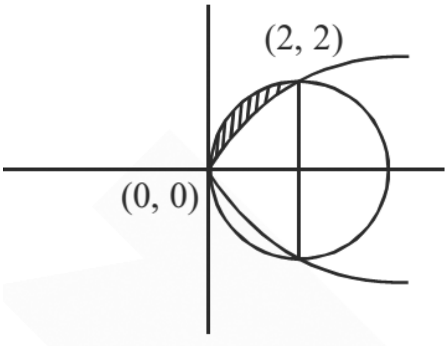
PART-B - MATHS

31. The area (in sq. units) of the region $\{(x, y) : y^2 \geq 2x \text{ and } x^2 + y^2 \leq 4x, x \geq 0, y \geq 0\}$ is :

(1) $\pi - \frac{4\sqrt{2}}{3}$ (2) $\frac{\pi}{2} - \frac{2\sqrt{2}}{3}$ (3) $\pi - \frac{4}{3}$ (4) $\pi - \frac{8}{3}$

Sol.



$$\text{Area} = \frac{1}{4}(\text{Area of circle}) - \int_0^2 \sqrt{2x}\,dx$$

$$= \pi - \frac{8}{3}$$

32. If $f(x) + 2f\left(\frac{1}{x}\right) = 3x$, $x \neq 0$ and $S = \{x \in R : f(x) = f(-x)\}$; then S :

(1) contains exactly two elements (2) contains more than two elements

(3) is an empty set (4) contains exactly one element

Sol. $f(x) + 2f(1/x) = 3x$ (1)

$f(1/x) + 2f(x) = \frac{3}{x}$ (2)

From (1) & (2)

$f(x) = \frac{2 - x^2}{x}$

$f(x) = f(-x)$

$x^2 = 2$

$x = \pm\sqrt{2}$

S contains exactly two elements

33. The integral $\int \frac{2x^{12} + 5x^9}{(x^5 + x^3 + 1)^3}dx$ is equal to :

(1) $\frac{x^5}{2(x^5 + x^3 + 1)^2} + C$ (2) $\frac{-x^{10}}{2(x^5 + x^3 + 1)^2} + C$

(3) $\dfrac{-x^5}{\left(x^5+x^3+1\right)^2}+C$ (4) $\dfrac{x^{10}}{2\left(x^5+x^3+1\right)^2}+C$

(where c is an arbitary constant)

Sol. $\displaystyle\int \frac{\left(2x^{12}+5x^9\right)dx}{x^{15}\left(1+x^{-2}+x^{-5}\right)^3}$

$1+x^{-2}+x^{-5}=t$

$I=-\displaystyle\int \frac{dt}{t^3}$

$=\dfrac{1}{2t^2}+C$

$=\dfrac{x^{10}}{2(x^5+x^3+1)^2}+C$

34. For $x \in R$, $f(x) = |\log 2 - \sin x|$ and $g(x) = f(f(x))$, then :

(1) $g'(0) = -\cos(\log 2)$ (2) g is differentiable at $x = 0$ and $g'(0) = -\sin(\log 2)$

(3) g is not differentiable at $x = 0$ (4) $g'(0) = \cos(\log 2)$

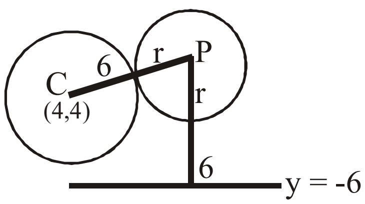
Sol. $g(x) = |\log 2 - \sin|\log 2 - \sin x||$
gx near by of $x = 0$
$g(x) = \log 2 - \sin(\log 2 - \sin x)$
$g'(x) = -(\cos(\cos 2 - \sin x))(-\cos x)$

$g'(0) = \cos(\log 2)$

35. The centres of those circles which touch the circle, $x^2 + y^2 - 8x - 8y - 4 = 0$, externally and also touch the x-axis, lie on :

(1) a hyperbola (2) a parabola (3) a circle (4) an ellipse which is not a circle

Sol.



