JEE Main September 2020

Question Paper With Text Solution

5 September| Shift-1

PHYSICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN SEP 2020 | 5 SEP SHIFT-1

1. A physical quantity z depends on four observables a, b, c and d, as z $\frac{a^2 b^{\frac{2}{3}}}{\sqrt{c}d^3}$.

The percentages of error in the measurement of a, b, c and d are 2%, 1.5%, 4% and 2.5% respectively. The percentage of error in z is :

(1) 13.5% (2) 16.5% (3) 14.5% (4) 12.25%

Ans (3)

Sol. $z = \frac{a^2 b^{\frac{2}{3}}}{\sqrt{c}d^3}$

$$\frac{\Delta z}{z} = 2\left(\frac{\Delta a}{a}\right) + \frac{2}{3}\left(\frac{\Delta b}{b}\right) + \frac{1}{2}\left(\frac{\Delta c}{c}\right) + 3\left(\frac{\Delta d}{d}\right)$$

$$\frac{\Delta z}{z}\% = 2\times 2 + \frac{2}{3}\times 1.5 + \frac{1}{2}\times 4 + 3\times 2.5$$

$$\frac{\Delta z}{z}\% = 14.5\%$$

2. Two capacitors of capacitances C and 2C are charged to potential differences V and 2V, respectively. These are then connected in parallel in such a manner that the positive terminal of one is connected to the negative terminal of the other. The final energy of this configuration is :

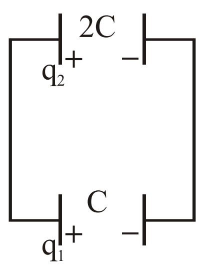
(1) zero (2) $\frac{9}{2}CV^2$ (3) $\frac{25}{6}CV^2$ (4) $\frac{3}{2}CV^2$

Ans (4)

Sol. Inital charge on capacitor C = CV

Initial charge on capacitor 2C = 4CV

after the capacitors are connected with each other



using conservation of charge

$q_1 + q_2 = 3CV$ ......(i)

Also potential on both capacitors should be same.

$\Rightarrow \quad \frac{q_1}{C} = \frac{q_2}{2C}$ ......(ii)

from (i) & (ii) $q_1 = CV$ & $q_2 = 2CV$

U_1 (final) = $\frac{q_1^2}{2C} = \frac{CV^2}{2}$

U_2 (final) = $\frac{q_2^2}{2(2C)} = CV^2$

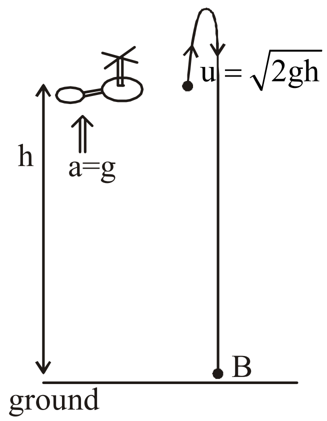
$\therefore$ U (final) = $\frac{CV^2}{2} + CV^2 = \frac{3}{2}CV^2$

3. A helicopter rises from rest on the ground vertically upwards with a constant acceleration g. A food packet is dropped from the helicopter when it is at a height h. The time taken by the packet to reach the ground is close to [g is the acceleration due to gravity]:

(1) $t = 3.4\sqrt{\left(\frac{h}{g}\right)}$ (2) $t = \sqrt{\left(\frac{2h}{3g}\right)}$ (3) $t = \frac{2}{3}\sqrt{\left(\frac{h}{g}\right)}$ (4) $t = 1.8\sqrt{\left(\frac{h}{g}\right)}$

Ans (1)

Sol. Velocity of packet when it is dropped from helicopter $u = \sqrt{2gh}$

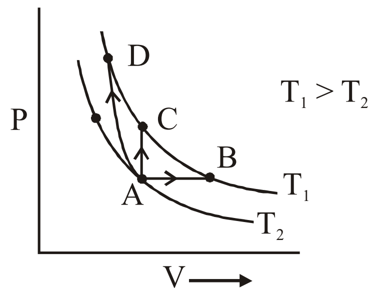


use $S = ut + \frac{1}{2}a\,t^2$ (for packet from A to B)

