 Ω. When the sliding contact is in the middle of the potentiometer wire, the potential drop across 2 Ω resistor is :



(1) 10 V

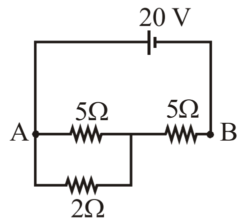
(2) 5 V

(3) 40/9 V

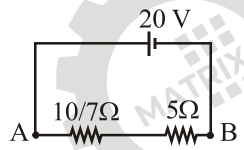
(4) 40/11 V

Ans. Official Answer NTA (3)

Sol. Equivalent circuit diagram



This can be reduced to



Potential drop on $\frac{10}{7}\Omega$ is $= \frac{20}{\left(\frac{10}{7}+5\right)} \times \frac{10}{7} = \frac{40}{9}$ V

8. The instantaneous velocity of a particle moving in a straight line is given as $v = \alpha t + \beta t^2$, where α and β are constants. The distance travelled by the particle between 1 s and 2 s is :

(1) $\frac{3}{2}\alpha + \frac{7}{3}\beta$

(2) $3\alpha + 7\beta$

(3) $\frac{3}{2}\alpha + \frac{7}{2}\beta$

(4) $\frac{\alpha}{2} + \frac{\beta}{3}$

Ans. Official Answer NTA (1)

Sol. $v = \alpha t + \beta t^2$

displacement $\frac{ds}{dt} = \alpha t + \beta t^2$

$\Rightarrow s = \left(\frac{\alpha t^2}{2} + \frac{\beta t^3}{3}\right)\Big|_1^2$

$\Rightarrow s = \frac{3}{2}\alpha + \frac{7}{3}\beta$

9. A force $\vec{F} = \left(40\hat{i} + 10\hat{j}\right)N$ acts on a body of mass 5 kg. If the body starts from rest, its position vector $\vec{r}$ at time t = 10 s, will be :

(1) $\left(100\hat{i} + 400\hat{j}\right)m$

(2) $\left(400\hat{i} + 400\hat{j}\right)m$

(3) $\left(100\hat{i} + 100\hat{j}\right)m$

(4) $\left(400\hat{i} + 100\hat{j}\right)m$

Ans. Official Answer NTA (4)

Sol. $\vec{a} = 8\hat{i} + 2\hat{j}$

$\therefore \vec{v} = 8t\,\hat{i} + 2t\,\hat{j}$

and position $\vec{r} = 4t^2\hat{i} + t^2\hat{j}$

$\therefore \vec{r}$ (at t = 10 s) = $\left(400\hat{i} + 100\hat{j}\right)m$

10. A balloon was moving upwards with a uniform velocity of 10 m/s. An object of finite mass is dropped from the balloon when it was at a height of 75 m from the ground level. The height of the balloon from the ground when object strikes the ground was around : (takes the value of g as 10 m/s^2)

(1) 125 m

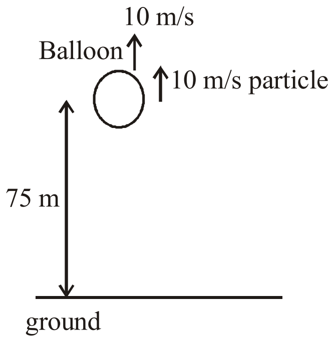
(2) 300 m

(3) 250 m

(4) 200 m

Ans. Official Answer NTA (1)

Sol.



Using $s = ut + \frac{1}{2} at^2$ for particle

$\Rightarrow -75 = 10\,t - 5\,t^2$

$\Rightarrow t = 5$ s

Height of balloon = 75 + 5 × 10 = 125 m

11. A 10 Ω resistance is connected across 220 V – 50 Hz AC supply. The time taken by the current to change from its maximum value to the rms value is :

(1) 4.5 ms

(2) 3.0 ms

(3) 1.5 ms

(4) 2.5 ms

Ans. Official Answer NTA (4)

Sol. $I = I_0 \cos(\omega t)$

Current is at peak value at t = 0

it will be at rms value $\frac{I_0}{\sqrt{2}}$ at time t

$\Rightarrow \frac{I_0}{\sqrt{2}} = I_0 \cos(\omega t)$

$\Rightarrow \omega t = \pi / 4$

$\Rightarrow t = \frac{1}{400} s = 2.5$ ms

12. If q_f is the free charge on the capacitor plates and q_b is the bound charge on the dielectric slab of dielectric constant k placed between the capacitor plates, then bound charge q_b can be expressed as :

(1) $q_b = q_f\left(1+\frac{1}{k}\right)$

(2) $q_b = q_f\left(1-\frac{1}{k}\right)$

(3) $q_b = q_f\left(1-\frac{1}{\sqrt{k}}\right)$

(4) $q_b = q_f\left(1+\frac{1}{\sqrt{k}}\right)$

Ans. Official Answer NTA (2)

