JEE Main September 2020

Question Paper With Text Solution

3 September| Shift-2

PHYSICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN SEP 2020 | 3 SEP SHIFT-2

1. Concentric metallic hollow spheres of radii R and 4R hold charges Q_1 and Q_2 respectively. Given that surface charge densities of the concentric spheres are equal, the potential difference V(R) – V(4R) is :

(1) $\frac{3Q_2}{4\pi\varepsilon_0 R}$ (2) $\frac{3Q_1}{4\pi\varepsilon_0 R}$ (3) $\frac{3Q_1}{16\pi\varepsilon_0 R}$ (4) $\frac{Q_2}{4\pi\varepsilon_0 R}$

Ans. (3)

Sol. $V_{inner} = \frac{KQ_1}{R} + \frac{KQ_2}{4R}$

$V_{outer} = \frac{KQ_1}{4R} + \frac{KQ_2}{4R}$

Potential difference

$$\Delta V = V_{inner} - V_{outer}$$

$$= \frac{3}{4} \cdot \frac{KQ_1}{R} = \frac{3}{16\pi \in_0} \cdot \frac{Q_1}{R}$$

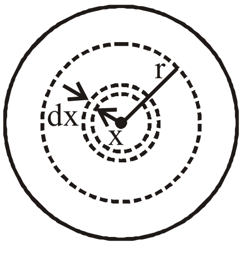
2. The mass density of a planet of radius R varies with the distance r from its centre as $\rho(r) = \rho_0\left(1 - \frac{r^2}{R^2}\right)$.

Then the gravitational field is maximum at :

(1) $r = \sqrt{\frac{5}{9}}R$ (2) $r = \sqrt{\frac{3}{4}}R$ (3) $r = \frac{1}{\sqrt{3}}R$ (4) $r = R$

Ans. (1)

Sol.



$dm = \rho \times 4\pi x^2 dx$

$$= \rho_0\left(1 - \frac{x^2}{R^2}\right) \times 4\pi x^2 dx$$

gravitational field due to small element, $dE = \frac{Gdm}{r^2}$

$$E = \int_0^r \frac{G\rho_0\left(1-\frac{x^2}{R^2}\right)4\pi x^2 dx}{r^2}$$

$$E = \frac{G\rho_0 4\pi}{r^2}\int_0^r\left(x^2 - \frac{x^4}{R^2}\right)dx$$

$$= \frac{G\rho_0 4\pi}{r^2}\left[\frac{r^3}{3} - \frac{r^5}{5R^2}\right]$$

$$= G\rho_0 4\pi\left[\frac{r}{3} - \frac{r^3}{5R^2}\right]$$

E is maximum when $\frac{dE}{dr} = 0 \Rightarrow \frac{dE}{dr} = 4\pi G\rho_0\left(\frac{1}{3} - \frac{3r^2}{5R^2}\right) = 0$

$$\Rightarrow \frac{3r^2}{5R^2} = \frac{1}{3} \Rightarrow r^2 = \frac{5R^2}{9} \Rightarrow r = \frac{\sqrt{5}}{3}R = \sqrt{\frac{5}{9}}R$$

3. The electric field of a plane electromagnetic wave propagating along the x direction in vacuum is $\vec{E} = E_0\hat{j}\cos(\omega t - kx)$. The magnetic field $\vec{B}$, at the moment t = 0 is :

(1) $\vec{B} = E_0\sqrt{\mu_0 \in_0}\cos(kx)\hat{j}$

(2) $\vec{B} = E_0\sqrt{\mu_0 \in_0}\cos(kx)\hat{k}$

(3) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \in_0}}\cos(kx)\hat{j}$

(4) $\vec{B} = \frac{E_0}{\sqrt{\mu_0 \in_0}}\cos(kx)\hat{k}$

Ans. (2)

Sol. $B_0 = \frac{E_0}{C} = \frac{E_0}{1/\sqrt{\mu_0 \in_0}} = E_0\sqrt{\mu_0 \in_0}$

As the light is propagating in x direction

& $\hat{E} \times \hat{B} \parallel \hat{C}$

$\therefore$ $\vec{B}$ should be in $\hat{k}$ direction

$\therefore \quad \vec{B} = B_0 \cos(\omega t - kx)\hat{k}$

At $\quad t = 0, \quad \vec{B} = B_0 \cos kx\,\hat{k} = E_0\sqrt{\mu_0 \in_0}\left(\cos kx\right)\hat{k}$

