JEE Main April 2023

Question Paper With Text Solution

08 April | Shift-1

MATHEMATICS

JEE MAIN APRIL 2023 | 08TH APRIL SHIFT-1

SECTION – A

Question ID : 3666942933

1. Let α, β, γ be the three roots of the eqation $x^3 + bx + c = 0$. if $\beta\gamma = 1 = -\alpha$, then $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ is equal to:

माना α, β, γ समीकरण $x^3 + bx + c = 0$ के तीन मूल है। यदि $\beta\gamma = 1 = -\alpha$, तो $b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$ बराबर है।

(1) 21 (2) 19 (3) $\frac{169}{8}$ (4) $\frac{155}{8}$

Ans. Official Answer NTA (2)

Sol. Roots of $x^3 + bx + c = 0$ are α, β, γ

$\because \beta\gamma = 1 = -\alpha$

$\therefore \alpha = -1$ ....(i)

and $\alpha + \beta + \gamma = 0$ ....(ii)

$\alpha\beta\gamma = -c$ ....(iii)

$\therefore c = 1$ ....(iv)

$\beta + \gamma = 1$ ....(v)

$\because \alpha\beta + \beta\gamma + \gamma\alpha = b$

$\Rightarrow \alpha(\beta + \gamma) + \beta\gamma = b$

$\therefore b = 0$ ....(vi)

$\therefore \beta = -\omega, \gamma = -\omega^2$ ....(vii)

$b^3 + 2c^3 - 3\alpha^3 - 6\beta^3 - 8\gamma^3$

$= 0 + 2 + 3 + 6 + 8$

$= 19$

Question ID : 3666942947

2. If the points with position vectors $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$, $6\hat{i} + 11\hat{j} + 11\hat{k}$, $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ are collinear, then $(19\alpha - 6\beta)^2$ is equal to :

यदि स्थिति सदिश $\alpha\hat{i} + 10\hat{j} + 13\hat{k}$, $6\hat{i} + 11\hat{j} + 11\hat{k}$, $\frac{9}{2}\hat{i} + \beta\hat{j} - 8\hat{k}$ के बिंदु एक रेखा में हैं, तो $(19\alpha - 6\beta)^2$ बराबर है :

(1) 16 (2) 36 (3) 49 (4) 25

Ans. Official Answer NTA (2)

Sol. If $A(\vec{a}), B(\vec{b}), C(\vec{c})$ are collinear then

$\overrightarrow{AB} \parallel \overrightarrow{BC} \parallel \overrightarrow{AC}$

Now $\overrightarrow{AB} = (6-\alpha)\hat{i} + \hat{j} - 2\hat{k}$

$\overrightarrow{BC} = -\frac{3}{2}\hat{i} + (\beta - 11)\hat{j} - 19\hat{k}$

$\overrightarrow{AB} \parallel \overrightarrow{BC} \Rightarrow \frac{6-\alpha}{-\frac{3}{2}} = \frac{1}{\beta-11} = +\frac{2}{19}$

$\Rightarrow 6-\alpha = \frac{-3}{19} \Rightarrow \alpha = 6 + \frac{3}{19} = \frac{117}{19} \Rightarrow 19\alpha = 117$

$\beta - 11 = +\frac{19}{2} \Rightarrow 11 + \frac{19}{2} = \frac{41}{2} \Rightarrow 6\beta = 123$

$(19\alpha - 6\beta)^2 = (-6)^2 = 36$

Question ID : 3666942941

3. Let $I(x) = \int \frac{(x+1)}{x\left(1+xe^x\right)^2}dx, x > 0$. If $\lim\limits_{x\to\infty} I(x) = 0$, then $I(1)$ is equal to :

माना $I(x) = \int \frac{(x+1)}{x\left(1+xe^x\right)^2}dx, x > 0$ है। यदि $\lim\limits_{x\to\infty} I(x) = 0$ है, तो $I(1)$ बराबर है :

(1) $\frac{e+2}{e+1} - \log_e(e+1)$
(2) $\frac{e+1}{e+2} + \log_e(e+1)$
(3) $\frac{e+1}{e+2} - \log_e(e+1)$
(4) $\frac{e+2}{e+1} + \log_e(e+1)$

