JEE Main September 2020

Question Paper With Text Solution

3 September| Shift-1

PHYSICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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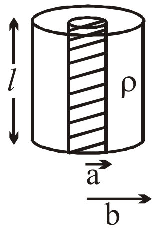
JEE MAIN SEP 2020 | 3 SEP SHIFT-1

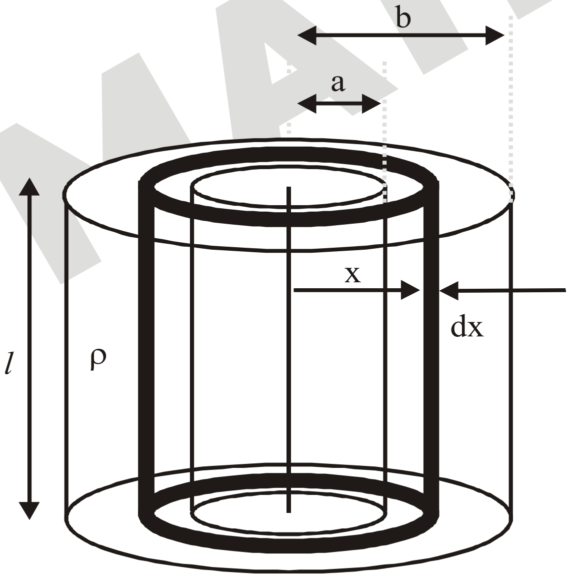
1. Model a torch battery of length l to be made up of a thin cylindrical bar of radius 'a' and a concentric thin cylindrical shell of radius 'b' filled in between with an electrolyte of resistivity ρ (sec figure). If the battery is connected to a resistance of value R, the maximum Joule heating in R will take place for :



(1) $R = \frac{\rho}{2\pi\ell}\left(\frac{b}{a}\right)$ (2) $R = \frac{\rho}{2\pi\ell}\ell n\left(\frac{b}{a}\right)$ (3) $R = \frac{2\rho}{\pi\ell}\ell n\left(\frac{b}{a}\right)$ (4) $R = \frac{\rho}{\pi\ell}\ell n\left(\frac{b}{a}\right)$

Ans. (1)

Sol.



The ressistance of small element

$$\Delta R = \frac{\rho dr}{2\pi r\ell}$$

$$R = \frac{\rho}{2\pi\ell}\int_a^b \frac{dr}{r}$$

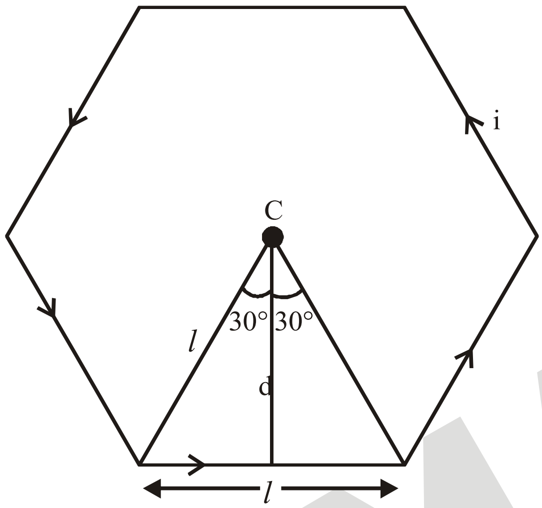
$$R = \frac{\rho}{2\pi\ell}\ell n\frac{b}{a}$$

2. Magnitude of magnetic field (in SI units) at the centre of a hexagonal shape coil of side 10 cm, 50 turns and carrying current 1 (Ampere) in units of $\frac{\mu_0 I}{\pi}$ is:

(1) $500\sqrt{3}$ (2) $50\sqrt{3}$ (3) $5\sqrt{3}$ (4) $250\sqrt{3}$

Ans. (1)

Sol.