Sol. $q_b = q_f\left(1-\frac{1}{k}\right)$

Derivation is done in class notes

13. A heat engine has an efficiency of $\frac{1}{6}$. When the temperature of sink is reduced by 62º C, its efficiency get doubled. The temperature of the source is :

(1) 62º C

(2) 99º C

(3) 124º C

(4) 37º C

Ans. Official Answer NTA (2)

Sol. $\eta = 1-\frac{T_2}{T_1}$

$\Rightarrow \frac{1}{6} = 1-\frac{T_2}{T_1} \quad \Rightarrow \frac{T_2}{T_1} = \frac{5}{6}$ ......(i)

When T_2 is reduced by 62ºC

$\Rightarrow \frac{1}{3} = 1-\frac{(T_2-62)}{T_1} \quad \Rightarrow \frac{2}{3} = \frac{T_2-62}{T_1}$ ......(ii)

from (1) and (2)

T_1 = 372 k = 99ºC

14. Two ions having same mass have charges in the ratio 1 : 2. They are projected normally in a uniform magnetic field with their speeds in the ratio 2 : 3. The ratio of the radii of their circular trajectories is :

(1) 2 : 3

(2) 3 : 1

(3) 4 : 3

(4) 1 : 4

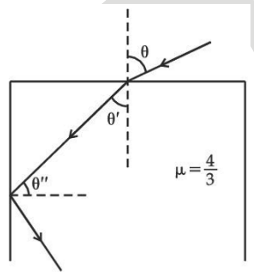
Ans. Official Answer NTA (3)

Sol. $\frac{q_1}{q_2} = \frac{1}{2}$ & $\frac{v_1}{v_2} = \frac{2}{3}$

$\because r = \frac{mv}{qB}$

$\therefore \frac{r_1}{r_2} = \frac{v_1 q_2}{q_1 v_2} = \frac{4}{3}$

15. A ray of light entering from air into a denser medium of refractive index $\frac{4}{3}$, as shown in figure. The light ray suffers total internal reflection at the adjacent surface as shown. The maximum value of angle θ should be equal to :



(1) $\sin^{-1}\frac{\sqrt{7}}{4}$

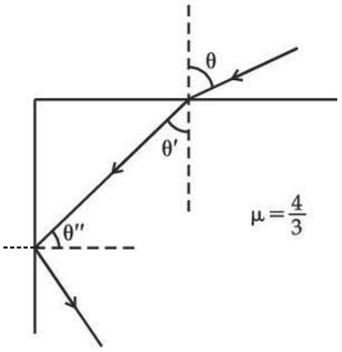
(2) $\sin^{-1}\frac{\sqrt{5}}{4}$

(3) $\sin^{-1}\frac{\sqrt{5}}{3}$

(4) $\sin^{-1}\frac{\sqrt{7}}{3}$

Ans. Official Answer NTA (4)

Sol.



For maximum value of θ the angle θ'' = θ_C (critical angle)

$\Rightarrow \theta' = \sin^{-1}\left(\frac{3}{4}\right)$

$\sin\theta'' = \frac{3}{4}$ ......(1)

$\because \theta' + \theta'' = 90^o$

$\Rightarrow \theta'' = 90^o - \theta'$

$\Rightarrow \sin(\theta'') = \cos\theta'$

$\Rightarrow \cos(\theta') = \frac{3}{4}$ and $\sin\theta' = \frac{\sqrt{7}}{4}$ Using(1)

For refraction at 1st boundary using snells law

$\sin\theta = \frac{4}{3}\sin(\theta')$

$\Rightarrow \theta = \sin^{-1}\left(\frac{\sqrt{7}}{3}\right)$

16. The force is given in terms of time t and displacement x by the equation

F = A cos Bx + C sin Dt

The dimensional formula of $\frac{AD}{B}$ is :

(1) $[M^1 L^1 T^{-2}]$

(2) $[M^2 L^2 T^{-3}]$

(3) $[M^1 L^2 T^{-3}]$

(4) $[M^0 L T^{-1}]$

Ans. Official Answer NTA (3)

Sol. A and C have dimensions same as force $[M^1 L^1 T^{-2}]$

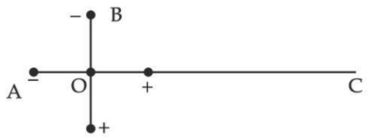
Dimension of B is $[L^{-1}]$

Dimension of D is $[T^{-1}]$

$\therefore$ dimension of $\frac{AD}{B}$ are $[M^1 L^2 T^{-3}]$

17. Two ideal electric dipoles A and B, having their dipole moment p_1 and p_2 respectively are placed on a plane with their centres at O as shown in the figure. At point C on the axis of dipole A, the resultant electric field is making an angle of 37º with the axis.