Ans. Official Answer NTA (1)

Sol. $I(x) = \int \frac{(x+1)}{x\left(1+xe^x\right)^2}dx$

$1 + xe^x = t$

$\left(xe^x + e^x\right)dx = dt$

$(x+1)dx = \frac{1}{e^x}dt$

$\therefore \int \frac{dt}{xe^x \cdot t^2} = \int \frac{dt}{(t-1)t^2} = \int \frac{dt}{t(t-1)\cdot t} \Rightarrow \int \frac{t-(t-1)}{t(t-1)t}dt$

$= \int \frac{dt}{t(t-1)} - \int \frac{dt}{t^2} = \int \frac{t-(t-1)}{t(t-1)}dt + \frac{1}{t} + c$

$= \ln|t-1| - \ln|t| + \frac{1}{t} + c$

$\Rightarrow \ln\left|xe^x\right| - \ln\left|1+xe^x\right| + \frac{1}{1+xe^x} + c$

$I(x) = \ln\left|\frac{xe^x}{1+xe^x}\right| + \frac{1}{1+xe^x} + c$

$\lim_{x\to\infty} I(x) = c = 0$

$\therefore I(1) = \ln\left|\frac{e}{1+e}\right| + \frac{1}{1+e}$

$= \ln e - \ln(1+e) + \frac{1}{1+e}$

$= \frac{e+2}{e+1} - \ln(1+e)$

Question ID : 3666942932

4. If for $z = \alpha + i\beta, |z+2| = z + 4(1+i)$, then $\alpha+\beta$ and $\alpha\beta$ are the roots of the equation :

यदि $z = \alpha + i\beta$ के लिए $|z+2| = z+4(1+i)$, तो $\alpha+\beta$ तथा $\alpha\beta$ किस समीकरण के मूल है ?

(1) $x^2 + 7x + 12 = 0$ (2) $x^2 + 2x - 3 = 0$ (3) $x^2 + x - 12 = 0$ (4) $x^2 + 3x - 4 = 0$

Ans. Official Answer NTA (1)

Sol. $z = \alpha + i\beta$

$|z+2| = z + 4(1+i)$

$|(\alpha+2) + i\beta| = \alpha + i\beta + 4 + 4i$

$\sqrt{(\alpha+2)^2 + \beta^2} = (\alpha+4) + i(\beta+4)$

compare real and imaginary part from both sides

$\beta + 4 = 0 \Rightarrow \beta = -4$

and $\sqrt{(\alpha+2)^2 + 16} = \alpha + 4$

$\Rightarrow \alpha^2 + 4 + 4\alpha + 16 = \alpha^2 + 16 + 8\alpha$

$\Rightarrow \alpha = 1, \beta = -4$

$\alpha + \beta = -3, \alpha\beta = -4$

Hence equation is $x^2 - (-3-4)x + (-3)(-4) = 0$

$x^2 + 7x + 12 = 0$

Question ID : 3666942939

5. Let $S_K = \frac{1+2+\ldots+K}{K}$ and $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}\left(Bn^2 + Cn + D\right)$, where $A, B, C, D \in \mathbb{N}$ and A has least value. Then :

(1) $A + B$ is divisible by D
(2) $A + B + C + D$ is divisible by 5
(3) $A + B = 5(D - C)$
(4) $A + C + D$ is not divisible by B

माना $S_K = \frac{1+2+\ldots+K}{K}$ तथा $\sum_{j=1}^{n} S_j^2 = \frac{n}{A}\left(Bn^2 + Cn + D\right)$ है, जहाँ $A, B, C, D \in \mathbb{N}$ है तथा A का मान न्यूनतम है। तो :

(1) A + B, D से विभाज्य है
(2) A + B + C + D, 5 से विभाज्य है
(3) $A + B = 5(D - C)$
(4) A + C + D, B से विभाज्य नहीं है

Ans. Official Answer NTA (1)

