PART - B
MATHEMATICS
SINGLE CORRECT CHOICE TYPE

Q.31 to Q.60 has four choices (A), (B), (C), (D) out of which ONLY ONE is correct.

31. If the tangent at (1, 7) to the curve $x^2 = y - 6$ touches the circle $x^2 + y^2 + 16x + 12y + c = 0$ then the value of c is

यदि वक्र $x^2 = y - 6$ के बिन्दु (1, 7) पर बनी स्पर्शरेखा वृत $x^2 + y^2 + 16x + 12y + c = 0$ को स्पर्श करती है, तो c का मान है –

(1) 85 (2) 95 (3) 195 (4) 185

Ans. 2

Sol. $x^2 = y - 6$

$\left(\frac{dy}{dx}\right)_{(1,7)} = 2$

$T(1, 7) = 2x - y + 5 = 0$

distance of center from tangent = radius

$r = \left|\frac{-16+6+5}{\sqrt{5}}\right| = \sqrt{5}$

$\sqrt{g^2 + f^2 - c} = \sqrt{5}$

$\Rightarrow c = 95$

32. If L_1 is the line of intersection of the planes $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ and L_2 is the line of intersection of the planes $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$, then the distance of the origin from the plane containing the lines L_1 and L_2 is

यदि समतलों $2x - 2y + 3z - 2 = 0$, $x - y + z + 1 = 0$ की परिच्छेदी रेखा L_1 है तथा समतलों $x + 2y - z - 3 = 0$, $3x - y + 2z - 1 = 0$ की परिच्छेदी रेखा L_2 है, तो मूल बिन्दु की दूरी उस सममतल से जो रेखाओं L_1 और L_2 का अंतर्विष्ट करता है, है –

(1) $\frac{1}{2\sqrt{2}}$ (2) $\frac{1}{\sqrt{2}}$ (3) $\frac{1}{4\sqrt{2}}$ (4) $\frac{1}{3\sqrt{2}}$

Ans. 4

Sol. required plane will pass though line of intersection of planes so equation of required plane will be

$x + 2y - z - 3 + \lambda(3x - y + 2z - 1) = 0$

take a fixed point on L_1 (y = 0)

P(–5, 0, 4)

point P will lie on required plane

$\Rightarrow \lambda = -\frac{3}{2}$

So equation of required plane will be

$7x - 7y + 8z + 3 = 0$

distance from origin $= \left|\frac{3}{\sqrt{162}}\right| = \frac{1}{3\sqrt{2}}$

33. If $\alpha, \beta \in C$ are the distinct roots, of the equation $x^2 - x + 1 = 0$, then $\alpha^{101} + \beta^{107}$ is equal to

यदि $\alpha, \beta \in C$ समीकरण $x^2 - x + 1 = 0$ के विभिन्न मूल है, तो $\alpha^{101} + \beta^{107}$ बराबर है –

(1) 1 (2) 2 (3) –1 (4) 0

Ans. 1

Sol. Roots of $x^2 - x + 1 = 0$ are $\alpha = -\omega, \beta = -\omega^2$

$\alpha^{101} + \beta^{107} = -\omega^{101} - \omega^{214}$

$-\omega^2 - \omega = 1$

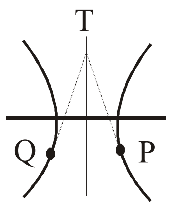
34. Tangent are drawn to the hyperbola $4x^2 - y^2 = 36$ at the points P and Q. If these tangents intersect at the point T(0, 3) then the area (in sq. units) of ΔPTQ is

एक अतिपरवलय $4x^2 - y^2 = 36$ के बिंदुओं P तथा Q पर स्पर्श रेखाएँ खींची जाती है। यदि यह स्पर्शरेखाएँ बिंदु T(0, 3) पर काटती है, तो ΔPTQ का क्षेत्रफल (वर्ग इकाइयों में) है –

(1) $60\sqrt{3}$ (2) $36\sqrt{5}$ (3) $45\sqrt{5}$ (4) $54\sqrt{3}$

Ans. 3

Sol.