$\Rightarrow \; -h = \sqrt{2gh}\;t + \frac{1}{2}(-g)t^2$

$\Rightarrow t = \left(\sqrt{2}+2\right)\sqrt{\frac{h}{g}} = 3.4\sqrt{\frac{h}{g}}$

4. Three different processes that can occur in an ideal monoatomic gas are shown in the P vs V diagram. The paths are labelled as A→B, A→C and A→D. the change in internal energies during these process are taken as E_{AB}, E_{AC} and E_{AD} and the workdone as W_{AB}, W_{AC} and W_{AD}. The correct relation between these parameter are :



(1) $E_{AB}>E_{AC}>E_{AD}, W_{AB}<W_{AC}<W_{AD}$

(2) $E_{AB}=E_{AC}=E_{AD}, W_{AB}>0, 2_{AC}=0, W_{AD}> 0$

(3) $E_{AB}=E_{AC}<E_{AD}, W_{AB}>0, W_{AC}=0, W_{AD}<0$

(4) $E_{AB}<E_{AC}<E_{AD}, W_{AB}>0, W_{AC}>W_{AD}$

Ans (2)

Sol. As initial & final temperatures for each process are same

$\therefore$ Change in internal energy for each process will be equal

$E_{AB} = E_{AC} = E_{AD}$

5. An electrical power line, having a total resistance of 2Ω, delivers 1 kW at 220 V. The efficiency of the transmission line is approximately:

(1) 96% (2) 85% (3) 72% (4) 91%

Ans (1)

Sol. Current $I = \frac{P}{V} = \frac{1000}{220}$

Power dissipated in wire $= I^2R = \left(\frac{1000}{220}\right)^2 \times 2 = 41.3 \text{ W}$

$\therefore$ efficiency (%) = $\left(\frac{1000-41.3}{1000}\right)\times 100\% \simeq 96\%$

6. Assume that the displacement (s) of air is proportional to the pressure difference (Δp) created by a sound wave. Displacement (s) further depends on the speed of sound (υ), density of air (ρ) and the frequency (f). If $\Delta f \sim 10$Pa, $\upsilon \sim 300$ m/s, $\rho \sim 1$ kg/m_3 and f ~ 1000 Hz, then s will be of the order of (take the multiplicative constant to be 1)

(1) $\frac{1}{10}$mm (2) 1 mm (3) 10 mm (4) $\frac{3}{100}$mm

Ans (4)

Sol. $S \propto (\Delta P)\,(V)^a\,(\rho)^b\,(f)^c$

$[S] = [\Delta P][V]^a[\rho]^b[f]^c$

$[L^1] = [M^1L^{-1}T^{-2}][L^1T^{-1}]^a[M^1L^{-3}]^b[T^{-1}]^c$

$\Rightarrow [L^1] = [M^{1+b}][L^{-1+a-3b}][T^{-2-a-c}]$

comparign the coefficients

$1+b = 0$

$-1 + a - 3b = 1$

$-2 - a - c = 0$

$\Rightarrow a = -1,\ b = -1,\ c = -1$

$$\therefore\ S = \frac{\Delta P}{V\rho g} = \frac{10^{-4}}{3}\text{m}$$

$$= \frac{1}{300}\text{mm}$$

nearest opion is $\frac{3}{100}$mm

7. A bullet of mass 5 g, travelling with a speed of 210 m/s, strikes a fixed wooden target. One half of its kinetic energy is converted into heat in the bullet while the other half is converted into heat in the wood. The rise of temperature of the bullet if the specific heat of its material is 0.030 cal/ (g –°C) (1 cal = 4.2 $\times 10^7$ ergs) close to :

(1) 87.5°C (2) 119.2°C (3) 83.3°C (4) 38.4°C

Ans (1)