Distance of P from (4, 4) = Distance of P from y = –6

$\Rightarrow$ locus of P is parabola

36. The sum of all real values of x satisfying the equation :

$\left(x^2-5x+5\right)^{x^2+4x-60}=1$ is :

(1) 6 (2) 5 (3) 3 (4) –4

Sol. $x^2 - 5x + 5 = 1$
$\Rightarrow x = 1, 4$

$x^2 + 4x - 60 = 0$
$x = -10, x = 6$
$x^2 - 5x + 5 = -1$
$x = 2$ $\quad\quad$ $x = 3$ (Rejected)
Sum = $1 + 4 + 6 - 10 + 2$
$= 3$

37. If the 2nd , 5th and 9th terms of a non-constant A.P. are in G.P., then the common ratio of this G.P. is :

(1) 1 (2) 7/4 (3) 8/5 (4) 4/3

Sol. $T_2 = a$
$T_5 = a + 3d$
$T_9 = a + 7d$
$(a + 3d)^2 = a(a + 7d)$

$d = \frac{a}{9}$

$r = \frac{a+3d}{a} = 1 + \frac{3d}{a}$

$r = 1 + \frac{3}{9}$

$= \frac{4}{3}$

38. The eccentricity of the hyperbola whose length of the latus rectum is equal to 8 and the length of its conjugate axis is equal to half of the distance between its foci, is :

(1) $\frac{2}{\sqrt{3}}$ (2) $\sqrt{3}$ (3) $\frac{4}{3}$ (4) $\frac{4}{\sqrt{3}}$

Sol. $2b = ae$
$4b^2 = a^2e^2$
$4a^2(e^2 - 1) = a^2e^2$

$e = \frac{2}{\sqrt{3}}$

39. If the number of terms in the expansion of $\left(1 - \frac{2}{x} + \frac{4}{x^2}\right)^n, x \neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is :

(1) 243 (2) 729 (3) 64 (4) 2187

Sol. Number of terms = $2n + 1 = 28$

$n = \frac{27}{2}$ thats why it is bonus

40. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to :

(1) $p \vee q$ (2) $p \vee \sim q$ (3) $\sim p \wedge q$ (4) $p \wedge q$

Sol. $(p \wedge \sim q) \cup (\sim p \wedge q) \vee q$

$(\sim(p \rightarrow q)) \vee (\sim(q \rightarrow p)) \vee q$

$\sim((p \rightarrow q) \wedge (q \rightarrow p)) \vee q$

$\sim (p \leftrightarrow q) \vee q$

$((\sim p) \leftrightarrow q) \vee q$

Now make truth table

41. Consider $f(x) = \tan^{-1}\left(\sqrt{\frac{1+\sin x}{1-\sin x}}\right), x \in \left(0, \frac{\pi}{2}\right)$. A normal to y = f(x) at $x = \frac{\pi}{6}$ also passes through the point :

(1) $\left(\frac{\pi}{6}, 0\right)$ (2) $\left(\frac{\pi}{4}, 0\right)$ (3) (0, 0) (4) $\left(0, \frac{2\pi}{3}\right)$

Sol. $f(x) = \tan^{-1}\left(\frac{1+\sin x}{\cos x}\right)$

$= \tan^{-1}\left(\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)\right)$

$f(x) = \frac{\pi}{4}+\frac{x}{2}$

$f'(x) = \frac{1}{2}$

Eq. of normal $y - \frac{\pi}{3} = -2\left(x - \frac{\pi}{6}\right)$

$2x + y = \frac{2\pi}{3}$

42. $\lim_{n\to\infty}\left(\frac{(n+1)(n+2)....3n}{n^{2n}}\right)^{1/n}$ is equal to :

(1) $\frac{9}{e^2}$ (2) 3 log 3 – 2 (3) $\frac{18}{e^4}$ (4) $\frac{27}{e^2}$

Sol. $L = \prod_{r=1}^{2n}\left(\frac{n+r}{n}\right)^{\frac{1}{n}}$

$\ln L = \frac{1}{n}\sum_{r=1}^{2n} \ln\left(1+\frac{r}{n}\right)$

$\int_0^2 \ln(1+x)\,dx$

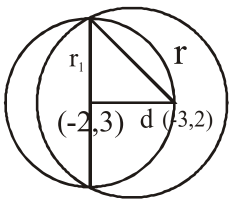
$\ln L = 3\ln 3 - 2$

$L = \frac{27}{e^2}$

43. If one of the diameters of the circle, given by the equation, $x^2 + y^2 - 4x + 6y - 12 = 0$, is a chord of a circle S, whose centre is at (–3, 2), then the radius of S is :

(1) 5 (2) 10 (3) $5\sqrt{2}$ (4) $5\sqrt{3}$

Sol.



