JEE Main April 2024

Question Paper With Text Solution

09 April | Shift-2

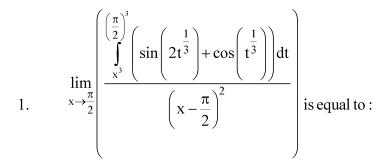# MATHEMATICS

JEE MAIN APRIL 2024 | 09TH APRIL SHIFT-2

SECTION – A

Question ID : 68019114612

1. $\lim\limits_{x \to \frac{\pi}{2}} \left(\dfrac{\displaystyle\int_{x^3}^{\left(\frac{\pi}{2}\right)^3} \left(\sin\left(2t^{\frac{1}{3}}\right) + \cos\left(t^{\frac{1}{3}}\right) \right) dt}{\left(x - \dfrac{\pi}{2}\right)^2} \right)$ is equal to :

(1) $\dfrac{11\pi^2}{10}$　　　　(2) $\dfrac{3\pi^2}{2}$　　　　(3) $\dfrac{5\pi^2}{9}$　　　　(4) $\dfrac{9\pi^2}{8}$

Ans.　Official answer NTA (4)

Sol.

Question ID : 68019114623

2.　If an unbiased dice is rolled thrice, then the probability of getting a greater number in the ith roll than the number obtained in the $(i-1)^{th}$ roll, $i = 2, 3$, is equal to :

(1) $\dfrac{1}{54}$　　　　(2) $\dfrac{3}{54}$　　　　(3) $\dfrac{2}{54}$　　　　(4) $\dfrac{5}{54}$

Ans.　Official answer NTA (4)

Sol.

Question ID : 68019114614

3.　The integral $\displaystyle\int_{\frac{1}{4}}^{\frac{3}{4}} \cos\left(2\cot^{-1} \sqrt{\dfrac{1-x}{1+x}} \right) dx$　is equal to :

(1) $\dfrac{1}{4}$　　　　(2) $-\dfrac{1}{2}$　　　　(3) $-\dfrac{1}{4}$　　　　(4) $\dfrac{1}{2}$

Ans.　Official answer NTA (3)

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Sol.

Question ID : 68019114610

4.   Let a, ar, ar², ... be an infinite G.P. If $\sum_{n=0}^{\infty} ar^n = 57$ and $\sum_{n=0}^{\infty} a^3 r^{3n} = 9747$, then a + 18r is equal to :

(1) 46   (2) 31   (3) 38   (4) 27

Ans.   Official answer NTA (2)

Sol.

Question ID : 68019114611

5.   If $\log_e y = 3 \sin^{-1} x$, then $(1 - x^2)y'' - xy'$ at $x = \dfrac{1}{2}$ is equal to :

(1) $9e^{\frac{\pi}{6}}$ (2) $3e^{\frac{\pi}{2}}$ (3) $3e^{\frac{\pi}{6}}$ (4) $9e^{\frac{\pi}{2}}$

Ans.   Official answer NTA (4)

Sol.

Question ID : 68019114618

6.   Let the foci of a hyperbola H coincide with the foci of the ellipse $E : \dfrac{(x-1)^2}{100} + \dfrac{(y-1)^2}{75} = 1$ and the eccentricity of the hyperbola H be the reciprocal of the eccentricity of the ellipse E. If the length of the transverse axis of H is α and the length of its cojugate axis is β, then $3\alpha^2 + 2\beta^2$ is equal to :

(1) 225   (2) 237   (3) 205   (4) 242

Ans.   Official answer NTA (1)

Sol.

Question ID : 68019114622

7.   If the variance of the frequency distribution

x	c	2c	3c	4c	5c	6c
f	2	1	1	1	1	1

is 160, then the value of $c \in N$ is :

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(1) 6   (2) 7   (3) 8   (4) 5

Ans. Official answer NTA (2)

Sol.

Question ID : 68019114605

8.   Let z be a complex number such that the real part of $\dfrac{z-2i}{z+2i}$ is zero. Then, the maximum value of $|z-(6+8i)|$ is equal to :

(1) ∞ (2) 8   (3) 10   (4) 12

Ans. Official answer NTA (4)

Sol.

Question ID : 68019114615

9.   The value of the integral $\displaystyle\int_{-1}^{2}\log_e\left(x+\sqrt{x^2+1}\right)dx$ is :

(1) $\sqrt{2}-\sqrt{5}+\log_e\left(\dfrac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$ (2) $\sqrt{5}-\sqrt{2}+\log_e\left(\dfrac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$

(3) $\sqrt{2}-\sqrt{5}+\log_e\left(\dfrac{9+4\sqrt{5}}{1+\sqrt{2}}\right)$ (4) $\sqrt{5}-\sqrt{2}+\log_e\left(\dfrac{7+4\sqrt{5}}{1+\sqrt{2}}\right)$

Ans. Official answer NTA (3)

Sol.