The ratio of the dipole moment of A and B, $\frac{p_1}{p_2}$ is : (take $37^o = \frac{3}{5}$)



(1) 3/8

(2) 3/2

(3) 2/3

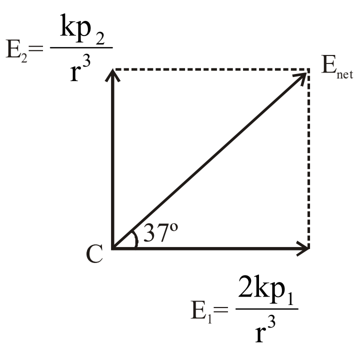
(4) 4/3

Ans. Official Answer NTA (3)

Sol. At point C

Electric field due to p_1 is $\frac{2kp_1}{r^3}$

Electric field due to p_2 is $\frac{kp_2}{r^3}$



$\tan 37^\circ = E_2/E_1$

$\Rightarrow \frac{p_1}{p_2} = \frac{2}{3}$

18. In a simple harmonic oscillation, what fraction of total mechanical energy is in the form of kinetic energy, when the particle is midway between mean and extreme position.

(1) 3/4

(2) 1/3

(3) 1/2

(4) 1/4

Ans. Official Answer NTA (1)

Sol. When particle is midway $KE = \frac{1}{2}k\left(A^2 - x^2\right)$ (x = A/2)

$= \frac{3}{8}kA^2$

$= \frac{3}{4}\left(\frac{1}{2}kA^2\right)$

$= \frac{3}{4}$ of mechanical energy

19. The relation between time t and distance x for a moving body is given as $t = mx^2 + nx$, where m and n are constants. The retardation of the motion is : (Where v stands for velocity)

(1) $2\,mnv^3$

(2) $2\,n^2v^3$

(3) $2\,nv^3$

(4) $2\,mv^3$

Ans. Official Answer NTA (4)

Sol. $mx^2 + nx = t$

differentiating

$\Rightarrow 2mx\frac{dx}{dt} + \frac{ndx}{dt} = 1$

$\Rightarrow \frac{dx}{dt} = \frac{1}{(2mx + n)} = v$ ....(1)

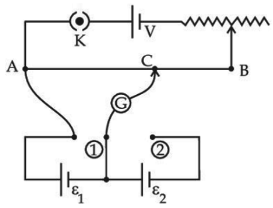
again differentiating

$$\Rightarrow \frac{dv}{dt} = \frac{-2m\ dx/dt}{(2mx+n)^2}$$

$\Rightarrow a = -2mv^3$ using (1)

$\therefore$ retardation = $2mv^3$

20. In the given potentiometer circuit arrangement, the balancing length AC is measured to be 250 cm. When the galvanometer connection is shifted from point (1) to point (2) in the given diagram, the balancing length becomes 400 cm. The ratio of the emf of two cells, $\frac{\varepsilon_1}{\varepsilon_2}$ is :



(1) 4/3

(2) 8/5

(3) 3/2

(4) 5/3

Ans. Official Answer NTA (4)

Sol. When galvanometer connection is at 1

ε_1 = (Potential gradient) × 250 cm ....(1)

When connection is at 2

$\varepsilon_1 + \varepsilon_2$ = (Potential gradient) × 400 cm ....(2)

From (1) and (2)

$\frac{\varepsilon_1}{\varepsilon_2} = 5/3$

SECTION - B

1. A message signal of frequency 20 kHz and peak voltage of 20 volt is used to modulate a carrier wave of frequency 1 MHz and peak voltage of 20 volt. The modulation index will be ———.

Ans. Official Answer NTA (1)

Sol. Modulation index = $\frac{A_m}{A_c} = \frac{20}{20} = 1$

2. A light beam of wavelength 500 nm is incident on a metal having work function of 1.25 eV, placed in a magnetic field of intensity B. The electrons emitted perpendicular to the magnetic field B, with maximum kinetic energy are bent into circular arc of radius 30 cm. The value of B is ——— $\times 10^{-7}$ T. Given hc = 20×10^{-26} J-m, mass of electron = 9×10^{-31} kg

Ans. Official Answer NTA (125)

Sol. $E_{ph} = \frac{hc}{\lambda} = 2.5\text{ eV}$

$\because K(\max) = E_{ph} - \phi = 1.25\text{ eV}$

Radius $r = \frac{\sqrt{2mK}}{qB}$

$\Rightarrow B^2 = \frac{2mK}{r^2q^2}$

$\Rightarrow B = 1.25 \times 10^{-5}$

3. A solid disc of radius 20 cm and mass 10 kg is rotating with an angular velocity of 600 rpm, about an axis normal to its circular plane and passing through its centre of mass. The retarding torque required to bring the disc at rest in 10 s is ——— p $\times 10^{-1}$ Nm.