4. Two sources of light emit X–rays of wavelength 1nm and visble light of wavelength 500 nm, respectively. Both the sources emit light of the same power 200W. The ratio of the number density of photons of X–rays to the number density of photons of the visible light of the given wavelengths is :

(1) 500 (2) $\frac{1}{500}$ (3) $\frac{1}{250}$ (4) 250

Ans. (2)

Sol. P_S – Power of sources

$P_S = n\frac{hc}{\lambda}$; n = no. of photons emitted /s

$\Rightarrow \quad n \propto \lambda \quad \Rightarrow \quad \frac{n_2}{n_1} = \frac{\lambda_2}{\lambda_1} = \frac{1}{500}$

5. Amount of solar energy received on the earth's surface per unit area per unit time is defined as solar constant. Dimension of solar constant is :

(1) ML^2T^{-2} (2) $M^2L^0T^{-1}$ (3) MLT^{-2} (4) ML^0T^{-3}

Ans. (4)

Sol. Solar constant $= \frac{\text{Energy}}{\text{Time Area}}$

$= \frac{M^1L^2T^{-2}}{TL^2} = M^1L^0T^{-3}$

6. Hydrogen ion and singly ionized helium atom are accelerated, from rest, through the same potential difference. The ratio of final speeds of hydrogen and helium ions is close to :

(1) 5 : 7 (2) 2 : 1 (3) 1 : 2 (4) 10 : 7

Ans. (2)

Sol. $K_H = K_{He}$

$\Rightarrow \frac{1}{2}m_H V_H^2 = \frac{1}{2}m_{He}V_{He}^2$

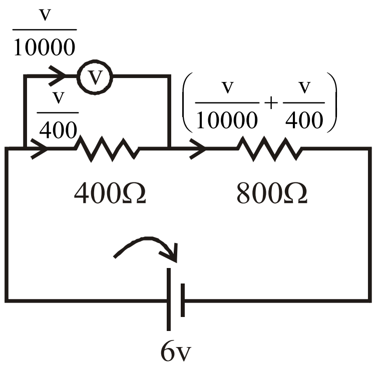
$$\Rightarrow \frac{V_H}{V_{He}} = \sqrt{\frac{m_{He}}{m_H}} = \frac{2}{1}$$

7. Two resistors 400Ω and 800Ω are connected in series across a 6V battery. The potential difference measured by a voltmeter of 10kΩ across 400Ω resistor is close to :

(1) 1.95 V (2) 2.05 V (3) 1.8 V (4) 2 V

Ans. (1)

Sol.



Let voltmeter reading is v

$$\frac{v}{400} \times 400 + \left(\frac{v}{10000} + \frac{v}{400}\right) 800 = 6$$

$$\Rightarrow \quad v + \frac{8v}{100} + 2v = 6$$

$$\frac{77v}{25} = 6$$

$$v = \frac{150}{77} = 1.95v$$

8. A calorimeter of water equivalent 20 g contains 180 g of water at 25ºC. 'm' grams of steam at 100ºC is mixed in it till the temperature of the mixure is 31ºC. The value of 'm' is close to (Latent heat of water = 540 cal g^{-1}, specific heat of water = 1 cal g^{-1} $ºC^{-1}$)

(1) 4 (2) 2 (3) 2.6 (4) 3.2

Ans. (2)

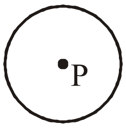
Sol. Heat loss = Heat gain

$\Rightarrow$ $(m \times 540) + m \times 1 \times (100 - 31)$
$= (180 + 20) \times 1 \times (31 - 25)$

$\Rightarrow$ $m\,(540 + 69) = 200 \times 6$

$\Rightarrow$ $m = \dfrac{200 \times 6}{609} = 1.97 \approx 2$

9. A perfectly diamagnetic sphere has a small spherical cavity at its centre, which if filled with a paramagnetic substance. The whole system is placed in a uniform magnetic field $\vec{B}$. Then the field inside the paramagnetic substance is :



(1) much large tha $|\vec{B}|$ but opposite to $\vec{B}$

(2) zero

(3) $\vec{B}$

(4) much large than $|\vec{B}|$ and parallel to $\vec{B}$

Ans. (2)

Sol. A perfect diamagnetic substance will completely expel the magnetic field. So no magnetic field line will enter inside the sphere. So net magnetic field at every point inside the sphere will be zero.