Eqaution of tangent at P

$\frac{x\sec\theta}{3} - \frac{y\tan\theta}{6} = 1$

it will pass though (0, 3)

$0 - \frac{3\tan\theta}{6} = 1$

$\tan\theta = -2 \Rightarrow \sec\theta = \sqrt{5}$

area $= \frac{1}{2}|6\sec\theta|(|6\tan\theta| + 3)$

$= 45\sqrt{5}$

35. If the curves $y^2 = 6x$, $9x^2 + by^2 = 16$ intersect each other at right angles, then the value of b is

यदि वक्र $y^2 = 6x$, $9x^2 + by^2 = 16$ समकोण पर प्रतिच्छेद करते हैं, तो b का मान है –

(1) 4 (2) $\frac{9}{2}$ (3) 6 (4) $\frac{7}{2}$

Ans. 2

Sol. $y^2 = 6x$

$2yy' = 6$

$y' = \frac{3}{y_1}$

$9x^2 + by^2 = 16$

$18x + 2byy' = 0$

$y' = \frac{-9x_1}{by_1}$

$m_1m_2 = -1$

$27x_1 = by_1^2$ ..... (i)

$6x_1 = y_1^2$ ..... (ii)

(i) ÷ (ii)

$b = \frac{9}{2}$

36. If the system of linear equations

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

has a non-zero solution (x, y, z), then $\frac{xz}{y^2}$ is equal to

यदि रैखिक समीकरण निकाय

$x + ky + 3z = 0$

$3x + ky - 2z = 0$

$2x + 4y - 3z = 0$

का एक शून्येतर हल (x, y, z) है, तो $\frac{xz}{y^2}$ बराबर है –

(1) –30 (2) 30 (3) –10 (4) 10

Ans. 4

Sol. $x + ky + 3z = 0$ ...... (i)

$3x + ky - 2z = 0$ .... (ii) system will have ∞ solutions

$2x + 4y - 3z = 0$ .... (iii)

(i) – (ii)

$x = \frac{5z}{2}$ then $y = \frac{-z}{2}$

$\frac{xy}{z^2} = 10$

37. Let $S = \{x \in R: x \geq 0$ and $2|\sqrt{x}+3| + \sqrt{x}(\sqrt{x}-6)+6=0\}$. Then S :

(1) contains exactly two elements (2) contains exactly four elements

(3) is an empty set (4) contains exactly one element

माना $S = \{x \in R: x \geq 0$ तथा $2|\sqrt{x}+3| + \sqrt{x}(\sqrt{x}-6)+6=0\}$ तो S

(1) में मात्र दो अवयव हैं (2) में मात्र चार अवयव हैं

(3) एक रिक्त समुच्चय हैं (4) में मात्र एक ही अवयव हैं

Ans. 1

Sol. $2\left|\sqrt{x}-3\right| + x - 6\sqrt{x} + 6 = 0$

Put $\left|\sqrt{x}-3\right| = t$

$2t + t^2 - 3 = 0$

$t = 1$ $t = -3$

$\left|\sqrt{x}-3\right| = 1$ $\left|\sqrt{x}-3\right| = -3$

$\sqrt{x} = 4, \sqrt{x} = 2$ $x = \phi$

$x = 16, x = 4$

38. If sum of all the solution of the equation $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ in $[0, \pi]$ is $k\pi$, then k is equal to

यदि समीकरण $8\cos x.\left(\cos\left(\frac{\pi}{6}+x\right).\cos\left(\frac{\pi}{6}-x\right)-\frac{1}{2}\right)=1$ के अंतराल $[0, \pi]$ में सभी हलों का योग $k\pi$ है, तो k बराबर है–

(1) $\frac{8}{9}$ (2) $\frac{20}{9}$ (3) $\frac{2}{3}$ (4) $\frac{13}{9}$

Ans. 4

Sol. $8\cos x.\cos\left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right) - 4\cos x = 1$