$$\cos 30 = \frac{d}{l}$$

$$\Rightarrow\ d = \frac{\sqrt{3}}{2} l$$

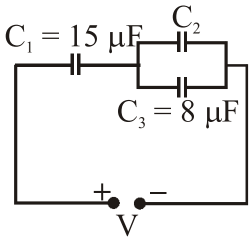
$$l = 0.1\text{m}$$

$$B_C = 50\left[6 \times \frac{\mu i}{4\pi\left(\frac{\sqrt{3}l}{2}\right)}(\sin 30^\circ + \sin 30^\circ\right]$$

$$= 300\frac{\mu o i}{2\sqrt{3}\pi l}\left(2 \times \frac{1}{2}\right) = \frac{150}{\sqrt{3}}\frac{\mu o i}{\pi l}$$

$$= 500\sqrt{3}\frac{\mu_0 i}{\pi}$$

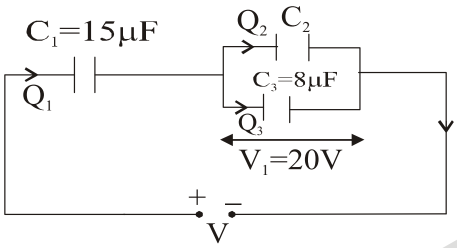
3. In the circuit shown in the figure, the total charge is 750 μC and the voltage across capacitor C_2 is 20 V. Then the charge on capacitor C_2 is :



(1) 590 μC (2) 450 μC (3) 650 μC (4) 160 μC

Ans. (1)

Sol.



$Q_1 = Q_2 + Q_3 = 750$ μc

$Q_3 = 8 \times 20 = 160$ μc

$\therefore Q_2 = 750 - 160 = 590$ μc

4. A uniform thin rope of length 12 m and mass 6 kg hangs vertically from a rigid support and a block of mass 2 kg is attached to its free end. A transverse short wave-train of wavelength 6 cm is produced at the lower end of the rope. What is the wavelength of the wavetrain (in cm) when it reaches the top of the rope ?

(1) 6 (2) 12 (3) 9 (4) 3

Ans. (2)

Sol. $V \propto \lambda$

$$\frac{V_1}{V_2} = \frac{\lambda_1}{\lambda_2}$$

$$\lambda_2 = \frac{V_1}{V_2}\lambda_1 = \sqrt{\frac{T_2}{T_1}} \quad T_2 = 8g \text{ (Top)}$$

$\sqrt{\frac{8g}{2g}}\lambda_1$ $\quad\quad T_1 = 2g$ (Bottom)

$= 2\lambda_1 = 12$cm

5. A charged particle carrying charge 1 μC is moving with velocity $\left(2\hat{i}+3\hat{j}+4\hat{k}\right)ms^{-1}$. If an external magnetic field of $\left(5\hat{i}+3\hat{j}-6\hat{k}\right)\times10^{-3}T$ exists in the region where the particle is moving then the force on the particle is $\vec{F}\times10^{-9}$ N. The vector $\vec{F}$ is :

(1) $-30\hat{i}+32\hat{j}-9\hat{k}$ (2) $-300\hat{i}+320\hat{j}-90\hat{k}$

(3) $-0.30\hat{i}+0.32\hat{j}-0.09\hat{k}$ (4) $-3.0\hat{i}+3.2\hat{j}-0.9\hat{k}$

Ans. (1)

Sol. $\overline{F}=\left(\overline{V}\times\overline{B}\right)=10^{-6}\times10^{-3}\begin{vmatrix}\hat{i} & \hat{J} & \hat{K}\\ 2 & 3 & 4\\ 5 & 3 & -6\end{vmatrix}$

$=10^{-9}\left[\hat{i}\left(-18-12\right)-\hat{J}\left(-12-20\right)+\hat{K}\left(6-15\right)\right]$

$=10^{-9}\left[-30\hat{i}+32\hat{J}-9\hat{K}\right]$

6. The magnetic field of a plane electromagnetic wave is

$\vec{B}=3\times10^{-8}\sin[200\pi(y+ct)]\,\hat{i}$ T

where $c = 3\times10^8$ ms^{-1} is the speed of light. The corresponding electric field is :

(1) $\vec{E}=9\sin[200\pi(y+ct)]\hat{k}$ V/m (2) $\vec{E}=-9\sin[200\pi(y+ct)]\hat{k}$ V/m