10. If a semiconductor photodiode can detect a photon with a maximum wavelength of 400 nm, then its band gap energy is :

Planck's constant $h = 6.63 \times 10^{-34}$ J.s.

Speed of light $c = 3 \times 10^{8}$ m/s

(1) 1.1 eV (2) 3.1 eV (3) 2.0 eV (4) 1.5 eV

Ans. (2)

Sol. $\lambda = 400\,\text{nm}$

Band gap energy $E_g = \dfrac{hc}{\lambda} = \dfrac{1240}{400} \approx 3.1\text{eV}$

11. A metallic sphere cools from 50ºC to 40ºC in 300 s. If atmospheric temperature around is 20ºC, the the sphere's temperature after the next 5 minutes will be close to :

(1) 31ºC (2) 33ºC (3) 28ºC (4) 35ºC

Ans. (2)

Sol. Using Newton's Law of cooling

$$\frac{50-40}{300} = K\left(\frac{50+40}{2} - 2\right) \quad ....(i)$$

Next 5 Min.

$$\frac{40-\theta}{300} = K\left(\frac{40+\theta}{2} - 20\right) \quad ....(ii)$$

Dividing (ii) / (i)

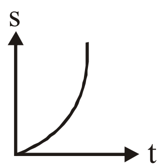
$$\frac{40-\theta}{10} = \frac{40+\theta-40}{50+40-40} = \frac{\theta}{50}$$

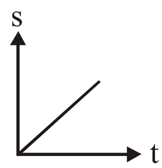
$$40 - \theta = \frac{\theta}{5}$$

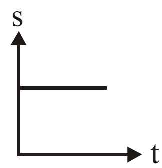
$$200 - 5\theta = \theta$$

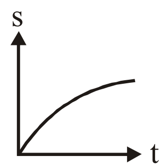
$$\therefore\ \theta = \frac{200}{6} = 33.3^{\circ}C$$

12. A particle is moving unidirectionally on a horizontal plane under the action of a constant power supplying energy source. The displacement (s) –time (t) graph that describes the motion of the particle is (graphs are drawn schematically and are not to scale) :

(1) 

(2) 

(3) 

(4) 

Ans. (1)

Sol. $P.t = \frac{1}{2}mv^2 \Rightarrow v = \left(\sqrt{\frac{2P}{m}}\right)t^{1/2}$

$$s = \int_0^t v dt = \sqrt{\frac{2P}{m}} \int_0^t t^{1/2} dt$$

$$= \sqrt{\frac{2P}{m}} . \frac{t^{3/2}}{3/2}$$

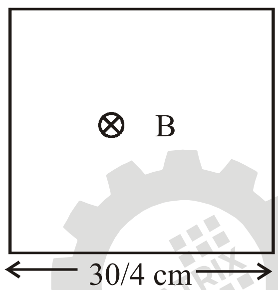
$$s = \sqrt{\frac{8P}{9m}} . t^{3/2}$$

13. A unifrom magnetic field B exists in a direction perpendicular to the plane of a square loop made of a metal wire. the wire has diameter of 4 mm and a total lenth of 30cm. The magnetic field changes with time at a steady rate dB/dt = 0.032 Ts^{-1}. The induced current in the loop is close to (Resistivity of the metal wire is $1.23 \times 10^{-8}\,\Omega m$)

(1) 0.61A (2) 0.43A (3) 0.53A (4) 0.34A

Ans. (1)

Sol.



Radius of cross section of wire = $\frac{d}{2} = 2mm = 2 \times 10^{-3}\,m$

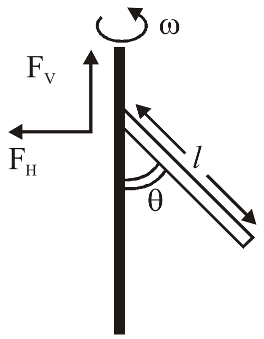
$$\frac{dB}{dt} = 0.032 \frac{T}{s}$$

$$R = \frac{\rho l}{a} = \frac{(1.23 \times 10^{-8})(0.3)}{\pi \times (2 \times 10^{-3})^2} = 0.0294 \times 10^{-2}\,\Omega$$

$$\phi = BA = B\left(\frac{0.3}{4}\right)^2$$

$$|E| = \frac{d\phi}{dt} = \frac{(0.3)^2}{16}\frac{dB}{dt} = 0.018 \times 10^{-2}$$

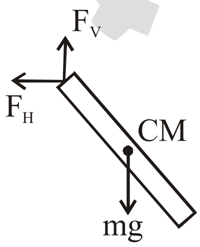
$$i = \frac{E}{R} = \frac{0.018\times10^{-2}}{0.0294\times10^{-2}} = 0.61A$$

14. 