$8\cos x\left(\frac{3}{4}-\sin^2 x\right) - 4\cos x = 1$

$2\cos x\left(\frac{4\cos^2 x - 1}{4}\right) - 4\cos x = 1$

$8\cos^3 x - 2\cos x - 4\cos x = 1$

$8\cos^3x - 6\cos x = 1$

$\cos 3x = 1/2$

$3x = 2n\pi \pm \frac{\pi}{3}$

$x = \frac{\pi}{9}, \frac{5\pi}{9}, \frac{7\pi}{9}$

$\text{Sum} = \frac{13\pi}{9}$

39. A bag contains 4 red and 6 black balls. A ball is drawn at random from the bag, its colour is observed and this ball along with two additional balls of the same colour are returened to the bag. If now a ball is drawn at random from the bag, then the probability that this drawn ball is red, is

एक थैले में 4 लाल तथा 6 काली गेंदें है। थैले में से यादृच्छया एक गेंद निकाली गयी, तथा उसका रंग देखकर, उस गेंद को, दो अन्य उसी रंग की गेंदों के साथ वापिस थैले में डाल दिया गया। अब यदि थैले में से यादृच्छया एक गेंद निकाली जाए, तो प्रायिकता कि उस गेंद का रंग लाल है, है –

(1) $\frac{1}{5}$ (2) $\frac{3}{4}$ (3) $\frac{3}{10}$ (4) $\frac{2}{5}$

Ans. 4

Sol. = P (Red in Ist turn & Red in IInd turn) + P(Black in Ist turn & Red in IInd turn)

$\left(\frac{4}{10}\times\frac{6}{12}\right) + \left(\frac{6}{10}\times\frac{4}{12}\right) = \frac{2}{5}$

40. Let $f(x) = x^2 + \frac{1}{x^2}$ and $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$. If $h(x) = \frac{f(x)}{g(x)}$, then the local minimum value of $h(x)$ is

माना $f(x) = x^2 + \frac{1}{x^2}$ तथा $g(x) = x - \frac{1}{x}$, $x \in R - \{-1, 0, 1\}$ हैं। यदि $h(x) = \frac{f(x)}{g(x)}$ है, तो $h(x)$ का स्थानीय न्यनृतम मान है –

(1) $-2\sqrt{2}$ (2) $2\sqrt{2}$ (3) 3 (4) –3

Ans. 2

Sol. $h(x) = \frac{x^2 + \frac{1}{x^2}}{1 - \frac{1}{x}}$

$h(x) = \frac{t^2 + 2}{t} = t + \frac{2}{t}$ where $t = x - \frac{1}{x}$

$h'(x)\left(1 - \frac{2}{t^2}\right)\left(\frac{dt}{dx}\right)$

$$h'(x) = \left(\frac{t^2-2}{t}\right)\left(1+\frac{1}{x^2}\right)$$

Minima at $t = \sqrt{2}$

$$h(x)|_{Min} = \frac{2+2}{\sqrt{2}} = 2\sqrt{2}$$

41. Two sets A and B are as under :

A = $\{(a, b) \in R \times R : |a-5| < 1 \text{ and } |b-5| < 1\}$: B $\{(a, b) \in R \times R: 4(a-6)^2 + 9(b-5)^2 \le 36\}$. Then

(1) $A \cap B = \phi$ (an empty set) (2) neither $A \subset B$ not $B \subset A$

(3) $B \subset A$ (4) $A \subset B$

दो समुच्चय A तथा B निम्न प्रकार के है –

A = $\{(a, b) \in R \times R : |a-5| < 1$ तथा $|b-5| < 1\}$: B $\{(a, b) \in R \times R: 4(a-6)^2 + 9(b-5)^2 \le 36\}$. तो

(1) $A \cap B = \phi$ (एक रिक्त समुच्चय) (2) न तो $A \subset B$ और न ही $B \subset A$

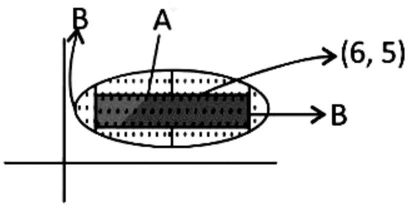
(3) $B \subset A$ (4) $A \subset B$

Ans. 4

Sol. Let $a - 6 = x$ $b - 5 = y$

$|x + 1| < 1$ $|y| < 1$ $4x^2 + 9y^2 \le 36$

$-2 < x < 0$ $-1 < y < 1$



Boundries of square are lying in ellipse

$\Rightarrow A \subset B$

42. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to

बूले के व्यंजक $\sim(p \vee q) \vee (\sim p \wedge q)$ के समतुल्य है –

(1) q (2) ~q (3) ~p (4) p

Ans. 3

Sol.