$r_1 = 5$

$d = \sqrt{50}$

$r = \sqrt{r_1^2 + d^2}$

$= \sqrt{75} = 5\sqrt{3}$

44. Let two fair six-faced dice A and B be thrown simultaneously. If E_1 is the event that die A shows up four E_2 is the event that die B shows up two and E_3 is the event that the sum of numbers on both dice is odd, then which of the following statements is NOT true ?

(1) E_1 and E_3 are independent (2) E_1, E_2 and E_3 are independent

(3) E_1 and E_2 are independent (4) E_2 and E_3 are independent

Sol. $P(E_1 \cap E_2 \cap E_3) = 0$

and $P(E_1) = 1/6$ $P(E_2) = 1/6$

$P(E_3) = 1/2$

So E_1, E_2 & E_3 are not independent

45. A value of θ for which $\frac{2+3i\sin\theta}{1-2i\sin\theta}$ is purely imaginary, is :

(1) $\sin^{-1}\left(\frac{\sqrt{3}}{4}\right)$ (2) $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$ (3) $\frac{\pi}{3}$ (4) $\frac{\pi}{6}$

Sol. $z = \frac{2+3i\sin\theta}{1-2i\sin\theta}$

$z + \bar{z} = 0$

$\Rightarrow \sin^2\theta = 1/3$

$\sin\theta = \frac{1}{\sqrt{3}}$

46. If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \ldots\ldots,$

is $\frac{16}{5}$m, then m is equal to :

(1) 100 (2) 99 (3) 102 (4) 101

Sol. $\frac{16m}{5} = \sum_{r=2}^{11}\left(\frac{4r}{5}\right)^2$

$\frac{16m}{5} = \frac{16}{25}\sum_{r=2}^{11} r^2 = \frac{16}{25}\times 505$

$m = 101$

47. The system of linear equations

$x + \lambda y - z = 0$

$\lambda y - y - z = 0$

$x + y - \lambda z = 0$

has a non-trivial solution for :

(1) exactly two values of λ (2) exactly three values of λ

(3) infinitely many values of λ (4) exactly one value of λ

Sol. $\begin{vmatrix} 1 & \lambda & 1 \\ \lambda & -1 & -1 \\ 1 & 1 & -\lambda \end{vmatrix} = 0$

$\lambda^3 - \lambda = 0$

$\lambda = 0,\quad \lambda = 1, \lambda = -1$

48. If the line, $\frac{x-3}{2} = \frac{y+2}{-1} = \frac{z+4}{3}$ lies in the plane, $lx + my - z = 9$, then $l^2 + m^2$ is equal to :

(1) 5 (2) 2 (3) 26 (4) 18

Sol. (3, –2, –4) will lie on plane

$3l - 2m + 4 = 6$ (1)

(2, –1, 3) is perpendicular to plane

$2l - m - 3 = 0$

$2l - m = 3$ (2)

$\Rightarrow l = 1, m = -1$

49. If all the words (with or without meaning) having five letters, formed using the letters of the word SMALL and arranged as in a dictionary; then the position of the word SMALL is :

(1) 52nd (2) 58th (3) 46th (4) 59th

Sol. $A(LLMS) \Rightarrow \frac{\lfloor 4}{\lfloor 2} = 12$

$L(ALMS) \Rightarrow \lfloor 4 = 24$

$M(ALLS) \Rightarrow \frac{\lfloor 4}{\lfloor 2} = 12$

$SA(LLM) \Rightarrow \frac{\lfloor 3}{\lfloor 2} = 3$

$SL(ALM) \Rightarrow \lfloor\underline{3} = 6$

$SMALL \Rightarrow 1$

58

50. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true ?

