JEE Main April 2024

Question Paper With Text Solution

09 April | Shift-1

MATHEMATICS

JEE MAIN APRIL 2024 | 09TH APRIL SHIFT-1

SECTION – A

Question ID : 87827056156

1. The parabola $y^2 = 4x$ divides the area of the circle $x^2 + y^2 = 5$ in two parts. The area of the smaller part is equal to :

(1) $\frac{1}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (2) $\frac{1}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (3) $\frac{2}{3}+5\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ (4) $\frac{2}{3}+\sqrt{5}\sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$

Ans. Official answer NTA (3)

Sol.

Question ID : 87827056158

2. The solution curve, of the differential equation $2y\frac{dy}{dx}+3=5\frac{dy}{dx}$, passing through the point (0, 1) is a conic, whose vertex lies on the line :

(1) $2x + 3y = 6$ (2) $2x + 3y = 9$ (3) $2x + 3y = -6$ (4) $2x + 3y = -9$

Ans. Official answer NTA (2)

Sol.

Question ID : 87827056148

3. If the domain of the function $f(x)=\sin^{-1}\left(\frac{x-1}{2x+3}\right)$ is $\mathbf{R} - (\alpha, \beta)$, then $12\alpha\beta$ is equal to :

(1) 24 (2) 36 (3) 40 (4) 32

Ans. Official answer NTA (4)

Sol.

Question ID : 87827056151

4. The coefficient of x^{70} in $x^2(1 + x)^{98} + x^3 (1+ x)^{97} + x^4 (1 + x)^{96} +....+ x^{54}(1 + x)^{46}$ is ${}^{99}C_p - {}^{46}C_q$.. Then a possible value of p + q is :

(1) 68 (2) 83 (3) 55 (4) 61

Ans. Official answer NTA (2)

Sol.

Question ID : 87827056166

5. The frequency distribution of the age of students in a class of 40 students is given below :

Age	15	16	17	18	19	20
No of Students	5	8	5	12	x	y

If the mean deviation about the median is 1.25, then $4x + 5y$ is equal to :

(1) 43 (2) 46 (3) 44 (4) 47

Ans. Official answer NTA (3)

Sol.

Question ID : 87827056152

6. If the sum of the series $\frac{1}{1.(1+d)}+\frac{1}{(1+d)(1+2d)}+.....+\frac{1}{(1+9d)(1+10d)}$ is to 5, then 50d is equal to :

(1) 10 (2) 5 (3) 15 (4) 20

Ans. Official answer NTA (2)

Sol.

Question ID : 87827056162

7. Let the line L intersect the lines $x-2=-y=z-1$, $2(x+1)=2(y-1)=z+1$ and be parallel to the line $\frac{x-2}{3}=\frac{y-1}{1}=\frac{z-2}{2}$. Then which of the following points lies on L ?

(1) $\left(-\frac{1}{3},-1,-1\right)$ (2) $\left(-\frac{1}{3},-1,1\right)$ (3) $\left(-\frac{1}{3},1,-1\right)$ (4) $\left(-\frac{1}{3},1,1\right)$

Ans. Official answer NTA (3)

Sol.

Question ID : 87827056161

8. A ray of light coming from the point P(1, 2) gets reflected from the point Q on the x-axis and then passes through the point R(4, 3). If the point S(h, k) is such that PQRS is a parallelogram, then hk^2 is equal to :

(1) 80 (2) 90 (3) 60 (4) 70

Ans. Official answer NTA (4)

Sol.

Question ID : 87827056159

9. Let $f(x) = x^2 + 9g(x) = \frac{x}{x-9}$ and a = fog(10), b = gof(3). If e and l denote the eccentricity and the length of the latus rectum of the ellipse $\frac{x^2}{a} + \frac{y^2}{b} = 1$, then $8e^2 + l^2$ is equal to :

(1) 6 (2) 12 (3) 16 (4) 8

Ans. Official answer NTA (4)

Sol.

Question ID : 87827056153

10. Let $f(x) = ax^3 + bx^2 + cx + 41$ be such that f(1) = 40, f'(1) = 2 and f'' (1) = 4. Then $a^2 + b^2 + c^2$ is equal to :

(1) 73 (2) 62 (3) 51 (4) 54

Ans. Official answer NTA (3)

Sol.