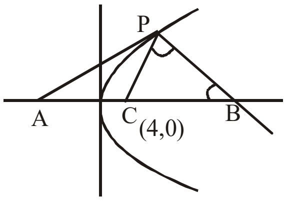
p	q	$p \cup q$	$\sim(p \cup q)$	~(p)	$(\sim p) \wedge q$	$(\sim(p \vee q)) \cup ((\sim p) \wedge q))$
T	T	T	F	F	F	F
T	F	T	F	F	F	F
F	T	T	F	T	T	T
F	F	F	T	T	F	T

43. Tangent and normal are drawn at P(16, 16) on the parabola $y^2 = 16x$, which intersect the axis of the parabola at A and B, respectively. If C is the centre of the circle through the points P, A and B and $\angle CPB = \theta$, then a value of $\tan\theta$ is

परवलय $y^2 = 16x$ के एक बिन्दु P(16, 16) पर स्पर्श रेखा तथा अभिलम्ब खींचे जाते है तो परवलय के अक्ष को बिन्दुओं क्रमशः A तथा B पर प्रतिच्छेद करते हैं। यदि बिन्दुओं P, A तथा B से होकर जाने वाले वृत का केन्द्र C है तथा $\angle CPB = \theta$ तो $\tan\theta$ का एक मान है –

(1) 3 (2) $\frac{4}{3}$ (3) $\frac{1}{2}$ (4) 2

Ans. 4

Sol. 

$\left(-\frac{dx}{dy}\right) = -t = -2$

Slope of normal at $P = -\tan\theta = -2$

$\tan\theta = 2$

44. If $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$, then the ordered pair (A, B) is equal to

यदि $\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$ तो क्रमित युग्म (A, B) बराबर है –

(1) (–4, 5) (2) (4, 5) (3) (–4, –5) (4) (–4, 3)

Ans. 1

Sol. $4x\begin{vmatrix} a & b & c \\ c & a & b \\ b & c & a \end{vmatrix} = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$

$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (5x-4)(x+4)^2$

$A = -4, B = 5$

45. The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ is

$\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x > 1)$ के प्रसार में सभी विषम घातों वाले पदों के गुणांको का योग है –

(1) 1 (2) 2 (3) –1 (4) 0

Ans. 2

Sol. $f(x)=\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$

$= 2(x^5 + 10x^3 (x^3 - 1) + 5x(x^3 - 1)^2)$

Sum of coefficients off add degree $= \dfrac{f(1)-f(-1)}{2} = 2$

46. Let $a_1, a_2, a_3, \ldots\ldots\ldots\ldots a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$. If $a_1^2 + a_2^2 + \ldots\ldots + a_{17}^2 = 140m$, then m is equal to

माना $a_1, a_2, a_3, \ldots\ldots\ldots\ldots a_{49}$ एक समांतर श्रेढ़ी में ऐसे है कि $\sum_{k=0}^{12} a_{4k+1} = 416$ तथा $a_9 + a_{43} = 66$ है। यदि $a_1^2 + a_2^2 + \ldots\ldots + a_{17}^2 = 140m$ है, तो m बराबर है –

(1) 34 (2) 33 (3) 66 (4) 68

Ans. 1

Sol. Let a_1 = a common difference = d

$\sum_{k=0}^{12} a_{4k+1} = 416 \Rightarrow a + 24d = 32$

$a_9 + a_{43} = 66 \Rightarrow a + 25d = 33$

$a = 8$

$d = 1$

$a_1^2 + a_2^2 + \ldots\ldots + a_{17}^2 = 8^2 + 9^2 + \ldots\ldots + 24^2$

$= (1^2 + 2^2 + \ldots\ldots + 24^2) - (1^2 + 2^2 + \ldots + 7^2)$

$4760 = 140\ m$

$m = 34$

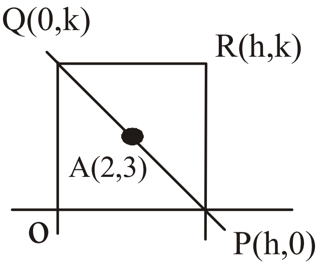
47. A straight line through a fixed point (2, 3) intersects the coordinate axes at distinct points P and Q. If O is the origin and the rectangle OPRQ is completed, then the locus of R is