Sol. $S_K = \frac{K\cdot(K+1)}{2K} = \frac{K+1}{2}$

$\sum_{j=1}^{n}\left(S_j\right)^2 = \sum_{j=1}^{n}\frac{(j+1)^2}{4} = \frac{1}{4}\left(2^2 + 3^2 + 4^2 \ldots (n+1)^2\right)$

$\Rightarrow \frac{1}{4}\left(\frac{(n+1)(n+2)(2n+3)}{6} - 1\right)$

$= \frac{1}{4}\left(\frac{\left(n^2+3n+2\right)(2n+3) - 6}{6}\right)$

$= \frac{1}{4}\left(\frac{2n^3 + 6n^2 + 4n + 3n^2 + 9n + 6 - 6}{6}\right)$

$=\frac{1}{4}\left(\frac{2n^3+9n^2+13n}{6}\right)$

$=\frac{n}{24}\left(2n^2+9n+13\right)$

$A=24, B=2, C=9, D=13$

$\frac{A+B}{D}=\frac{26}{13}=2$

Question ID : 3666942934

6. Let $A=\begin{bmatrix}2 & 1 & 0\\ 1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$. If $|\text{adj}(\text{adj}(\text{adj } 2A))|=(16)^n$, then n is equal to :

माना $A=\begin{bmatrix}2 & 1 & 0\\ 1 & 2 & -1\\ 0 & -1 & 2\end{bmatrix}$ है। यदि $|\text{adj}(\text{adj}(\text{adj } 2A))|=(16)^n$ है, तो n बराबर है :

(1) 8 (2) 9 (3) 10 (4) 12

Ans. Official Answer NTA (3)

Sol. $\left|\text{adj}\left(\text{adj}\left(\text{adj } 2A\right)\right)\right|$

$=\left|\text{adj}\left(\text{adj}\left(2A\right)\right)\right|^2$

$=|\text{adj } 2A|^4$

$=|2A|^8=(8\,|A|)^8=16^{10}$ $\{\because\ |A|=4\}$

$\Rightarrow n=10$

Question ID : 3666942946

7. The shortest distance between the lines $\frac{x-4}{4}=\frac{y+2}{5}=\frac{z+3}{3}$ and $\frac{x-1}{3}=\frac{y-3}{4}=\frac{z-4}{2}$ is :

रेखाओं $\frac{x-4}{4}=\frac{y+2}{5}=\frac{z+3}{3}$ तथा $\frac{x-1}{3}=\frac{y-3}{4}=\frac{z-4}{2}$ के बीच न्यूनतम दूरी है :

(1) $3\sqrt{6}$ (2) $6\sqrt{3}$ (3) $6\sqrt{2}$ (4) $2\sqrt{6}$

Ans. Official Answer NTA (1)

Sol. $S_d = \left|\frac{(\vec{a}-\vec{b})\times(\vec{n}_1\times\vec{n}_2)}{|\vec{n}_1\times\vec{n}_2|}\right|$

$\bar{a} = (4,-2,-3)$

$\bar{b} = (1,3,4)$

$\bar{n}_1 = (4,5,3)$

$\bar{n}_2 = (3,4,2)$

$$\bar{n}_1\times\bar{n}_2 = \begin{vmatrix} i & j & k \\ 4 & 5 & 3 \\ 3 & 4 & 2 \end{vmatrix} = \hat{i}(-2)-\hat{j}(-1)+\hat{k}(1) = (-2,1,1)$$

$$S_d = \frac{(3,-5,-7)\cdot(-2,1,1)}{\sqrt{6}} = \left|\frac{-6-5-7}{\sqrt{6}}\right| = 3\sqrt{6}$$

Question ID : 3666942937

8. The number of ways, in which 5 girls and 7 boys can be seated at a round table so that no two girls sit together, is :

5 लड़कियों तथा 7 लड़कों को एक गोल मेज पर इस प्रकार बैठाने, कि कोई भी दो लड़कियाँ एक साथ न बैठें, के तरीकों की संख्या है :

(1) $7(360)^2$ (2) $7(720)^2$ (3) 720 (4) $126(5!)^2$

Ans. Official Answer NTA (4)