Sol. Kinetic energy of bullet $= \frac{1}{2}mv^2$

Heat energy received by bullet $= \frac{mv^2}{4}$

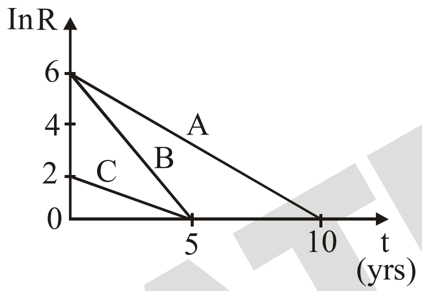
Heat = $ms\Delta T$

$\Rightarrow\ ms\Delta T = \frac{mv^2}{4}$

$\Rightarrow\ (0.03\times1000)\times4.2\times\Delta T = \frac{(210)^2}{4}$

$\Rightarrow\ \Delta T = 87.5°C$

8. Activities of three radioactive substances A, B and C are represented by the curves A, B and C, in the figure. Then their half-lives $T_{\frac{1}{2}}(A)$: $T_{\frac{1}{2}}(B)$: $T_{\frac{1}{2}}(C)$ are in the ratio :



(1) 2 : 1 : 3 (2) 3 : 2 : 1 (3) 2 : 1 : 1 (4) 4 : 3 : 1

Ans (1)

Sol. Activity of a radioactive sample $R = R_0\, e^{-\lambda t}$ [R_0 is initial activity]

$\ell n(R) = -\lambda t + \ell n(R_0)$

Comparing with equation of straight line

$y = mx + c$

Slope of $\ell n(R)$ versus t is $-\lambda$

$\Rightarrow \lambda_A = \frac{6}{10}$, $\lambda_B = \frac{6}{5}$, $\lambda_C = \frac{2}{5}$

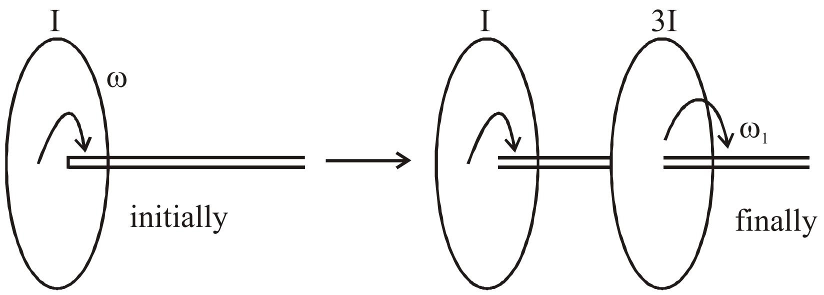
$\because\ t_A : t_B : t_C = \frac{1}{\lambda_A} : \frac{1}{\lambda_B} : \frac{1}{\lambda_C} = 2:1:3$

9. A wheel is rotating freely with an angular speed ω on a shaft. The moment of inertia of the wheel is I and the moment of inertia of the shaft is negligible. Another wheel of moment of inertia 3I initially at rest is suddenly coupled to the same shaft. The resultant fractional loss in the kinetic energy of the system is:

(1) 0 (2) $\frac{5}{6}$ (3) $\frac{1}{4}$ (4) $\frac{3}{4}$

Ans (4)

Sol.



Using conservation of angular momentum (about shaft)

$I\omega = (I + 3I)\omega_1 \quad \Rightarrow \quad \omega_1 = \frac{\omega}{4}$

$\text{K.E.(initial)} = \frac{1}{2}I\omega^2$

$\text{K.E.(final)} = \frac{1}{2}(4I)\omega_1^2 = \frac{I\omega^2}{8}$

$\therefore$ fractional loss $= \frac{K.E_i - K.E_f}{K.E_i} = \frac{3}{4}$

10. In a resonance tube experiment when the tube is filled with water up to a height of 17.0 cm from bottom,it resonates with a given tuning fork. When the water level is raised the next resonance with the same tuning fork occurs at a height of 24.5 cm. If the velocity of sound in air is 330 m/s, the tuning fork frequency is ;