एक सरल रेखा, जो एक अचर बिन्दु (2, 3) से होकर जाती है, निर्देशांक अक्षों को दो विभिन्न बिन्दुओं P तथा Q पर प्रतिच्छेद करती है। यदि O मूल बिन्दु है तथा आयत OPRQ को पूरा किया जाता है तो R का बिन्दुपथ है –

(1) $3x + 2y = xy$ (2) $3x + 2y = 6xy$ (3) $3x + 2y = 6$ (4) $2x + 3y = xy$

Ans. 1

Sol.



P, A, Q will be collinear

$$\begin{vmatrix} h & 0 & 1 \\ 2 & 3 & 1 \\ 0 & k & 1 \end{vmatrix} = 0$$

$3h + 2k = hk$

locus of R (h, k) will be

$3x + 2y = xy$

48. The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ is

$\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$ का मान है –

(1) 4π (2*) $\frac{\pi}{4}$ (3) $\frac{\pi}{8}$ (4) $\frac{\pi}{2}$

Ans. 2

Sol. $I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$

Use king & Add

$$2I = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \sin^2 x\, dx = 2\int_{0}^{\frac{\pi}{2}} \sin^2 x\, dx$$

$I = \frac{\pi}{4}$

49. Let $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, and α, β ($\alpha < \beta$) be the roots of the quadratic equation $18x^2 - 9\pi x + \pi^2 = 0$. Then the area (in sq. unit) bounded by the curve $y = (gof)(x)$ and the lines $x = \alpha$, $x = \beta$ and $y = 0$ is

माना $g(x) = \cos x^2$, $f(x) = \sqrt{x}$, तथा α, β ($\alpha < \beta$) द्विघाती समीकरण $18x^2 - 9\pi x + \pi^2 = 0$ के मूल हैं। तो वक्र $y = (gof)(x)$ तथा रेखाओं $x = \alpha$, $x = \beta$ तथा $y = 0$ द्वारा घिरे क्षेत्र का क्षेत्रफल (वर्ग इकाइयों में) है–

(1) $\frac{1}{2}\left(\sqrt{3}-\sqrt{2}\right)$ (2) $\frac{1}{2}\left(\sqrt{2}-1\right)$ (3) $\frac{1}{2}\left(\sqrt{3}-1\right)$ (4) $\frac{1}{2}\left(\sqrt{3}+1\right)$

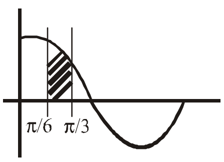
Ans. 3

Sol. $gof(x) = \cos x$

$18x^2 - \pi x + \pi^2 = 0$

$(6x - \pi)(3x - \pi) = 0$

$x = \frac{\pi}{6}, x = \frac{\pi}{3}$



$$A = \int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x \, dx = \frac{\sqrt{3}-1}{2}$$

50. For each $t \in R$, let $[t]$ be the greatest integer less than or equal to t. Then $\lim_{x\to 0+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+......+\left[\frac{15}{x}\right]\right)$

(1) is equal to 120 (2) does not exist (in R)

(3) is equal to 0 (4) is equal to 15

प्रत्येक $t \in R$ के लिए माना $[t]$, t से छोटा महतम पूर्णांक है, तो $\lim_{x\to 0+} x\left(\left[\frac{1}{x}\right]+\left[\frac{2}{x}\right]+......+\left[\frac{15}{x}\right]\right)$