Sol. 7 boys can sit = 6 !

which create 7 gap between then in which 5 girls have to set

No of ways = $6!\,.\times{}^7C_5\,5! = 126(5!)^2$

Question ID : 3666942948

9. In a bolt factory, machines A, B and C manufacture respectively 20%, 30% and 50% of the total bolts. Of their output 3, 4 and 2 percent are respectsively defective bolts. A bolt is drawn at random from the product. If the bolt drawn is found the defective, then the probability that it is manufactured by the C is:

एक बोल्ट बनाने के कारखाने में मशीन A, B तथा C कुल उत्पादन का क्रमशः 20%, 30% तथा 50% बोल्ट बनाती है। इन मशीनों के उत्पादन का क्रमशः 3, 4 तथा 2 प्रतिशत बोल्ट खराब हैं। बोल्टों के उत्पादन में से एक बोल्ट यादृच्छया निकाला जाता है। यदि निकाला गया बोल्ट खराब पाया जाता है, तो इसके मशीन C द्वारा बनाये जाने की प्रायिकता है

(1) $\frac{3}{7}$ (2) $\frac{2}{7}$ (3) $\frac{9}{28}$ (4) $\frac{5}{14}$

Ans. Official Answer NTA (4)

Sol. Using Bayes' Theorem

Required probability $= \frac{50\times 2}{20\times 3+30\times 4+50\times 2}$

$= \frac{10}{6+12+10}$

$= \frac{10}{28}$

$= \frac{5}{14}$

Question ID : 3666942945

10. If the equation of the plane containing the line $x+2y+3z-4=0=2x+y-z+5$ and perpendicular to the plane $\vec{r}=(\hat{i}-\hat{j})+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3\hat{k})$ is $ax+by+cz=4$, then $(a-b+c)$ is equal to :

यदि समतल, जिसमें रेखा $x + 2y + 3z - 4 = 0 = 2x + y - z + 5$ स्थित है तथा जो समतल $\vec{r}=(\hat{i}-\hat{j})+\lambda(\hat{i}+\hat{j}+\hat{k})+\mu(\hat{i}-2\hat{j}+3\hat{k})$ के लंबवत् है, का समीकरण $ax + by + cz = 4$ है, तो $(a - b + c)$ बराबर है :

(1) 18 (2) 22 (3) 20 (4) 24

Ans. Official Answer NTA (2)

Sol. Equation of plane containing line of intersection of plane $x+2y+3z-4=0$ and $2x+y-z+5=0$ will be

$P_1 + \lambda P_2 = 0 \Rightarrow (1 + 2\lambda)x + (2 + \lambda)y + (3 - \lambda)z + (5\lambda - 4) = 0$ _______(1)

This plane is $\perp$ to plane

$\vec{r}=\left(\vec{i}-\vec{j}\right)+\lambda\left(\vec{i}+\vec{j}+\vec{k}\right)+\mu\left(\vec{i}-2\vec{j}+3\vec{k}\right)$ _______(2)

Normal of plane $(2)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ 1 & -2 & 3\end{vmatrix}=5\hat{i}-2\hat{j}-3\hat{k}$

Now both plane (1) and (2) $\perp$ Hence their normal also $\perp$

$5(1+2\lambda)-2(2+\lambda)-3(3-\lambda)=0$

$-8+11\lambda=0\Rightarrow\lambda=\frac{8}{11}$

Now equation of plane $\frac{27}{11}x+\frac{30}{11}y+\frac{25}{11}z-\frac{4}{11}=0$

$27x + 30y + 25z = 4$

Hence $a - b + c = 27 - 30 + 25 = 22$

Question ID : 3666942940

11. $\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right)$ is equal to :

$\lim_{x\to 0}\left(\left(\frac{(1-\cos^2(3x))}{\cos^3(4x)}\right)\left(\frac{\sin^3(4x)}{(\log_e(2x+1))^5}\right)\right)$ बराबर है :

(1) 15 (2) 24 (3) 9 (4) 18

Ans. Official Answer NTA (4)