Question ID : 87827056167

11. Let $\left|\cos\theta \cdot \cos(60+\theta)\right| \le \frac{1}{8}, \theta \in [0, 2\pi]$. Then, then sum of all $\theta \in [0, 2\pi]$, where $\cos 3\theta$ attains its maximum value of is :

(1) 18π (2) 9π (3) 15π (4) 6π

Ans. Official answer NTA (4)

Sol.

Question ID : 87827056160

12. Let a circle passing through (2, 0) have its centre at the point (h, k). Let (x_c, y_c) be the point of intersection of the lines $3x + 5y = 1$ and $(2 + c)x + 5c^2y = 1$. If $h = \lim_{c\to 1} x_c$ and $k = \lim_{c\to 1} y_c$, then the equation of the circle is :

(1) $25x^2 + 25y^2 - 2x + 2y - 60 = 0$
(2) $5x^2 + 5y^2 - 4x + 2y - 12 = 0$
(3) $25x^2 + 25y^2 - 20x + 2y - 60 = 0$
(4) $5x^2 + 5y^2 - 4x - 2y - 12 = 0$

Ans. Official answer NTA (3)

Sol.

Question ID : 87827056157

13. The solution of the differential equation $(x^2 + y^2)dx - 5xy\,dy = 0$, $y(1) = 0$, is :

(1) $|x^2 - 4y^2|^5 = x^2$ (2) $|x^2 - 2y^2|^5 = x^2$ (3) $|x^2 - 4y^2|^6 = x$ (4) $|x^2 - 2y^2|^6 = x$

Ans. Official answer NTA (1)

Sol.

Question ID : 87827056149

14. Let α, β be the roots of the equation $x^2 + 2\sqrt{2}x - 1 = 0$. Then quadratic equation, whose roots are $\alpha^4 + \beta^4$ and $\frac{1}{10}\left(\alpha^6 + \beta^6\right)$, is :

(1) $x^2 - 180x + 9506 = 0$
(2) $x^2 - 195x + 9466 = 0$
(3) $x^2 - 195x + 9506 = 0$
(4) $x^2 - 190x + 9466 = 0$

Ans. Official answer NTA (3)

Sol.

Question ID : 87827056150

15. Let $\lambda, \mu \in \mathbf{R}$. If the system of equations

$3x + 5y + \lambda z = 3$

$7x + 11y - 9z = 2$

$97x + 155y - 189z = \mu$

has infinitely many solutions, then $\mu + 2\lambda$ is equal to :

(1) 27 (2) 24 (3) 25 (4) 22

Ans. Official answer NTA (3)

Sol.

Question ID : 87827056163

16. The shortest distance between the lines $\frac{x-3}{4}=\frac{y+7}{-11}=\frac{z-1}{5}$ and $\frac{x-5}{3}=\frac{y-9}{-6}=\frac{z+2}{1}$ is :

(1) $\frac{185}{\sqrt{563}}$ (2) $\frac{187}{\sqrt{563}}$ (3) $\frac{178}{\sqrt{563}}$ (4) $\frac{179}{\sqrt{563}}$

Ans. Official answer NTA (2)

Sol.

Question ID : 87827056154

17. A variable line L passes through the point (3, 5) and intersects the positive coordinate axes at the points A and B. The minimum area of the triangle OAB, where O is origin, is :

(1) 35 (2) 25 (3) 40 (4) 30

Ans. Official answer NTA (4)

Sol.

Question ID : 87827056155

18. Let $\int \frac{2-\tan x}{3+\tan x}dx=\frac{1}{2}\left(\alpha x+\log_e|\beta\sin x+\gamma\cos x|\right)+C$, where C is the constant of integration. Then $\alpha+\frac{\gamma}{\beta}$ is equal to :

(1) 3 (2) 1 (3) 7 (4) 7

Ans. Official answer NTA (4)

Sol.

Question ID : 87827056165

19. Let three vectors $\vec{a}=\alpha\hat{i}+4\hat{j}+2\hat{k}, \vec{b}=5\hat{i}+3\hat{j}+4\hat{k}, \vec{c}=x\hat{i}+y\hat{i}+z\hat{k}$ form a triangle such that $\vec{c}=\vec{a}-\vec{b}$ and the area of the triangle is $5\sqrt{6}$. If α is a positive real number, then $|\vec{c}|^2$ is equal to :

(1) 14 (2) 10 (3) 12 (4) 16

Ans. Official answer NTA (1)

Sol.