(1) 120 के बराबर है (2) (R में) इसका अस्तित्व नहीं है

(3) 0 के बराबर है (4) 15 के बराबर है

Ans. 1

Sol. $x - 1 < [x] \le x$

$\left(\frac{1}{x}-1\right) < \left[\frac{1}{x}\right] \le \frac{1}{x}$

$x\left(\frac{1}{x}-1\right) < x\left[\frac{1}{x}\right] \le \frac{x}{x}$

use sandwich

$\lim\limits_{x\to 0} x\left[\frac{1}{x}\right] = 1$

Simillarly $\lim\limits_{x\to 0} x\left[\frac{2}{x}\right] = 2$

⋮

⋮

⋮

$\lim\limits_{x\to 0} x\left[\frac{15}{x}\right] = 15$

Ans = 1 + 2 + ... + 15 = 120

51. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is

यदि $\sum_{i=1}^{9}(x_i - 5) = 9$ तथा $\sum_{i=1}^{9}(x_i - 5)^2 = 45$ है, तो नौ प्रेक्षणों $x_1, x_2, ..., x_9$ का मानक विचलन है –

(1) 2 (2) 3 (3) 9 (4) 4

Ans. 1

Sol. There is no effect on S.D. on shiffing of origin

$x_i - 5 = x_i$

$\sum x_i = 9$

$\sum (x_i)^2 = 45$

$$\text{S.D.} = \sqrt{\frac{\sum(x_i^2)}{9} - \left(\frac{\sum x_i}{9}\right)^2}$$

$$= \sqrt{\frac{45}{9} - \left(\frac{9}{9}\right)^2} = 2$$

52. The integral $\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x\right)^2} dx$ is equal to

समाकलन $\int \frac{\sin^2 x \cos^2 x}{\left(\sin^5 x + \cos^3 x \sin^2 x + \sin^3 x \cos^2 x + \cos^5 x\right)^2} dx$ बराबर है –

(1) $\frac{1}{1+\cot^3 x} + C$ (2) $\frac{-1}{1+\cot^3 x} + C$ (3) $\frac{1}{3(1+\tan^3 x)} + C$ (4) $\frac{-1}{3(1+\tan^3 x)} + C$

(Where C is a constant of integration)

(जहाँ C एक समाकलन अचर है)

Ans. 4

Sol. Divide numberator & denominator by $\cos^{10}x$

$$\int \frac{\tan^2 x \sec^6 x\, dx}{(\tan^5 x + \tan^2 x + \tan^3 x + 1)^2}$$

$$\int \frac{\tan^2 x \sec^6 x\, dx}{\left(1+\tan^3 x\right)^2 \left(1+\tan^2 x\right)^2}$$

$$\int \frac{\tan^2 x \sec^6 x\, dx}{\left(1+\tan^3 x\right)^2}$$

Put $1 + \tan^3 x = t$

$$I = \frac{1}{3}\int \frac{dt}{t^2}$$

$$= -\frac{1}{3t} + C$$

$$-\frac{1}{3(1+\tan^3 x)} + C$$

53. Let $S = \{t \in R: f(x) = |x - \pi| \,.\, (e^{|x|} - 1) \sin |x|$ is not differentiable at $t\}$. Then the set S is equal to

(1) $\{\pi\}$ (2) $\{0, \pi\}$ (3) ϕ (an empty set) (4) $\{0\}$

माना $S = \{t \in R: f(x) = |x - \pi| \,.\, (e^{|x|} - 1) \sin |x|$ जो पर अवकलनीय नहीं है$\}$, तो समुच्चय S बराबर है –

(1) $\{\pi\}$ (2) $\{0, \pi\}$ (3) ϕ (एक रिक्त समुच्चय) (4) $\{0\}$

Ans. 3

Sol. We will check at $x = 0, \pi$

$$f(x) \begin{cases} (x-\pi)(e^x-1)\sin x & x \geq \pi \\ (\pi-x)(e^x-1)\sin x & 0 \leq x \pi \\ -(\pi-x)(e^{-x}-1)\sin x & x < 0 \end{cases}$$

at $x = 0$
$f'(0^+) = 0$
$f'(0^-) = 0$

at $x = \pi$

$f'(\pi^+) = 0$

$f'(\pi^-) = 0$

It is differentiable at $x = 0, \pi$

$S = \{\ \} = \phi$

54. Let $y = y(x)$ be the solution o f the differential equation $\sin x\frac{dy}{dx} + y\cos x = 4x$, $x \in (0, \pi)$. If $y\left(\frac{\pi}{2}\right) = 0$, then $y\left(\frac{\pi}{6}\right)$ is equal to