A uniform rod of lenght 'l' is pivoted at one of its ends on a vertical shaft of negligible radius. When the shaft rotates at angular speed ω the rod makes an angle θ with it (see figure). To find θ equate the rate of change of angular momentum (direction going into the paper) $\frac{ml^2}{12}\omega^2 \sin\theta \cos\theta$ about the centre of mass (CM) to the torque provided by the horizontal and vertical forces F_H and F_V about the CM. The value of θ is then such that :

(1) $\cos\theta = \frac{g}{2l\omega^2}$
(2) $\cos\theta = \frac{2g}{2l\omega^2}$
(3) $\cos\theta = \frac{2g}{2l\omega^2}$
(4) $\cos\theta = \frac{g}{l\omega^2}$

Ans. (2)

Sol. 

$F_V = mg$

$F_H = F_{centripetal} = m\omega^2 \frac{l}{2} \sin\theta$

$\tau_{cm} = 0$

$$F_V \frac{l}{2}\sin\theta - F_H\frac{l}{2}\cos\theta = \frac{ml^2}{12}\omega^2 \sin\theta\cos\theta$$

$$\Rightarrow mg\ \frac{l}{2}\sin\theta - m\omega^2\ \frac{l}{2}\sin\theta\left(\frac{l}{2}\cos\theta\right) = \frac{ml^2}{12}\omega^2\sin\theta\cos\theta$$

$$\Rightarrow \cos\theta = \frac{3g}{2l\omega^2}$$

15. A bock of mass 1.9 kg is at rest at the edge of a table, of height 1m. A bullet of mass 0.1 kg collides with the block and sticks to it. If the velocity of the bullet is 20m/s in the horizontal direction just before the collision then the kinetic energy just before the combined system strikes the floor, is [Take g = 10 m/s^2. Assume there is no rotational motion and loss of energy after the collision is negligable.]

(1) 19 J (2) 23 J (3) 21 J (4) 20 J

Ans. (3)

Sol. Conservation of linear momentum

$0.1 \times 20 = (0.1 + 1.9) \times v$

$v = 1$ m/s

Using work energy theorem

$W_g = \Delta k$

$2 \times g \times 1 = k - \frac{1}{2} \times 2 \times 1^2$

$\therefore\ k = 21$ J

16. The radius R of a nucleus of mass number A can be estimated by the formula $R = (1.3 \times 10^{-15})\,A^{1/3}$ m. It follows that the mass density of a nucleus is of the order of ; ($M_{prot.} \cong M_{neut.} \simeq 1.67 \times 10^{-27}$ kg)

(1) 10^{24} kg m^{-3} (2) 10^{10} kg m^{-3} (3) 10^{3} kg m^{-3} (4) 10^{17} kg m^{-3}

Ans. (4)

Sol. density $= \dfrac{\text{mass}}{\text{volume}}$

$$= \frac{\left(1.67 \times 10^{-27}\right)A}{\frac{4}{3}\pi\left[1.3 \times 10^{-15}\right]^3 \times A}$$

$$\approx 10^{17}\,\frac{kg}{m^3}$$

17. To raise the temperature of a certain mass of gas by 50ºC at a constant pressure, 160 calories of heat is required. When the same mass of gas is cooled by 100ºC at constant volume, 240 calories of heat is released. How many degrees of freedom does each molecule of this gas have (assume gas to be ideal)?