(1) $3a^2 - 34a + 91 = 0$ (2) $3a^2 - 23a + 44 = 0$ (3) $3a^2 - 26a + 55 = 0$ (4) $3a^2 - 32a + 84 = 0$

Sol. $\sigma^2 = (3.5)^2 = 12.25$

$$\sigma^2 = \frac{1}{n}\sum x^2 - (\bar{x})^2$$

$$12.25 = \frac{1}{4}(2^2 + 3^2 + a^2 + 11^2) - \left(\frac{2+3+a+11}{4}\right)^2$$

$3a^2 - 32a + 84 = 0$

51. A wire of length 2 units is cut into two parts which are bent respectively to form a square of side = x units and a circle of radius = r units. If the sum of the areas of the square and the circle so formed is minimum, then :

(1) $x = 2r$ (2) $2x = r$ (3) $2x = (\pi + 4)r$ (4) $(4 - \pi)x = \pi r$

Sol. |— 4x —|— 2-4x —|

Perimeter of square = 4x and circumference of circle = 2 – 4x

$2\pi r = (2 - 4x)$

$$A = x^2 + \pi\left(\frac{2-4x}{2\pi}\right)^2$$

$$\frac{dA}{dx} = 0$$

$$2x = \frac{2}{\pi}(2 - 4x)$$

$$2x = \frac{2(2\pi r)}{\pi}$$

$x = 2r$

52. Let $p = \lim_{x \to 0^+}\left(1 + \tan^2\sqrt{x}\right)^{\frac{1}{2x}}$ then log p is equal to :

(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) 2 (4) 1

Sol. 1^∞ form

$$P = e^{\frac{1}{2x}\left(1 + \tan^2\sqrt{x} - 1\right)}$$

$$P = e^{\frac{1}{2}}$$

$\log P = \frac{1}{2}$

53. Let P be the point on the parabola, $y^2 = 8x$ which is at a minimum distance from the centre C of the circle, $x^2 + (y+6)^2 = 1$. Then the equation of the circle, passing through C and having its centre at P is :

(1) $x^2 + y^2 - \frac{x}{4} + 2y - 24 = 0$
(2) $x^2 + y^2 - 4x + 9y + 18 = 0$
(3) $x^2 + y^2 - 4x + 8y + 12 = 0$
(4) $x^2 + y^2 - x + 4y - 12 = 0$

Sol. $P(2t^2, 4t)$

Normal at P passes through (0, –6)

$tx + y = 4t + 2t^3$

$-6 = 4t + 2t^3$

$t^3 + 2t + 3 = 0$

$t = -1$

$P(2, -4)$

54. If a curve $y = f(x)$ passes through the point $(1, -1)$ and satisfies the differential equation, $y(1 + xy)dx = x\,dy$, then $f\left(-\frac{1}{2}\right)$ is equal to :

(1) 2/5
(2) 4/5
(3) $-\frac{2}{5}$
(4) $-\frac{4}{5}$

Sol. $y\,dx + xy^2dx = xdy$

$ydx - xdy = -xy^2dx$

$\int \frac{ydx - xdy}{y^2} = -\int xdx$

$\frac{x}{y} = \frac{-x^2}{2} + C$

$(1, -1) \Rightarrow \left(\frac{x^2+1}{2}\right)$

$y = -\frac{2x}{x^2+1}$

55. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three unit vectors such that $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2}(\vec{b} + \vec{c})$. If $\vec{b}$ is not parallel to $\vec{c}$, then the angle between $\vec{a}$ and $\vec{b}$ is :

(1) $\frac{2\pi}{3}$
(2) $\frac{5\pi}{6}$
(3) $\frac{3\pi}{4}$
(4) $\frac{\pi}{2}$

Sol. $(\vec{a}.\vec{c})\vec{b} - (\vec{a}.\vec{b})\vec{c} = \frac{\sqrt{3}}{2}\vec{b} + \frac{\sqrt{3}}{2}\vec{c}$