Sol. $\lim_{x\to 0}\left[\frac{1-\cos^2 3x}{9x^2}\right]\frac{9x^2}{\cos^3 4x}\cdot\frac{\left(\frac{\sin 4x}{4x}\right)^3\times 64x^3}{\left[\frac{\ln(1+2x)}{2x}\right]^5\times 32x^5}$

$\lim_{x\to 0}\left(\frac{9}{1}\times\frac{1\times 64}{1\times 32}\right)=18$

Question ID : 3666942949

12. Let $f(x)=\frac{\sin x+\cos x-\sqrt{2}}{\sin x-\cos x}, x\in[0,\pi]-\left\{\frac{\pi}{4}\right\}$. Then $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$ is equal to :

यदि $f(x)=\frac{\sin x+\cos x-\sqrt{2}}{\sin x-\cos x}, x\in[0,\pi]-\left\{\frac{\pi}{4}\right\}$ है, तो $f\left(\frac{7\pi}{12}\right)f''\left(\frac{7\pi}{12}\right)$ बराबर है :

(1) $\frac{2}{9}$ (2) $\frac{-2}{3}$ (3) $\frac{2}{3\sqrt{3}}$ (4) $\frac{-1}{3\sqrt{3}}$

Ans. Official Answer NTA (1)

Sol. $f(x)=\sqrt{2}\dfrac{\left(\frac{1}{\sqrt{2}}\sin x+\frac{1}{\sqrt{2}}\cos x\right)-1}{\sqrt{2}\left(\frac{1}{\sqrt{2}}\sin x-\frac{1}{\sqrt{2}}\cos x\right)}=\dfrac{\sin(x+\pi/4)-1}{\sin(x-\pi/4)}$

$$f'(x)=\frac{\cos(x+\pi/4)\sin(x-\pi/4)-\cos(x-\pi/4)(\sin(x+\pi/4)-1)}{\sin^2(x-\pi/4)}$$

$$f'(x)=\frac{\cos(x-\pi/4)-1}{\sin^2(x-\pi/4)}=\frac{-(1-\cos(x-\pi/4))}{1-\cos^2(x-\pi/4)}$$

$$f'(x)=-\frac{1}{1+\cos(x-\pi/4)}$$

$$=f''(x)=-\frac{\sin(x-\pi/4)}{(1+\cos(x-\pi/4))^2}$$

$$f(7\pi/12)=-\frac{1}{\sqrt{3}}$$

$$f''(7\pi/12)=-\frac{2\sqrt{3}}{9}$$

$$f(7\pi/12)f''(7\pi/12)=\frac{2}{9}$$

Question ID : 3666942950

13. Negation of $(p\Rightarrow q)\Rightarrow(q\Rightarrow p)$ is :

$(p\Rightarrow q)\Rightarrow(q\Rightarrow p)$ का निषेधन है :

(1) $p\vee(\sim q)$ (2) $(\sim q)\wedge p$ (3) $q\wedge(\sim p)$ (4) $(\sim p)\vee q$

Ans. Official Answer NTA (3)

Sol. $(p\rightarrow q)\rightarrow(q\rightarrow p)$

$\Rightarrow(p'\vee q)'\vee(q'\vee p)$

$\Rightarrow(p\wedge q')\vee(q'\vee p)$

$=p\vee q'$

Now $(p\vee q')'$

$= p' \wedge q$

Question ID : 3666942944

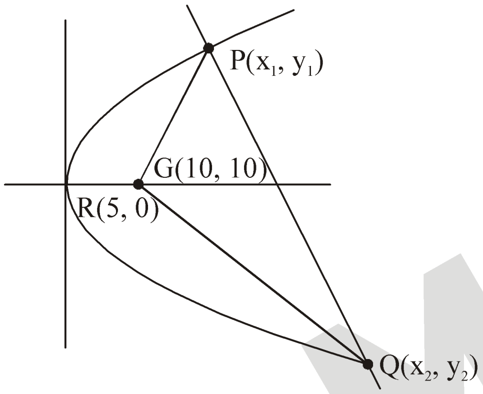
14. Let R be the focus of the parabola $y^2 = 20x$ and the line $y = mx + c$ intersect the parabola at two points P and Q. Let the point G(10, 10) be the centroid of the triangle PQR. If $c - m = 6$, then $(PQ)^2$ is :