माना अवकल समीकरण $\sin x\frac{dy}{dx} + y\cos x = 4x$, $x \in (0, \pi)$ का $y = y(x)$ एक हल है। यदि $y\left(\frac{\pi}{2}\right) = 0$ है, तो $y\left(\frac{\pi}{6}\right)$ बराबर है –

(1) $-\frac{8}{9}\pi^2$ (2) $-\frac{4}{9}\pi^2$ (3) $\frac{4}{9\sqrt{3}}\pi^2$ (4) $\frac{-8}{9\sqrt{3}}\pi^2$

Ans. 1

Sol. $\sin x\frac{dy}{dx} + 4\cos x = 4x$

$\int d(y\sin x) = \int 4x\,dx$

$4\sin x = 2x^2 + C$

$x = 0, y = \frac{\pi}{2} \Rightarrow C = -\frac{\pi^2}{2}$

Put $x = \frac{\pi}{6}$

$y \times \frac{1}{2} = 2\left(\frac{\pi^2}{36}\right) - \frac{\pi^2}{2}$

$y = \frac{-8\pi^2}{9}$

55. Let $\vec{u}$ be a vector coplanar with the vectors $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ and $\vec{b} = \hat{j} + \hat{k}$. If $\vec{u}$ is perpendicular to $\vec{a}$ and $\vec{u}.\vec{b} = 24$, then $|\vec{u}|^2$ is equal to

माना $\vec{u}$ एक ऐसा सदिश है जो सदिशों $\vec{a} = 2\hat{i} + 3\hat{j} - \hat{k}$ तथा $\vec{b} = \hat{j} + \hat{k}$ के साथ समतलीय है। यदि $\vec{u}$, $\vec{a}$ पर लंबवत् है तथा $\vec{u}.\vec{b} = 24$ है, तो $|\vec{u}|^2$ बराबर है –

(1) 256 (2) 84 (3) 336 (4) 315

Ans. 3

Sol. $\vec{u} = \lambda\left(\vec{a} \times \left(\vec{a} \times \vec{b}\right)\right)$

$= \lambda\left(\left(\vec{a}.\vec{b}\right)\vec{a} - \left(\vec{a}.\vec{a}\right)\vec{b}\right)$

$= \lambda\left(2\vec{a} - 14\vec{b}\right)$

$\vec{u} = \lambda\left(4\hat{i} - 8\hat{j} - 16\hat{k}\right)$

$\vec{u}.\vec{b} = 24 \Rightarrow \lambda = -1$

$\Rightarrow \vec{u} = -4\hat{i} + 8\hat{j} + 16\hat{k}$

$\Rightarrow |\vec{u}|^2 = 16 + 64 + 256 = 336$

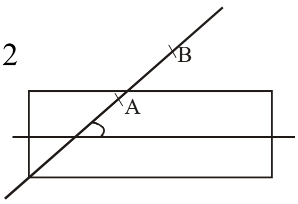
56. The length of the projection of the line segment joining the points (5, –1, 4) and (4, –1, 3) on the plane, x + y + z = 7 is

बिन्दुओं (5, –1, 4) तथा (4, –1, 3) को मिलाने वाले रेखाखण्ड का समतल x + y + z = 7 पर डाले गए प्रक्षेप की लम्बाई है –

(1) $\frac{1}{3}$ (2) $\sqrt{\frac{2}{3}}$ (3) $\frac{2}{\sqrt{3}}$ (4) $\frac{2}{3}$

Ans. 2

Sol.