$-\vec{a}.\vec{b} = \frac{\sqrt{3}}{2}$

$-\cos\theta = \frac{\sqrt{3}}{2}$

$\theta = \frac{5\pi}{6}$

56. If $A = \begin{bmatrix} 5a & -b \\ 3 & 2 \end{bmatrix}$ and $A \text{ adj } A = A A^T$, then $5a + b$ is equal to :

(1) 4 (2) 13 (3) –1 (4) 5

Sol. $A \text{ adj } A = AA^T$

$\text{adj } A = A^T$

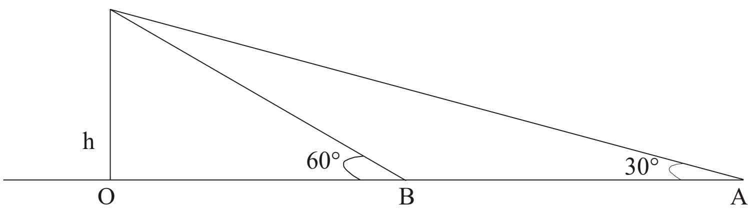
$\begin{bmatrix} 2 & b \\ -3 & 5a \end{bmatrix} = \begin{bmatrix} 5a & 3 \\ -b & 2 \end{bmatrix}$

$2 = 5a, b = 3$

57. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30º. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60º. Then the time taken (in minutes) by him, from B to reach the pillar, is :

(1) 20 (2) 5 (3) 6 (4) 10

Sol.



$OA = h\cot 30°$

$OB = h\cot 60°$

$V = \frac{AB}{10} = \frac{h\cot 30° - h\cot 60°}{10}$

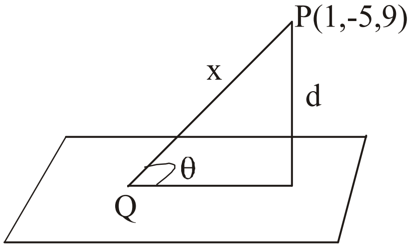
$= \frac{h}{5\sqrt{3}}$

$\text{time} = \frac{OB}{V} = \frac{h\cot 60°}{h/5\sqrt{3}} = 5$

58. The distance of the point (1, –5, 9) from the plane $x - y + z = 5$ measured along the line $x = y = z$ is :

(1) $\frac{10}{\sqrt{3}}$ (2) $\frac{20}{3}$ (3) $3\sqrt{10}$ (4) $10\sqrt{3}$

Sol.



$\sin\theta = (1,-1,1) \wedge (1,1,1)$

$= \left|\frac{(1-1+1)}{\sqrt{3}\sqrt{3}}\right| = \frac{1}{\sqrt{3}}$

$d = \left|\frac{1+5+9-5}{\sqrt{3}}\right| = \frac{10}{\sqrt{3}}$

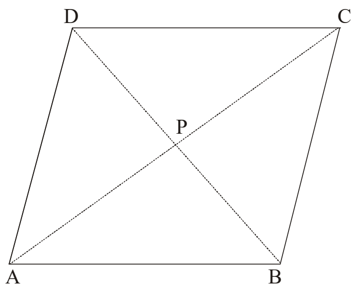
$x\sin\theta = d$

$x = \frac{10}{\sqrt{3}} \times 3 = 10\sqrt{3}$

59. Two sides of a rhombus are along the lines, $x - y + 1 = 0$ and $7x - y - 5 = 0$. If its diagonals intersect at $(-1, -2)$, then which one of the following is a vertex of this rhombus ?

(1) $\left(\frac{1}{3}, -\frac{8}{3}\right)$ (2) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$ (3) $(-3, -9)$ (4) $(-3, -8)$

Sol.



AB = $7x - y - 5 = 0$
AD = $x - y + 1 = 0$
A(+1, 2)
P(–1, – 2)
$\Rightarrow$ C(–3, –6)

BC = $x - y - 3 = 0$

CD = $7x - y + 15 = 0$

$\Rightarrow B\left(\frac{1}{3}, \frac{-8}{3}\right)$

60. If $0 \le x < 2\pi$, then the number of real values of x, which satisfy the equation

$\cos x + \cos 2x + \cos 3x + \cos 4x = 0$, is :

(1) 7 (2) 9 (3) 3 (4) 5

Sol. $\cos x + \cos 2x + \cos 3x + \cos 4x = 0$

$$2\cos\frac{5x}{2}\cos x\cos\frac{x}{2} = 0$$

$$\cos\frac{5x}{2} = 0$$

$$x = \frac{\pi}{5}, \frac{3\pi}{5}, \pi, \frac{7\pi}{5}, \frac{9\pi}{5}$$

$\cos x = 0$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\cos\frac{x}{2} = 0$$

$x = \pi$

Total = 7 solution