(1)3300Hz (2) 2200Hz (3) 550Hz (4) 1100Hz

Ans (2)

Sol. For resonance tube experiment

speed of sound, $V = 2f(\ell_2 - \ell_1)$

$\Rightarrow f = \frac{V}{2(\ell_2 - \ell_1)} = \frac{330}{2\times(7.5\times10^{-2})} = 2200 \text{ Hz}$

11. A square loop of side 2a, and carrying current I, is kept in XZ plane with its centre at origin. A long wire carrying the same current I is placed parallel to the z-axis and passing through the point (0, b, 0), (b >> a). The magnitude of the torque on the loop about z-axis is given by :

(1) $\frac{2\mu_0 I^2 a^3}{\pi b^2}$ (2) $\frac{\mu_0 I^2 a^2}{2\pi b}$ (3) $\frac{\mu_0 I^2 a^3}{2\pi b^2}$ (4) $\frac{2\mu_0 I^2 a^2}{\pi b}$

Ans (4)

Sol. Magnetic field at position of loop,

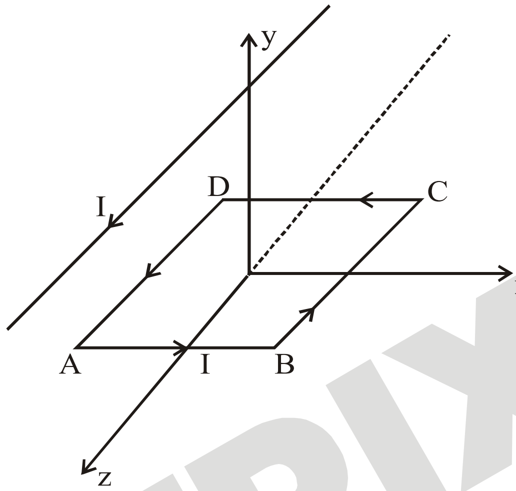
$B = \frac{\mu_0 I}{2\pi b}$ (parallel to plane of loop)

$\because$ torque, $\vec{\tau} = \vec{M} \times \vec{B}$

$\vec{M}$ & $\vec{B}$ are perpendicular

$\therefore \tau = MB = \left\{(2a)^2 I\right\} \frac{\mu_0 I}{2\pi b}$

$\tau = \frac{2\mu_0 I^2 a^2}{\pi b}$



12. A hollow spherical shell of outer radius R floats just submerged under the water surface. The inner radius of the shell is r. If the specific gravity of the shell material is $\frac{27}{8}$ w.r.t water, the value of r is :

(1) $\frac{4}{9}R$ (2) $\frac{2}{3}R$ (3) $\frac{8}{9}R$ (4) $\frac{1}{3}R$

Ans (3)

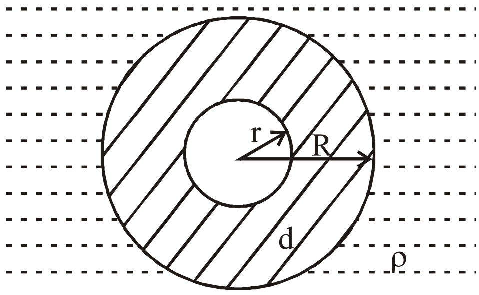
Sol. $\frac{d}{\rho} = \frac{27}{8}$

For floating,
Buoyant force = weight of shell

$\Rightarrow \frac{4}{3}\pi R^3 \rho g = \frac{4}{3}\pi (R^3 - r^3) dg$

$\Rightarrow \frac{R^3}{(R^3 - r^3)} = \frac{d}{\rho} = \frac{27}{8}$

$\Rightarrow \frac{r}{R} = \frac{(19)^{\frac{1}{3}}}{3} = 0.886 \simeq \frac{8}{9}$