(3) $\vec{E}=-10^{-6}\sin[200\pi(y+ct)]\hat{k}$ V/m (4) $\vec{E}=3\times10^{-8}\sin[200\pi(y+ct)]\hat{k}$ V/m

Ans. (2)

Sol. $\overline{E}=E_0\text{Sin}\left[200\pi(y+ct\right]\hat{E}$

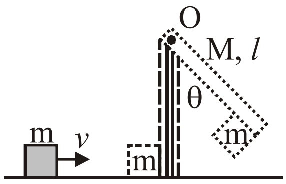
$E_0=B_0C=3\times10^{-8}\times3\times10^{8}=9\frac{v}{m}$ and $\quad\quad \hat{E}\times\hat{B}\parallel\hat{C}$

$\hat{B}=\hat{i}\ \&\ \hat{C}=-\hat{J}$

$\therefore \hat{E}=-\hat{K}$

So $\overline{E}=-9\sin\left[200\pi(y+ct)\right]\hat{K}\frac{v}{m}$

7. A block of mass m = 1 kg slides with velocity v = 6 m/s on a frictionless horizontal surface and collides with a uniform vertical rod and sticks to it as shown. The rod is pivoted about O and swings as a result of the collision making angle θ before momentarily coming to rest. If the rod has mass M = 2 kg, and length l = 1 m, the value of θ is approximately : (take g = 10 m/s²)



(1) 69° (2) 63° (3) 49° (4) 55°

Ans. (2)

Sol. ∴ Angular impluse on the system (Rod + Particle) about Hinge is zero during collision.

$\therefore L_i = L_f$

$$\Rightarrow mul = \left(\frac{Ml^2}{3} + ml^2\right)w$$

$$\Rightarrow 1\times6\times1 = \left[\frac{2\times(1)^2}{3} + 1(1)^2\right]^{\omega}$$

$$\Rightarrow \omega = \frac{18}{5}\frac{\text{Red}}{\text{see}}$$

Let Rod rotates upto angle θ.

From energy conservation

$$\frac{1}{2}\left(\frac{Ml^2}{3} + ml^2\right)\omega^2 = mgl(1-\cos\theta) + \frac{Mgl}{2}(1-\cos\theta)$$

$$\Rightarrow \frac{1}{2}\left(\frac{2}{3}+1\right)\left(\frac{18}{5}\right)^2 = (1-\cos\theta)\left[(1)10\times1 + \frac{2\times10\times1}{2}\right]$$

$\Rightarrow 1 - \cos\theta = 0.54$

$\Rightarrow \cos\theta = 0.46$

$\Rightarrow \cos\theta \approx 63°$

8. Pressure inside two soap bubbles are 1.01 and 1.02 atmosphere, respectively. The ratio of their volumes is :

(1) 0.8 : 1 (2) 4 : 1 (3) 8 : 1 (4) 2 : 1

Ans. (3)

Sol. Excess pressure (Δp) = Pressure inside – Atmospheric pressure

$$\therefore \frac{\Delta P_1}{\Delta P_2} = \frac{\frac{4T}{R_1}}{\frac{4T}{R_2}}$$

$$\Rightarrow \frac{R_2}{R_1} = \frac{1.01-1}{1.02-1} = \frac{0.01}{0.02} = \frac{1}{2}$$

$$\text{So } \frac{V_1}{V_2} = \left(\frac{R_1}{R_2}\right)^3 = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$$

9. A balloon filled with helium (32°C and 1.7 atm.) bursts. Immediately afterwards the expansion of helium can be considered as :

(1) irreversiblc isothermal (2) reversible isothermal

(3) reversible adiabatic (4) irreversible adiabatic

Ans. (4)

Sol. Process is fast so it will be irreversible adiabatic.

10. Using screw gauge of pitch 0.1 cm and 50 divisions on its circular scale, the thickness of an object is measured. It should correctly be recorded as :

(1) 2.123cm (2) 2.121 cm (3) 2.124cm (4) 2.125cm

Ans. (3)

Sol. $LC = \frac{0.1}{50} = 0.002$ cm per division

Thickness = Main Scale Reading + [Circular Scale Reading (L.C.)]

So Last digit should be multiple of L.C.