(1) 6 (2) 7 (3) 3 (4) 5

Ans. (1)

Sol. At constant pressure :

$$\Delta Q = nC_P\Delta T$$

$\Rightarrow$ $160 = nC_P\, 50$ ...(1)

At constant volume

$$\Delta Q = nC_V\Delta T$$

$\Rightarrow$ $240 = nC_V\, 100$ ...(2)

Equation (1) divided by (2)

$$\frac{160}{240} = \frac{C_P}{C_V}\frac{50}{100}$$

$$\frac{C_P}{C_V} = \frac{4}{3} = 1 + \frac{2}{f}$$

f = 6

18. Two light waves having the same wavelength λ in vacuum are in phase initially. Then the first wave travels a path L_1 thorugh a medium of refractive index n_1 while the second wave travels a path of length L_2 through a medium of refractive index n_2. After this the phase difference between the two waves is :

(1) $\frac{2\pi}{\lambda}(n_2L_1 - n_1L_2)$ (2) $\frac{2\pi}{\lambda}\left(\frac{L_1}{n_1} - \frac{L_2}{n_2}\right)$

(3) $\frac{2\pi}{\lambda}(n_1L_1 - n_2L_2)$ (4) $\frac{2\pi}{\lambda}\left(\frac{L_2}{n_1} - \frac{L_1}{n_2}\right)$

Ans. (3)

Sol. Here, optical path for first ray = n_1L_1

Optical path for second ray = n_2L_2

Path difference = $n_1L_1 - n_2L_2$

Now, phase difference

$$= \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} \times (n_1L_1 - n_2L_2)$$

19. A block of mass m attached to a massles spring is performing oscillatory motion of amplitude 'A' on a frictionless horizontal plane. If half of the mass of the block breaks off when it is passing through its equilibrium point, the amplitude of oscillation for the remaining system become fA. The value of f is:

(1) 1 (2) $\sqrt{2}$ (3) $\frac{1}{2}$ (4) $\frac{1}{\sqrt{2}}$

Ans. (4)

Sol. $V_i = A\omega = A\sqrt{\frac{k}{m}}; V_f = A'\omega' = A'\sqrt{\frac{k}{\frac{m}{2}}}$

and at mean position $V_1 = V_2$

$\Rightarrow \omega_1 A_1 = \omega_2 A_2$

$\Rightarrow \sqrt{\frac{K}{m}}\, A = \sqrt{\frac{2K}{m}}\, A_2$

$\Rightarrow A_2 = \frac{A}{\sqrt{2}}$

20. Which of the following will NOT be observed when a multimeter (operating in resistance measuring mode) probes connected across a component, are just reversed ?

(1) Multimeter shows a deflection, accompanied by a splash of light out of connected component in one direction and NO deflection on reversing the probes if the chosen component is LED.

(2) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is capacitor.

(3) Multimeter shows an equal deflection in both cases i.e. before and after reversing the probes if the chosen component is resistor.

(4) Multimeter shows NO deflection in both cases i.e. before and after reversing the probes if the chosen component is metal wire.

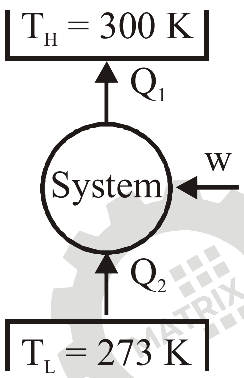
Ans. (2)

Sol. (1) If we assume that LED has negligible resistance then multimeter shows no deflection for forward bias but when it connects in reverse direction, its break down occurs so it shows a deflection, accompained by a splash of light out.
(2) Multimeter shows deflection when it connects with capacitor.
(3) No matter, how we connect the resistance across multimeter, it shows same deflection.
(4) The resistance of metal wire may be taken zero, so deflection in multimeter.

21. If minimum possible work is done by a refrigerator in converting 100 grams of water at 0ºC to ice, how much heat (in calories) is released to the surroundings at temperature 27ºC (Latent heat of ice=80 Cal/gram) to the nearest integer ?

Ans. 8791

Sol.



$Q_2 = ML = 8000$ Cal.

$w = Q_1 - Q_2$

and

$$\frac{w}{Q_2} = \left(\frac{T_H}{T_L} - 1\right)$$

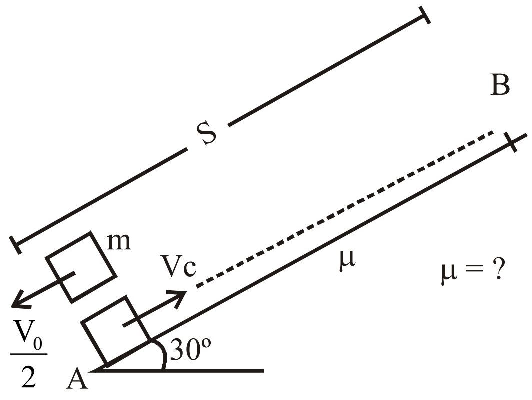
$$\Rightarrow \frac{Q_1}{Q_2} - 1 = \frac{T_H}{T_L} - 1$$