13. The value of the acceleration due to gravity is g_1 at a height $h = \frac{R}{2}$ (R=radius of the earth) from the surface of the earth. It is again equal to g_1 at a depth d below the surface of the earth. The ratio $\left(\frac{d}{R}\right)$

equals :

(1) $\frac{1}{3}$ (2) $\frac{7}{9}$ (3) $\frac{4}{9}$ (4) $\frac{5}{9}$

Ans (4)

Sol. g_1 at height $h = \frac{g}{\left(1+\frac{h}{R}\right)^2}$

at $h = \frac{R}{2}$ $g_1 = \frac{4}{9}g$ ....(i)

g_1 at depth d is, $g\left(1-\frac{d}{R}\right)$

$$g_1 = g\left(1-\frac{d}{R}\right) \quad \text{........(ii)}$$

From (i) & (ii) $\frac{4}{9}g = g\left(1-\frac{d}{R}\right)$

$$\Rightarrow \frac{d}{R} = \frac{5}{9}$$

14. With increasing biasing voltage of a photodiode, the photocurrent magnitude :

(1) Increases linearly

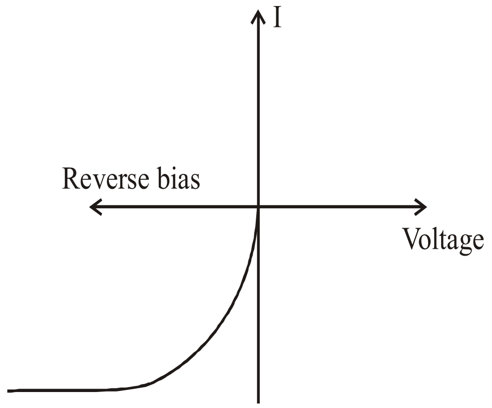
(2) increases intially and saturates finally

(3) remains constant

(4) increases initially and after attaining certain value, it decreases

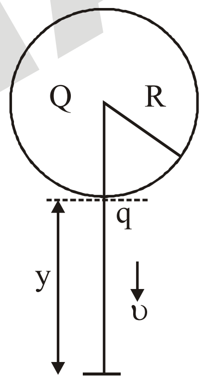
Ans (2)

Sol. For a photodiode current versus biasing voltage graph is



Initially increases and saturates finally

15. A solid sphere of radius R carries a chare Q + q distributed uniformly over its volume. A very small point like piece of it of mass m gets detached from the bottom of the sphere and falls down vertically under gravity. This piece carries charge q. If it acquires a speed υ when it has fallen through a vertical height y (see figure), then : (assume the remaining portion to be sphercal).



(1) $\upsilon^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R^2 ym} + g\right]$

(2) $\upsilon^2 = y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

(3) $\upsilon^2 = 2y\left[\frac{qQR}{4\pi\varepsilon_0 R(R+y)^3 m} + g\right]$

(4) $\upsilon^2 = 2y\left[\frac{qQ}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

Ans (4)

Sol. Apply conservation of mechanical energy

$\Rightarrow \Delta U + \Delta K = 0$

$\Rightarrow \Delta U_{gpe} + \Delta U_{epe} + \Delta K = 0$

$\Rightarrow -mgy + \left(\frac{kQq}{R+y} - \frac{kQq}{R}\right) + \frac{1}{2}mv^2 = 0$

$\Rightarrow v^2 = 2y\left[\frac{Qq}{4\pi\varepsilon_0 R(R+y)m} + g\right]$

16. A gavanometer of resistance G is converted into a voltmeter of range 0 – 1V by connecting a resistance R_1 in series with it. The additional resistance that should be connected in series with R_1 to incerease the range of the voltmeter to 0 – 2V will be :

(1) $R_1 + G$ (2) R_1 (3) G (4) $R_1 - G$

Ans (1)