11. When the wavelength of radiation falling on a metal is changed from 500 nm to 200 nm, the maximum kinetic energy of the photoelectrons becomes three times larger. The work function of the metal is close to :

(1) 0.81 eV (2) 0.52 eV (3) 1.02 eV (4) 0.61 eV

Ans. (4)

Sol. $\text{Kmax} = \frac{hc}{\lambda} - \phi$

$$\therefore \frac{k_1}{k_2} = \frac{\frac{hc}{\lambda_1} - \phi}{\frac{hc}{\lambda_2} - \phi} \Rightarrow \frac{k}{3k} = \frac{\frac{1242}{500} - \phi}{\frac{1242}{200} - \phi}$$

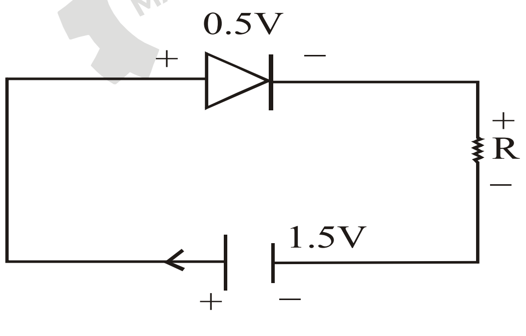
$\Rightarrow \phi \approx 0.61$

12. When a diode is forward biased, it has a voltage drop of 0.5 V. The safe limit of current through the diode is 10 mA. If a battery of emf 1.5 V is used in the circuit, the value of minimum resistance to be connected in series with the diode so that the current does not exceed the safe limit is :

(1) 200 Ω (2) 100 Ω (3) 50 Ω (4) 300 Ω

Ans. (2)

Sol.



$V_{diode} = 0.5$ volt

$V_R = 1.5 - 0.5 = 1$ volt

$iR = 1$

$R = \frac{1}{i} = \frac{1}{10^{-2}} = 100\Omega$

13. Moment of inertia of a cylinder of mass M, length L and radius R about an axis passing through its centre and perpendicular to the axis of the cylinder is $I = M\left(\frac{R^2}{4}+\frac{L^2}{12}\right)$. If such a cylinder is to be made for a given mass of a material, the ratio L/R for it to have minimum possible I is :

(1) $\frac{3}{2}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\frac{2}{3}$ (4) $\sqrt{\frac{3}{2}}$

Ans. (4)

Sol. For given mass, volume of material will be constant.

$$I = \frac{MR^2}{4}+\frac{ML^2}{12} \text{ and } v = \pi R^2 L$$

$$\Rightarrow I = \frac{M}{4}\left(\frac{v}{\pi L}\right)+\frac{ML^2}{12}$$

for minimum value $\Rightarrow \frac{dI}{dL} = 0$

$$\Rightarrow \frac{MV}{4\pi}\left(-\frac{1}{12}\right)+\frac{m}{12}(2L) = 0$$

$$\Rightarrow \frac{V}{4\pi L^2} = \frac{2L}{12}$$

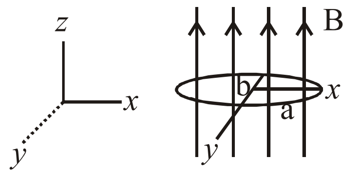
$$\Rightarrow V = \frac{2\pi L^3}{3}$$

$$\Rightarrow \pi R^2 L = \frac{2}{3}\pi L^3$$

$$\Rightarrow \left(\frac{L}{R}\right)^2 \frac{3}{2}$$

$$\Rightarrow \frac{L}{R} = \sqrt{\frac{3}{2}}$$

14. An elliptical loop having resistance R, of semi major axis a, and semi minor axis b is placed in a magnetic field as shown in the figure. If the loop is rotated about the x-axis with angular frequency ω, the average power loss in the loop due to Joule heating is:



(1) $\frac{\pi abB\omega}{R}$ (2) $\frac{\pi^2a^2b^2B^2\omega^2}{2R}$ (3) $\frac{\pi^2a^2b^2B^2\omega^2}{R}$ (4) zero

Ans. (2)