Question ID : 87827056164

20. Let $\overrightarrow{OA} = 2\vec{a}, \overrightarrow{OB} = 6\vec{a} + 5\vec{b}$ and $\overrightarrow{OC} = 3\vec{b}$, where O is the origin. If the area of the parallelogram with adjacent sides $\overrightarrow{OA}$ and $\overrightarrow{OC}$ is 15 sq. units, then the area (in sq. units) of the quadrilateral OABC is equal to :

(1) 40 (2) 35 (3) 38 (4) 32

Ans. Official answer NTA (2)

Sol.

SECTION – B

Question ID : 87827056176

21. Let the centre of a circle, passing through the points (0, 0), (1, 0) and touching the circle $x^2 + y^2 = 9$, be (h, k). Then for all possible values of coordinates of the centre (h, k), $4(h^2 + k^2)$ is equal to ________.

Ans. Official answer NTA (9)

Sol.

Question ID : 87827056172

22. The remainder when 428^{2024} is divided by 21 is________.

Ans. Official answer NTA (1)

Sol.

Question ID : 87827056170

23. Let A be a non - singular matrix of order 3. If det (3adj(2adj)((det A)(A))) = $3^{-13} \cdot 2^{-10}$ and det (3adj (2a)) = $2^m \cdot 3^n$, then |3m + 2n| is equal to ________.

Ans. Official answer NTA (14)

Sol.

Question ID : 87827056169

24. The sum of the square of the modulus of the elements in the set $\{z = a + ib : a, b \in Z, z \in C, |z-1| \leq 1, |z-5| \leq |z-5i|\}$ is ________.

Ans. Official answer NTA (9)

Sol.

Question ID : 87827056171

25. Let f: $(0, \pi) \rightarrow R$ be a function given by $f(x) = \begin{cases} \left(\frac{8}{7}\right)^{\frac{\tan 8x}{\tan 7x}}, & 0 < x < \frac{\pi}{2} \\ a-8, & x = \frac{\pi}{2} \\ (1+|\cot x|)^{\frac{b}{a}|\tan x|}, & \frac{\pi}{2} < x < \pi \end{cases}$ where a, b $\in$ Z. If f is continuous at $x = \frac{\pi}{2}$, then $a^2 + b^2$ is equal to________.

Ans. Official answer NTA (81)

Sol.

Question ID : 87827056168

26. Let A = {2,3,6,7} and B = {4,5,6,8}. Let R be a relation on A × B by $(a_1, b_1)R(a_2, b_2)$ if and only if $a_1 + a_2 = b_1 + b_2$. Then the number of elements in R is ________.

Ans. Official answer NTA (25)

Sol.

Question ID : 87827056174

27. Let the set of all positive values of λ, for which the point of local minimum of the function $(1 + x(\lambda^2 - x^2))$ satisfies $\frac{x^2 + x + 2}{x^2 + 5x + 6} < 0$, be (α, β). Then $\alpha^2 + \beta^2$ is equal to ________.

Ans. Official answer NTA (39)

Sol.

Question ID : 87827056175

28. Let $\lim_{n\to\infty}\left(\frac{n}{\sqrt{n^4+1}}-\frac{2n}{\left(n^2+1\right)\sqrt{n^4+1}}+\frac{n}{\sqrt{n^4+16}}-\frac{8}{\left(n^2+4\right)\sqrt{n^4+16}}+....+\frac{n}{\sqrt{n^4+n^4}}-\frac{2n\cdot n^2}{\left(n^2+n^2\right)\sqrt{n^4+n^4}}\right)$

be $\frac{\pi}{k}$, using only the principal values of the inverse trigonometric functions. Then k^2 is equal to ________.

Ans. Official answer NTA (32)

Sol.

Question ID : 87827056177

29. Let a, b and c denote the outcome of three independent rolls of a fair tetrahedral die, whose four faces are marked 1,2,3,4. If the probability that $ax^2 + bx + c = 0$ has all real roots is $\frac{m}{n}$, gcd(m, n) = 1, then m + n is equal to ________.

Ans. Official answer NTA (19)

Sol.

Question ID : 87827056173

30. If a function f satisfies f(m + n) = f(m) + f(n) for all m, n ∈ N and f(1) = 1, then the largest natural number λ such that $\sum_{k=1}^{2022} f(\lambda+k)\le(2022)^2$ is equal to ________.

Ans. Official answer NTA (1010)

Sol.