Question ID : 68019114620

10.   Between the following two statements :

Statement I : Let $\vec{a}=\hat{i}+2\hat{j}-3\hat{k}$ and $\vec{b}=2\hat{i}+\hat{j}-\hat{k}$. Then the vector $\vec{r}$ satisfying $\vec{a}\times\vec{r}=\vec{a}\times\vec{b}$ and $\vec{a}\cdot\vec{r}=0$ is of magnitude $\sqrt{10}$.

Statement II : In a triangle ABC, $\cos 2A+\cos 2B+\cos 2C\geq-\dfrac{3}{2}$.

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(1) Statement I is correct but Statement II is incorrect.

(2) Statement I is incorrect but Statement II is correct.

(3) Both Statement I and Statement II are correct.

(4) Both Statement I and Statement II are incorrect.

Ans.

Ans. Official answer NTA (2)

Question ID : 68019114608

11. $\lim\limits_{x \to 0} \dfrac{e - (1+2x)^{\frac{1}{2x}}}{x}$ is equal to :

(1) e

(2) $\dfrac{-2}{e}$

(3) 0

(4) $e - e^2$

Ans. Official answer NTA (1)

Sol.

Question ID : 68019114613

12. The area (in square units) of the region enclosed by the ellipse $x^2 + 3y^2 = 18$ in the first quadrant below the line $y = x$ is :

(1) $\sqrt{3}\pi - \dfrac{3}{4}$

(2) $\sqrt{3}\pi$

(3) $\sqrt{3}\pi + \dfrac{3}{4}$

(4) $\sqrt{3}\pi + 1$

Ans. Official answer NTA (2)

Sol.

Question ID : 68019114616

13. Let $\displaystyle\int_{0}^{x} \sqrt{1 - \big(y'(t)\big)^2}\, dt = \int_{0}^{x} y(t)\, dt$, $0 \le x \le 3$, $y \ge 0$, $y(0) = 0$. Then at $x = 2$, $y'' + y + 1$ is equal to :

(1) 1

(2) $\sqrt{2}$

(3) 2

(4) $\dfrac{1}{2}$

Ans. Official answer NTA (1)

Sol.

Question ID : 68019114609

14. The sum of the coefficient of $x^{\frac{2}{3}}$ and $x^{-\frac{2}{5}}$ in the binomial expansion $\left(x^{\frac{2}{3}} + \frac{1}{2} x^{-\frac{2}{5}} \right)^9$ is :

(1) $\frac{63}{16}$ (2) $\frac{21}{4}$ (3) $\frac{19}{4}$ (4) $\frac{69}{16}$

Ans. Official answer NTA (2)

Sol.

Question ID : 68019114621

15. Let $\vec{a} = 2\hat{i} + \alpha\hat{j} + \hat{k}$, $\vec{b} = -\hat{i} + \hat{k}$, $\vec{c} = \beta\hat{j} - \hat{k}$, where α and β are integers and $\alpha\beta = -6$. Let the values of the

ordered pair (α, β), for which the area of the parallelogram of diagonals $\vec{a} + \vec{b}$ and $\vec{b} + \vec{c}$ is $\frac{\sqrt{21}}{2}$, be (α_1, β_1)

and (α_2, β_2). Then $\alpha_1^2 + \beta_1^2 - \alpha_2\beta_2$ is equal to:

(1) 21   (2) 17   (3) 19   (4) 24

Ans. Official answer NTA (3)

Sol.

Question ID : 68019114607

16. Let $B = \begin{bmatrix} 1 & 3 \\ 1 & 5 \end{bmatrix}$ and A be a 2 × 2 matrix such that $AB^{-1} = A^{-1}$. If $BCB^{-1} = A$ and $C^4 + \alpha C^2 + \beta I = O$, then

$2\beta - \alpha$ is equal to :

(1) 16   (2) 2   (3) 10   (4) 8

Ans. Official answer NTA (3)

Sol.

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Question ID : 68019114619

17. Consider the line L passing through the points $(1,2,3)$ and $(2,3,5)$. The distance of the point $\left(\dfrac{11}{3}, \dfrac{11}{3}, \dfrac{19}{3}\right)$ from

the line L along the line $\dfrac{3x-11}{2} = \dfrac{3y-11}{1} = \dfrac{3z-19}{2}$ is equal to :

(1) 6   (2) 5   (3) 3   (4) 4

Ans. Official answer NTA (3)

Sol.

Question ID : 68019114604

18. Let the range of the function $f(x) = \dfrac{1}{2 + \sin 3x + \cos 3x}$, $x \in R$ be $[a, b]$. If α and β are respectively the A.M.

and the G.M. of a and b, then $\dfrac{\alpha}{\beta}$ is equal to :

(1) $\sqrt{\pi}$ (2) π (3) 2   (4) $\sqrt{2}$

Ans. Official answer NTA (4)

Sol.