Sol. Area(A) = πab

$\phi = BA\cos\omega t$

$\varepsilon = \frac{-d\phi}{dt} = AB\omega\cos\omega t$

$\langle P \rangle = \left\langle \frac{\varepsilon^2}{R} \right\rangle$

$= \left\langle \frac{A^2B^2\omega^2\cos^2 wt}{R} \right\rangle$

$= \frac{A^2B^2\omega^2}{R}\left(\frac{1}{2}\right)$

$= \frac{\pi^2a^2b^2B^2}{2R}\left(\omega^2\right)$

15. In a radioactive material, fraction of active material remaining after time t is 9/16. The fraction that was remaining after t/2 is :

(1) $\frac{3}{5}$ (2) $\frac{3}{4}$ (3) $\frac{7}{8}$ (4) $\frac{4}{5}$

Ans. (2)

Sol. $N = N_0 e^{-\lambda t}$ ....(1)

$N' = N_0 e^{-\lambda t/2}$ ....(2)

Form (1) & (2)

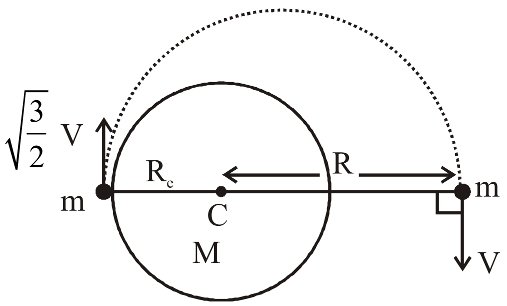
$\left(\frac{N'}{N_0}\right) = \left(\frac{N}{N_0}\right)^{\frac{1}{2}} = \left(\frac{9}{16}\right)^{\frac{1}{2}} = \frac{3}{4}$

16. A satellite is moving in a low nearly circular orbit around the earth. Its radius is roughly equal to that of the earth's radius R_e. By firing rockets attached to it, its speed is instantaneously increased in the direction of its motion so that it become $\sqrt{\frac{3}{2}}$ times larger. Due to this the farthest distance from the centre of the earth that the satellite reaches is R. Value of R is:

(1) $4R_e$ (2) $2.5R_e$ (3) $2R_e$ (4) $3R_e$

Ans. (4)

Sol.



$\Rightarrow V = \sqrt{\frac{GM}{R_e}}$

from Angular momentum conservation about C :-

$m\sqrt{\frac{3GM}{2R_e}}R_e = mv'R$

$\Rightarrow V' = \sqrt{\frac{3GMR_e}{2}}/R$

Now from mechanical energy conservation,

$\frac{GMm}{R_e} + \frac{1}{2}m\left(\frac{3GM}{2R_e}\right) = -\frac{GMm}{R} + \frac{1}{2}mv'^2$

$\Rightarrow -\frac{GMm}{4R_e} = -\frac{GMm}{R} + \frac{1}{2}m\left(\frac{3GMR_e}{2R^2}\right)$

$\Rightarrow -\frac{1}{4R_e} = -\frac{1}{R} + \frac{3R_e}{4R^2}$

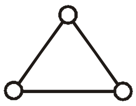
$\Rightarrow -R^2 = -4R_eR + 3R_e^2$

$\Rightarrow R^2 - 4R_eR + 3R_e^2 = 0$

$\Rightarrow (R - R_e)(R - 3R_e) = 0$

$\because R \neq R_e$

$\because R = 3R_e$

17. 

Consider a gas of triatomic molecules. The molecules are assumed to be triangular and made of mass-less rigid rods whose vertices are occupied by atoms. The internal energy of a mole of the gas at temperature T is :

(1) $\frac{5}{2}RT$ (2) 3RT (3) $\frac{3}{2}RT$ (4) $\frac{9}{2}RT$

Ans. (2)

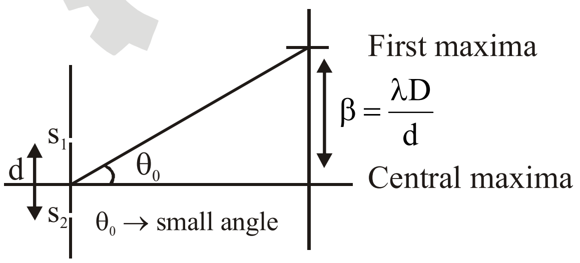
Sol. Degree of freedom for non-linear triatomic molecule = 6

$\therefore$ Internal energy $= \frac{f}{2}RT = \frac{6}{2}RT = 3RT$

18. In a Young's double slit experiment, light of 500 nm is used to produce an interference pattern. When the distance between the slits is 0.05 mm, the angular width (in degree) of the fringes formed on the distance screen is close to :

(1) 0.17º (2) 0.07º (3) 1.7º (4) 0.57º

Ans. (4)

Sol. 