$$\Rightarrow Q_1 = \frac{T_H}{T_L} Q_2$$

$$= \frac{300}{273} \times 8000 = 8791 \, \text{Cal}$$

22. A block starts moving up an inclined plane of inclination 30º with an initial velocity of v_0. It comes back to its initial position with velocity $\frac{v_0}{2}$. The value of the coefficient fo kinetic friction between the block and the inclined plane is close to $\frac{I}{1000}$, The nearest integer to I is ________.

Ans. 346

Sol. 

For up the inclined motion (A → B)

$a_1 = g\sin30^o + \mu g\cos30^o = 5+5\sqrt{3}\mu$

and

$V_0^2 - 2a_1(s) = 0$

$s = \frac{V_0^2}{a_1}$ ....(i)

For down the inclined motion (B → A)

$a_2 = g\sin30^o - \mu g\cos30^o = 5-5\sqrt{3}\mu$

and

$\left(\frac{V_0}{2}\right)^2 = 2a_2\,(s)$

$s = \frac{V_0^2}{4a_2}$ ....(ii)

From equation (i) and (ii)

$\frac{V_0^2}{a_1} = \frac{V_0^2}{4a_2}$

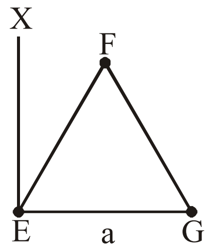
$\Rightarrow a_1 = 4a_2$

$\Rightarrow 5 + 5\sqrt{3}\mu = 4\{5 - 5\sqrt{3}\mu\}$

$\Rightarrow\ 5 + 5\sqrt{3}\mu = 20 - 20\sqrt{3}\mu \Rightarrow 25\sqrt{3}\mu = 15 \Rightarrow \mu = \dfrac{\sqrt{3}}{5} = 0.346 = \dfrac{I}{1000}$

$\therefore\ I = 346$

23. An massless equilateral triangle EFG of side 'a' (as shown in figure) has three particles of mass m situated at its vertices. The moment of inertia of the system about the line EX perpendicular to EG in the plane of EFG is $\dfrac{N}{20}ma^2$ where N is an integer. The value of N is _______.



Ans. 25

Sol. $I = m \times O^2 + ma^2 + m\left(\dfrac{a}{2}\right)^2$

$= \dfrac{5}{4}ma^2 = \dfrac{25}{20}ma^2$

N = 25

24. When an object is kept at a distance of 30 cm from a concave mirror, the image is formed at a distance of 10 cm from the mirror. If the object is moved with a speed of 9 cm^{-1}, the speed (in cms^{-1}) with which image moves at that instant is _________.

Ans. 1

Sol. Velocity of image,

$v_i = -\dfrac{v^2}{u^2}v_0$

$= -\left(\dfrac{10}{30}\right)^2(9)$

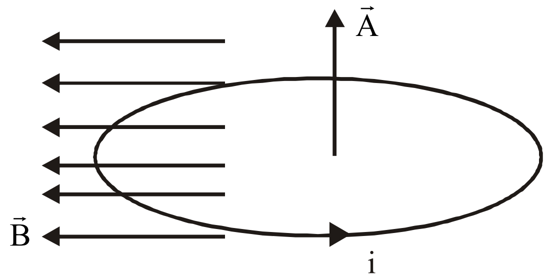
$= -1$cm/sec

$\therefore$ Speed of image = 1cm/sec

25. A galvanometer coil has 500 turns and each turn has an average area of 3×10^{-4} m^2. If a torque of 1.5 Nm is required to keep this coil parallel to a magnetic field when a current of 0.5 A is flowing through it, the strength of the field (in T) is _______.

Ans. 20

Sol.



$\tau = \left|\vec{M} \times \vec{B}\right| = NiAB\sin(90^\circ)$

$= NiAB$

$\Rightarrow \quad B = \dfrac{\tau}{NiS} = \dfrac{1.5}{500 \times 0.5 \times 3 \times 10^{-4}} = \dfrac{10^4}{500} = 20\,T$