Sol. For converting galvanometer into voltmeter of range V

$V = I_g(R_1 + G)$

For doubling the range connect R_2 in series

$2V = I_g(R_1 + G + R_2)$

This gives $R_2 = R_1 + G$

17. Number of molecules in a volume of 4 cm^3 of a perfect monoatomic gas at some temperature T and at a pressure of 2 cm of mercury is close to ? (given, mean kinetic energy of a molecule (at T) is 4×10^{-14} erg, g = 980 cm/s^2, density of mercury = 13.6 g/cm^3)

(1) 5.8×10^{16} (2) 4.0×10^{16} (3) 5.8×10^{18} (4) 4.0×10^{18}

Ans (4)

Sol. For monoatomic gas kinetic energy $K = \frac{3RT}{2N_A}$ [N_A is avogadro number]

$\Rightarrow RT = \frac{2KN_A}{3}$ ....(1)

Ideal gas equation PV = nRT

$\Rightarrow n = \frac{PV}{RT}$ $\Rightarrow \frac{N}{N_A} = \frac{PV}{RT}$ $\Rightarrow N = \frac{3PV}{2K}$ (using (i))

$$\Rightarrow N = \frac{3}{2}\frac{\left(13600\times9.8\times\frac{2}{100}\right)(4\times10^{-6})}{4\times10^{-14}\times10^{-7}} = 3.998\times10^{18} \simeq 4\times10^{18}$$

18. An electron is constrained to move along the y-axis with the speed of 0.1 c (c is the speed of light)in the presence of electromagnetic wave, whose electric field is $\vec{E} = 30\hat{j}\sin(1.5\times10^{7}t - 5\times10^{-2}x)$ V/m. The maximum magnetic force experienced by the electron will be :

(given)c = $\times 10^{8}$ ms^{-1} and electron charge = 1.6×10^{-19}C)

(1) 4.8×10^{-19} N (2) 1.6×10^{-19} N (3) 2.4×10^{-18} N (4) 3.2×10^{-18} N

Ans (1)

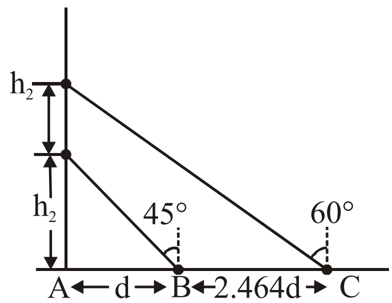
Sol. $E_0 = 30$ V/m

& $B_0 = \frac{E_0}{C}$

F_B (maximum) = $qVB_0 = 1.6\times10^{-19}\times0.1\times30$

$= 4.8\times10^{-19}$ N

19. A balloon is moving up in air vertically above a point A on the ground. When it is at a height h_1, a girl standing at a distance d (point B) from A (see figure) sees it at an angle 45° with respect to the vertical. When the balloon climbs up a further height h_2, it is seen at an angle 60° with respect to the vertical if the girl moves further by a distance 2.464 d (point C). Then the height h_2 is (given tan 30° = 0.5774) :



(1) 1.464 d (2) 0.464 d (3) 0.732 d (4) d

Ans (4)

Sol. $\tan 45^{o} = \frac{h_1}{d}$ $\Rightarrow h_1 = d$

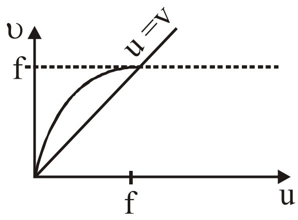
In new position

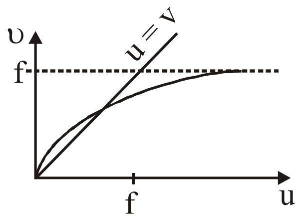
$\tan 30^{o} = \frac{h_1 + h_2}{d + 2.464d}$

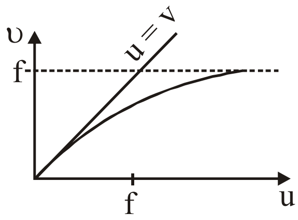
$\Rightarrow h_2 = (3.464 \times 0.5774 - 1)d$