Angular fringe width, $\theta_0 \approx \tan\theta_0 = \frac{\beta}{D}$

$\theta_0 = \frac{\lambda}{d} = \frac{500\times10^{-9}}{5\times10^{-5}} = 10^{-2}$ Radian = 0.57°

19. Two isolated conducting spheres S_1 and S_2 of radius $\frac{2}{3}R$ and $\frac{1}{3}R$ have 12 μC and –3 μC charges, respectively, and are at a large distance from each other. They are now connected by a conducting wire. A long time after this is done the charges on S_1 and S_2 are respectively :

(1) + 4.5 μC and –4.5μC (2) 3 μC and 6μC

(3) 6 μC and 3μC (4) 4.5 μC on both

Ans. (3)

Sol. Total charge = 12 – 3 = 9μC

Let final charges are q_1 & q_2

$$q_1 + q_2 = 9\mu C$$

and after connecting potential of S_1 & S_2 becomes same.

$$V_{S_1} = V_{S_2} \Rightarrow \frac{Kq_1}{\frac{2R}{3}} = \frac{Kq_2}{\frac{R}{3}}$$

$$\Rightarrow \frac{q_1}{q_2} = \frac{2}{1} \qquad .....(ii)$$

from (i) & (ii)

$\Rightarrow q_1 = 6\mu C$ & $q_2 = 3\mu C$

20. A 750 Hz, 20 V (rms) source is connected to a resistance of 100 Ω, an inductance of 0.1803 H and a capacitance of 10 μF all in series. The time in which the resistance (heat capacity 2 J/°C) will get heated by 10°C. (assume no loss of heat to the surroundings) is close to :

(1) 348 s (2) 418 s (3) 245 s (4) 365 s

Ans. (1)

Sol. Heat required Q = ms Δ T = 2 × 10 = 20 Joule

$\omega = 2\pi \times 750 = 1500\pi$, $X_L = \omega l = 1500\pi \times 0.1803 = 849.2\ \Omega$

$$X_C = \frac{1}{\omega c} = \frac{10^6}{1500\pi \times 10} = 21.2\Omega$$

$$i = \frac{V}{Z} = \frac{V}{\sqrt{R^2 + (X_C - X_C)^2}} = \frac{20}{\sqrt{(100)^2 + (828)^2}} = \frac{20}{834}$$

and $Q = i^2 Rt$

$$\Rightarrow 20 = \left(\frac{20}{834}\right)^2 \times 100 \times t \Rightarrow t = 348 \text{ sec}$$

21. A person of 80 kg mass is standing on the rim of a circular platform of mass 200 kg rotating about its axis at 5 revolutions per minute (rpm). The person now starts moving towards the centre of the platform. What will be the rotational speed (in rpm) of the platform when the person reaches its centre ______.

Ans. 9

Sol. $M = 200\text{kg}, m = 80 \text{ kg}, w_i = 5\text{rpm}, w_f = ?$

from angular momentum conservation,

$L_i = L_f$

$\Rightarrow I_i w_i = I_f w_f$

$$\Rightarrow \left[\frac{MR^2}{2} + mR^2\right] 5 \left[= \frac{MR^2}{2} + 0\right] w_f$$

$$\Rightarrow \left(\frac{200}{2} + 80\right) 5 = \left(\frac{200}{2}\right) w_f$$

$$\Rightarrow w_f = \frac{180 \times 5}{100} = 9\text{rpm}$$

22. A bakelite beaker has volume capacity of 500 cc at 30°C. When it is partially filled with V_m volume (at 30°C) of mercury, it is found that the unfilled volume of the beaker remains constant as temperature is varied. If $\gamma_{(beaker)} = 6 \times 10^{-6}$ ºC^{-1} and $\gamma_{(mercury)} = 1.5 \times 10^{-4}$ ºC^{-1} where γ is the coefficient of volume expansion, then V_m (in cc) is close to ....