Ans. Official Answer NTA (4)

Sol. I(disc) = $\frac{mr^2}{2} = \frac{10 \times 0.2^2}{2} = 0.2$

L(Ang. mom) = $I\omega = 0.2\left(\frac{600 \times 2\pi}{60}\right) = 4\pi$

$\because \tau = \frac{\Delta L}{\Delta t} = \frac{4\pi}{10} = 4\pi \times 10^{-1}$

4. From the given data, the amount of energy required to break the nucleus of aluminium

$^{27}_{13}Al$ is ———————— x $\times 10^{-3}$ J.

Mass of neutron = 1.00866 u

Mass of proton = 1.00726 u

Mass of Aluminium nucleus = 27.18846 u

(Assume 1 u corresponds to x J of energy)

(Round off to the nearest integer)

Ans. Official Answer NTA (27)

sol. Mass defect = [(13×1.00726 + 14×1.00866)–27.18846]u

$\Delta m = 0.02722$ u

$\therefore$ E(required) = (27.22×10^{-3})x

Answer = 27 (Nearest integer)

5. A system consists of two types of gas molecules A and B having same number density $2 \times 10^{25}/m^3$. The diameter of A and B are 10 Å and 5 Å respectively. They suffer collision at room temperature. The ratio of average distance covered by the molecule A to that of B between two successive collision is ——————— $\times 10^{-2}$.

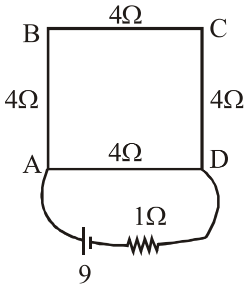
Ans. Official Answer NTA (25)

Sol. Mean free path $\lambda \propto \frac{1}{d^2}$ (d is diameter of molecule)

$$\frac{\lambda_A}{\lambda_B} = \frac{d_B^2}{d_A^2} = \frac{1}{4} = 25 \times 10^{-2}$$

6. A 16 Ω wire is bend to form a square loop. A 9 V supply having internal resistance of 1 Ω is connected across one of its sides. The potential drop across the diagonals of the square loop is ———— $\times 10^{-1}$ V.

Ans. Official Answer NTA (45)

Sol. 

Equivalent resistance of the circuit = 3 + 1 = 4 Ω

Current through cell = $\frac{9}{4}$ A

Current in AB = $\frac{9}{4}\left(\frac{1}{4}\right) = \frac{9}{16}$ A

Potential drop across AC = $\frac{9}{16} \times 8 = 4.5$ V

7. In a semiconductor, the number density of intrinsic charge carriers at 27º C is $1.5 \times 10^{16}/m^3$. If the semiconductor is doped with impurity atom, the hole density increases to $4.5 \times 10^{22}/m^3$. The electron density in the doped semiconductor is ———— $\times 10^9/m^3$.

Ans. Official Answer NTA (5)

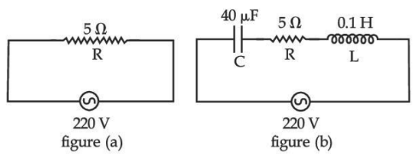
sol. For doped semiconductor

$n_e n_h = n_i^2$

$\Rightarrow n_e(4.5 \times 10^{22}) = (1.5 \times 10^{16})^2$

$\Rightarrow n_e = 5 \times 10^{11}$

8. Two circuits are shown in the figure (a) and (b). At a frequency of ———— rad/s the average power dissipated in one cycle will be same in both the circuits.



Ans. Official Answer NTA (500)

Sol. Power dissipated will be same when circuit (b) is in resonance.

$w_r = \frac{1}{\sqrt{LC}}$

$w_r = 500 \text{ rad/s}$

9. The nuclear activity of a radioactive element becomes $\left(\frac{1}{8}\right)^{th}$ of its initial value in 30 years.

The half-life of radioactive element is ———— years.

Ans. Official Answer NTA (10)

Sol. Nuclear activity $A \propto N$

$\Rightarrow \frac{N_0}{N} = 8$

$\Rightarrow N_0 = 8N$

$\Rightarrow N = \frac{N_0}{2^3}$

$\therefore$ 3 half life = 30 years

10. A force of F = (5y + 20) $\hat{j}$ N acts on a particle. The workdone by this force when the particle is moved from y = 0 m to y = 10 m is ———— J.

Ans. Official Answer NTA (450)

Sol. $\omega = \int_0^{10} (5y + 20)\,dy = 450 \text{ J}$