माना परवलय $y^2 = 20x$ की नाभि R है तथा रेखा $y = mx + c$ परवलय को दो बिंदुओं P तथा Q पर काटती है। माना त्रिभुज PQR का केन्द्रक, बिंदु G(10, 10) है। यदि $c - m = 6$ है, तो $(PQ)^2$ बराबर है :

(1) 325 (2) 346 (3) 296 (4) 317

Ans. Official Answer NTA (1)

Sol.



$y^2 = 20x,\ y = mx + c$

$$y^2 = 20\left(\frac{y-c}{m}\right)$$

$$y^2 - \frac{20y}{m} + \frac{20c}{m} = 0 \qquad \frac{y_1 + y_2 + y_3}{3} = 10$$

$$\frac{20}{m} = 30$$

$$m = \frac{2}{3}$$

and $c - m = 6$

$$c = \frac{2}{3} + 6 \Rightarrow \frac{20}{3} = c$$

$$y^2 - 30y + \frac{20 \times \frac{20}{3}}{\frac{2}{3}} = 0 \Rightarrow y^2 - 30y + 200 = 0$$

y = 10 $\Rightarrow$ x = 5
y = 20 $\Rightarrow$ x = 20
P(5, 10); (20, 20)Q
$PQ^2 = 15^2 + 10^2 = 225 + 100 = 325$

Question ID : 3666942931

15. Let the number of elements in sets A and B be five and two respectively. Then the number of subsets of $A \times B$ each having at least 3 and at most 6 elements is :

माना समुच्चयों A तथा B में अवयवों की संख्या क्रमशः पाँच तथा दो है। तो $A \times B$ के उपसमुच्चयों, जिनमें कम से कम 3 तथा अधिक से अधिक 6 अवयव हो, की संख्या है :

(1) 792 (2) 772 (3) 752 (4) 782

Ans. Official Answer NTA (1)

Sol. n(A) = 5, n(B) = 2

$\therefore\ n(A \times B) = 10$

Number of sub sets = ${}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 = 792$

Question ID : 3666942936

16. The number of arrangements of the letters of the word "INDEPENDENCE" in which all the vowels always occur together is :

शब्द "INDEPENDENCE" के अक्षरों को लिखने के तरीकें, जिनमें सभी स्वर हमेशा एक साथ हों, की संख्या है :

(1) 33600 (2) 16800 (3) 18000 (4) 14800

Ans. Official Answer NTA (2)

Sol. Vol. I, , E, E, E, E

Constant N, D, P, N, D, N, C

No of ways = $\frac{8!}{3!2!} \cdot \frac{5!}{4!} = 16800$

Question ID : 3666942938

17. If the coefficients of three consecutive terms in the expansion of $(1+x)^n$ are in the ratio 1 : 5 : 20, then the coefficient of the fourth term is :

यदि $(1+x)^n$ के प्रसार में तीन क्रमागत पदों के गुणांकों का अनुपात 1 : 5 : 20 है, तो चौथे पद का गुणांक है :

(1) 1827 (2) 2436 (3) 5481 (4) 3654

Ans. Official Answer NTA (4)

Sol. $\frac{{}^nC_{r-1}}{1} = \frac{{}^nC_r}{5} = \frac{{}^nC_{r+1}}{20}$

$\because \frac{{}^nC_r}{{}^nC_{r-1}} = 5 \Rightarrow \frac{\frac{\lfloor n}{\lfloor n-r \lfloor r}}{\frac{\lfloor n}{\lfloor n-r+1 \lfloor r-1}} = 5$