Ans. 20

Sol. $\Delta V_{Beakar} = \Delta V_{mercury}$

$\Rightarrow 500\, \gamma_{Beakar}\, \Delta T = V_M\, \gamma_{Hg} \Delta T$

$\Rightarrow V_M = \frac{500\times6\times10^{-6}}{1.5\times10^{-4}} = 20cc$

23. When a long glass capillary tube of radius 0.015 cm is dipped in a liquid, the liquid rises to a height of 15 cm within it. If the contact angle between the liquid and glass to close to 0°, the surface tension of the liquid, in milliNewton m^{-1}, is $[\rho_{(liquid)} = 900\ kgm^{-3}, g = 10\ ms^{-2}]$ (Give answer in closest integer) ....

Ans. 101

Sol. $h = \frac{2S\cos\theta}{\rho gr}$

$\Rightarrow S = \frac{\rho grh}{2\cos\theta} = \frac{900\times10\times0.015\times10^{-2}\times15\times10^{-2}}{2\cos\theta}$

$= 101.25\times10^{-3}\ \frac{N}{M}$

$\approx 101\frac{\text{milli Newton}}{m}$

24. A cricket ball of mass 0.15 kg is thrown vertically up by a bowling machine so that it rises to a maximum height of 20 m after leaving the machine. If the part pushing the ball applies a constant force F on the ball and moves horizontally a distance of 0.2 m while launching the ball, the value of F(in N)is(g = 10 ms^{-2})______.

Ans. 150

Sol. From work energy theorem,

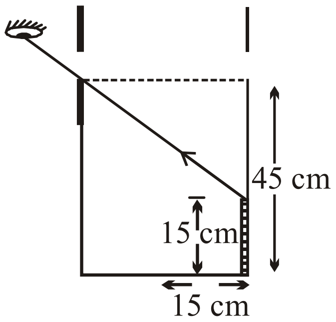
$w_F + w_g = \Delta K$

$F(0.2) - mg(20) = 0$

$F = mg\frac{(20)}{0.2}$

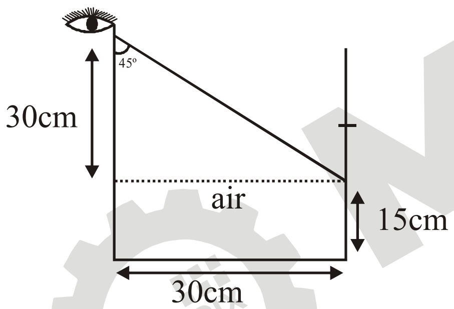
$= 0.15\times10\times\frac{20}{0.2} = 150N$

25. An observer can see through a small hole on the side of a jar (radius 15 cm) at a point at height of 15 cm from the bottom (see figure). The hole is at a height of 45 cm. When the jar is filled with a liquid up to a height of 30 cm the same observer can see the edge at the bottom of the jar. If the refractive index of the liquid is N/100, where N is an integer, the value of N is __________ .

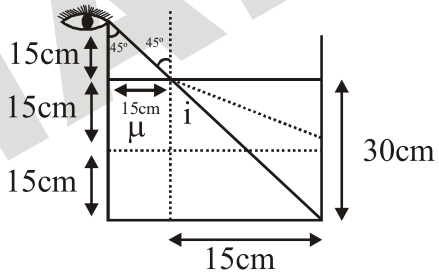


Ans. 158

Sol.



(Before filling)



(after filling water)

$\tan i = \frac{15}{30} = \frac{1}{2}$

$\sin i = \frac{1}{\sqrt{5}}$

and from snell's law : -

(i) $\sin 45° = \mu \sin i \Rightarrow \mu = \frac{\sin 45°}{\sin i} = \sqrt{\frac{5}{2}} = 1.58 = \frac{158}{100}$

$\therefore$ N = 158