Question ID : 68019114617

19. Two vertices of a triangle ABC are $A(3,-1)$ and $B(-2,3)$, and its orthocentre is $P(1,1)$. If the coordinates of the point C are (α, β) and the centre of the circle circumscribing the triangle PAB is (h, k), then the value of $(\alpha + \beta) + 2(h + k)$ equals :

(1) 5   (2) 51   (3) 15   (4) 81

Ans. Official answer NTA (1)

Sol.

Question ID : 68019114606

20. Let α, β; $\alpha > \beta$, be the roots of the equation $x^2 - \sqrt{2}x - \sqrt{3} = 0$. Let $P_n = \alpha^n - \beta^n$, $n \in N$. Then

$\left(11\sqrt{3} - 10\sqrt{2}\right)P_{10} + \left(11\sqrt{2} + 10\right)P_{11} - 11P_{12}$ is equal to :

(1) $11\sqrt{2}P_9$ (2) $11\sqrt{3}P_9$ (3) $10\sqrt{3}P_9$ (4) $10\sqrt{2}P_9$

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Ans. Official answer NTA (3)

Sol.

SECTION – B

Question ID : 68019114628

21. Let the set of all values of p, for which $f(x) = (p^2 - 6p + 8)(\sin^2 2x - \cos^2 2x) + 2(2 - p)x + 7$ does not have any critical point, be the interval (a, b). Then 16ab is equal to _____.

Ans. Official answer NTA (252)

Sol.

Question ID : 68019114626

22. The number of integers between 100 and 1000 having the sum of their digits equals to 14, is _____.

Ans. Official answer NTA (70)

Sol.

Question ID : 68019114630

23. Consider the circle C : $x^2 + y^2 = 4$ and the parabola P: $y^2 = 8x$. If the set of all values of α, for which three chords of the circle C on three distinct lines passing through the point (α, 0) are bisected by the parabola P is the interval (p, q), then $(2q - p)^2$ is equal to _____.

Ans. Official answer NTA (80)

Sol.

Question ID : 68019114625

24. Consider the matrices : $A = \begin{bmatrix} 2 & -5 \\ 3 & m \end{bmatrix}, B = \begin{bmatrix} 20 \\ m \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \end{bmatrix}$. Let the set of all m, for which the system of equations AX = B has a negative solution (i.e., x < 0 and y < 0), be the interval (a, b). Then $8\int_a^b |A| dm$ is _____.

Ans. Official answer NTA (450)

Sol.

Question ID : 68019114629

25.   For a differentiable function $f : R \to R$, suppose $f'(x) = 3f(x) + \alpha$, where $\alpha \in R$, $f(0) = 1$ and $\lim\limits_{x \to -\infty} f(x) = 7$.

Then $9f(-\log_e 3)$ is equal to _____.

Ans.   Official answer NTA (61)

Sol.

Question ID : 68019114627

26.   If $\left(\dfrac{1}{\alpha+1} + \dfrac{1}{\alpha+2} + \ldots\ldots + \dfrac{1}{\alpha+1012} \right) - \left(\dfrac{1}{2\cdot 1} + \dfrac{1}{4\cdot 3} + \dfrac{1}{6\cdot 5} + \ldots\ldots + \dfrac{1}{2024\cdot 2023} \right) = \dfrac{1}{2024}$, then α is equal

to _____.

Ans.   Official answer NTA (1011)

Sol.

Question ID : 68019114632

27.   The square of the distance of the image of the point $(6,1,5)$ in the line $\dfrac{x-1}{3} = \dfrac{y}{2} = \dfrac{z-2}{4}$, from the origin is

_____.

Ans.   Official answer NTA (62)

Sol.

Question ID : 68019114633

28.   Let the inverse trigonometric functions take principal values. The number of real solutions of the equation

$2\sin^{-1} x + 3\cos^{-1} x = \dfrac{2\pi}{5}$ is _____.

Ans.   Official answer NTA (0)

Sol.

Question ID : 68019114624

29. Let $A = \{(x, y) : 2x + 3y = 23, x, y \in N\}$ and $B = \{x : (x, y) \in A\}$. Then the number of one-one functions from A to B is equal to _____.

Ans. Official answer NTA (24)

Sol.

Question ID : 68019114631

30. Let A, B and C be three points on the parabola $y^2 = 6x$ and let the line segment AB meet the line L through C parallel to the x-axis at the point D. Let M and N respectively be the feet of the perpendiculars from A and B on L. Then $\left(\dfrac{AM \cdot BN}{CD}\right)^2$ is equal to _____.

Ans. Official answer NTA (36)

Sol.

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