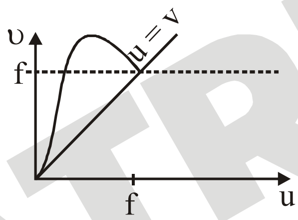
$\Rightarrow h_2 = d$

20. For a concave lens of focal length f, the relation between object and image distances u and υ, respectively, from its pole can best be represented by (u = υ is the reference line) :

(1) 

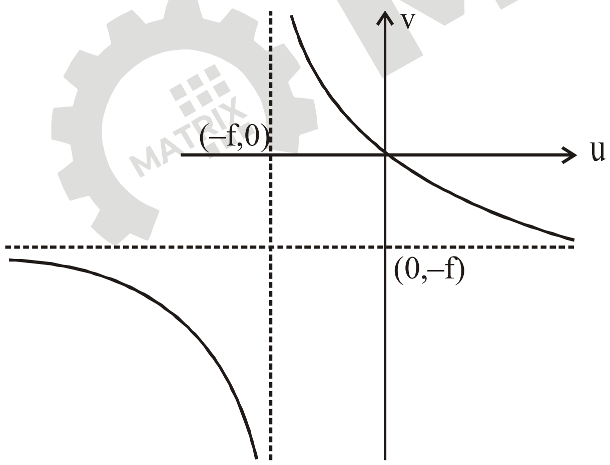
(2) 

(3) 

(4) 

Ans (3)

Sol. For concave lens graph of v verses u



As in the question u & v are distance we will plot |v| vs |u|

21. A force $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$ N acts at a point $(4\hat{i} + 3\hat{j} - \hat{k})$ m. Then the magnitude of torque about the point $(\hat{i} + 2\hat{j} + \hat{k})$ m will be $\sqrt{x}$ N-m. The value of x is_______.

Ans. 195

Sol. $\vec{F} = (\hat{i} + 2\hat{j} + 3\hat{k})$

$\vec{r} = (4\hat{i} + 3\hat{j} - \hat{k}) - (\hat{i} + 2\hat{j} + \hat{k})$

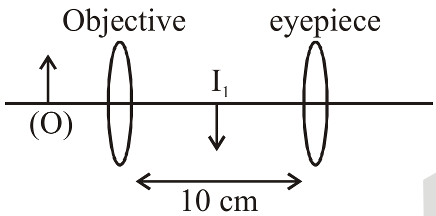
$\Rightarrow \vec{r} = 3\hat{i} + \hat{j} - 2\hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F} = 7\hat{i} - 11\hat{j} + 5\hat{k}$

$|\vec{\tau}| = \sqrt{195}$

22. A compound microscope consists of an objective lens of focal length 1 cm and an eye piece of focal length 5 cm with a separation of 10 cm. The distance between an object and the objective lens, at which the strain on the eye is minimum is $\frac{n}{40}$ cm. The value of n is ____.

Ans. 50

Sol. 

I_1 should be formed at focus of eyepiece so that final image is formed at infinity (least strain on eye)

So image from objective should be formed at 5 cm distance.

for objective lens using lens formula

$$\frac{1}{v_0} - \frac{1}{u_0} = \frac{1}{f_0} \Rightarrow \frac{1}{5} - \frac{1}{4} = \frac{1}{1}$$

$$\Rightarrow u_0 = \frac{-5}{4} \text{cm}$$

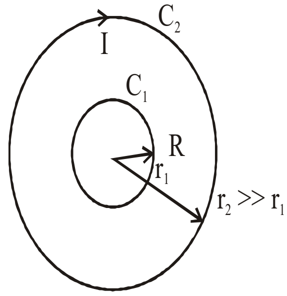
$$\therefore \text{ distance} = \frac{50}{40} \text{cm}$$

$\therefore$ n = 50

23. Two concentric circular coils, C_1 and C_2, are placed in the XY plane. C_1 has 500 turns, and a radius of 1 cm. C_2 has 200 turns and radius of 20 cm. C_2 carries a time dependent current $I(t) = (5t^2 - 2t + 3)$A where t is in s. The emf induced in C_1 (in mV), at the instant t = 1s is $\frac{4}{x}$. The value of x is______.