A (5,–1,4)
B (4,–1,3)

θ = angle between plane & line

$$\cos(90-\theta) = \left|\frac{\overrightarrow{AB}.\left(\hat{i}+\hat{j}+\hat{k}\right)}{\left(\overrightarrow{AB}\right)\left|\hat{i}+\hat{j}+\hat{k}\right|}\right| = \frac{2}{\sqrt{6}}$$

Req. length $= |\overrightarrow{AB}|\cos\theta$

$$= \sqrt{2} \times \frac{\sqrt{2}}{\sqrt{6}} = \sqrt{\frac{2}{3}}$$

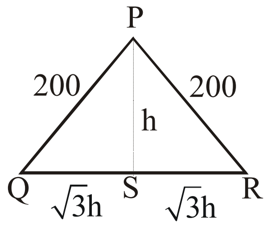
57. PQR is a triangular park with PQ = PR = 200m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P, Q and R are respectively 45º and 30º and 30º, then the height of the tower (in m) is

PQR एक त्रिकोणाकार पार्क है जिसमें PQ = PR = 200m है। QR के मध्य बिन्दु पर एक टीवी टावर स्थित है। यदि बिन्दुओं P, Q, R से टावर के शिखर के उन्नयन कोण क्रमशः 45º, 30º तथा 30º है तो टावर की ऊँचाई (m में) है –

(1) $100\sqrt{3}$ (2) $50\sqrt{2}$ (3) 100 (4) 50

Ans. 3

Sol.



height of tower be h

$\left(\sqrt{3}h\right)^2 + (h)^2 = (200)^2$

h = 100

58. From 6 different novels and 3 different dictionaries, 4 novels and 1 dictionary are to be selected and arranged in a row on a shelf so that the dictionary is always in the middle. The number of such arrangements is

(1) at least 500 but less than 750 (2) at least 750 but less than 1000

(3) at least 1000 (4) less than 500

6 भिन्न उपन्यासों तथा 3 भिन्न शब्दकोंशों में से 4 उपन्यासों तथा 1 शब्दकोश को चुनकर एक पंक्ति में एक शैल्फ पर इस प्रकार सजाया जाना है कि शब्द कोश सदा मध्य में हो। इस प्रकार के विन्यासों की संख्या है –

(1) कम से कम 500 लेकिन 750 से कम (2) कम से कम 750 लेकिन 1000 से कम

(3) कम से कम 1000 (4) 500 से कम

Ans. 3

Sol. ${}^{6}C_4\,{}^{3}C_1 \times 4!$

= 1080

59. Let A be the sum of the first 20 terms and B be sum of the first 40 terms of the series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ If $B - 2A = 100\lambda$, then λ is equal to

माना श्रेणी $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ के प्रथम 20 पदों का योग A है तथा प्रथम 40 पदों का योग B है। यदि $B - 2A = 100\lambda$, तो λ बराबर है –

(1) 464 (2) 496 (3) 232 (4) 248

Ans. 4

Sol. $f(x) = 1^2 + 2(2)^2 + 3^2 + ... + 2(x)^2$

$(1^2 + 2^2 + ... + n^2) + 2(2^2 + 4^2 + ... + n^2)$

$$\frac{n(n+1)(2n+1)}{6} + 4\frac{\left(\frac{1}{2}\left(\frac{n}{2}+1\right)(n+1)\right)}{6}$$

$f(x) = \dfrac{n(n+1)^2}{2}$; $B - 2A = f(40) - 2f(20)$

$= \frac{40+(41)^2}{2} - 2\left(\frac{20\times(21)^2}{2}\right)$; $\qquad 100\,\lambda = 24800$

$\lambda = 248$

60. Let the orthocentre and centroid of a triangle be A(–3, 5) and B(3, 3) respectively. If C is the circumcentre of this triangle, then the radius of the circle having line segment AC as diameter, is

माना एक त्रिभुज का लम्ब केन्द्र तथा केन्द्रक क्रमशः A(–3, 5) तथा B(3, 3) है। यदि इस त्रिभुज का परिकेन्द्र C है, तो रेखाखण्ड AC को व्यास मान कर बनाए जाने वाने वृत की त्रिज्या है –

(1) $3\sqrt{\frac{5}{2}}$ (2) $\frac{3\sqrt{5}}{2}$ (3) $\sqrt{10}$ (4) $2\sqrt{10}$

Ans. 1

Sol.

B divides AC in 2 : 1

$AC = \frac{3AB}{2}$; $\qquad r = \frac{AC}{2} = \frac{3AB}{4}$

$= \frac{3\sqrt{40}}{4}$; $\qquad = 3\sqrt{\left(\frac{5}{2}\right)}$