Ans. 5

Sol. Magnetic field at centre of C_2, $B = \left(\frac{\mu_0 I}{2r_2}\right)N_2$



flux through C_1 $\phi = (B\pi r_1^2)N_1$

$$|\varepsilon_{ind}| = \frac{\mu_0 \pi r_1^2 N_1 N_2}{2r_2}\left(\frac{dI}{dt}\right)$$

$$\Rightarrow |\varepsilon_{ind}| = \frac{\mu_0 \pi r_1^2 N_1 N_2}{2r_2}(10t-2)$$

$$\Rightarrow |\varepsilon_{ind}|(t = 1s) = \frac{4\pi \times 10^{-7} \times \pi\left(\frac{1}{100}\right)^2 \times 500 \times 200 \times 8}{2 \times (0.2)}$$

$$|\varepsilon_{ind}|_{t=1s} = 0.8\text{mv}$$

$\therefore x = 5$

24. A beam of electrons of energy E scatters from a target having atomic spacing of 1Å. The first maximum intensity occurs at $\theta = 60°$. The E (in eV) is ____.

(Planck constant $h = 6.64 \times 10^{-34}$Js, 1 eV = 1.6×10^{-19}J, electron mass $m = 9.1 \times 10^{-31}$ kg)

Ans 50

Sol. $\lambda = \frac{h}{\sqrt{2mE}}$ (debroglie wave length)

for maxima , $n\lambda = 2d \sin\left(90° - \frac{\theta}{2}\right)$

For 1st maxima $2d\sin 60° = \lambda$ (n = 1)

$$\Rightarrow \sqrt{3}d = \frac{h}{\sqrt{2mE}}$$

$$\Rightarrow E = \frac{h^2}{6d^2m} = \frac{(6.64 \times 10^{-34})^2}{6 \times (10^{-10})^2(9.1 \times 10^{-31})}$$

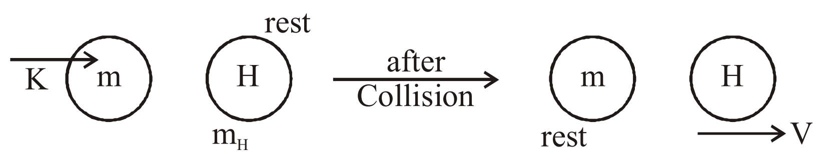
$$\Rightarrow E = 0.8075 \times 10^{-17}\text{ J}$$

$$E(\text{in ev}) = \frac{E}{1.6 \times 10^{-19}} = 50.47 \text{ ev} \simeq 50\text{ eV}$$

25. A particle of mass 200 MeV/C^2 collides with a hydrogen atom at rest. Soon after the collision the particle comes to rest, and the atom recoils and goes to its first exited state. The initial kinetic energy of theparticle (in eV) is $\frac{N}{4}$. The value of N is :

(Given the mass of the hydrogen atom to be 1 GeV/c^2)______.

Sol.



m = 200 MeV/C^2

m_H = 1000 MeV/C^2

$\Rightarrow \frac{m}{m_H} = 0.2$

applying conservation of linear momentum

$$\sqrt{2mK} = m_H V$$

$$\Rightarrow V = \frac{\sqrt{2mK}}{m_H} \qquad ....(i)$$

also loss in K.E = 1^{st} excitation energy of hydrogen

$$\Rightarrow K - \frac{m_H V^2}{2} = 13.6\left(1 - \frac{1}{2^2}\right)eV$$

$$\Rightarrow K - \frac{mK}{m_H} = 10.2 \text{ eV}$$

$$\Rightarrow K = \frac{10.2\,eV}{\left(1 - \frac{m}{m_H}\right)} = \frac{10.2}{1 - 0.2} = 12.75 \text{ eV}$$

$$= \frac{51}{4} \text{ eV}$$

$\therefore$ Ans. N = 51