$\Rightarrow \frac{n-r+1}{r} = 5 \Rightarrow n = 6r - 1 \ldots \text{ (i)}$

$\because \frac{{}^nC_{r+1}}{{}^nC_r} = 4 \Rightarrow \frac{n-r}{r+1} = 4 \Rightarrow n = 5r + 4.$

from (i) and (ii), $r = 5, n = 29$

$\Rightarrow$ Coefficient of fourth term $= {}^{29}C_3 = 3654$

Question ID : 3666942943

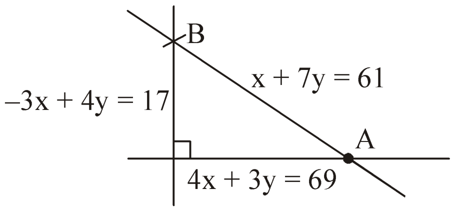
18. Let $C(\alpha, \beta)$ be the circumcenter of the triangle formed by the lines
$4x + 3y = 69$
$4y - 3x = 17$, and
$x + 7y = 61$.
Then $(\alpha - \beta)^2 + \alpha + \beta$ is equal to :

माना रेखाओं
$4x + 3y = 69$
$4y - 3x = 17$, तथा
$x + 7y = 61$.
द्वारा निर्मित त्रिभुज का परिकेन्द्र $C(\alpha, \beta)$ है। तो $(\alpha - \beta)^2 + \alpha + \beta$ बराबर है :

(1) 18 (2) 17 (3) 16 (4) 15

Ans. Official Answer NTA (2)

Sol.



$4x + 28y = 244$

$4x + 3y = 69$
$- \quad - \quad -$
$25y = 175$
$y = 7, x = 12$
$A(12, 7)$
$-3x + 4y = 17$
$3x + 21y = 183$
$25y = 200$
$y = 8, x = 5$
$B(5, 8)$
$\therefore$ Circumcenter

$\alpha = \frac{17}{2} \beta = \frac{15}{2}$

$\left(\frac{17}{2}, \frac{15}{2}\right)$

$(\alpha - \beta)^2 + \alpha + \beta$

$1 + 16 = 17$

Question ID : 3666942942

19. The area of the region $\{(x, y) : x^2 \le y \le 8 - x^2, y \le 7\}$ is :

क्षेत्र $\{(x, y) : x^2 \le y \le 8 - x^2, y \le 7\}$ का क्षेत्रफल है :

(1) 18 (2) 24 (3) 20 (4) 21

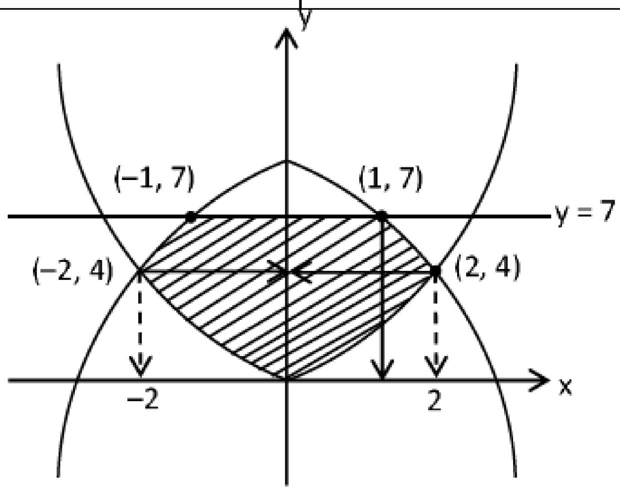
Ans. Official Answer NTA (3)

Sol. $y \ge x^2 \quad y \le 8 - x^2 \quad y \le 7$

$x^2 = 8 - x^2$

$x^2 = 4$

$x = \pm 2$



$$2\left(1.7+\int_1^2\left(8-2x^2\right)dx\right)-2\int_0^1\left(x^2\right)dx$$

$$=2\left[7+\left(8x-\frac{2x^3}{3}\right)_1^2\right]-2\left(\frac{x^3}{3}\right)_0^1$$

$$=2\left[7+\left(16-\frac{16}{3}\right)-\left(8-\frac{2}{3}\right)\right]-2\left(\frac{1}{3}\right)$$

$$=2\left[7+\frac{32}{3}-\frac{22}{3}\right]=2\left[7+\frac{10}{3}\right]\frac{-2}{3}$$

$$=\frac{60}{3}=20$$

Question ID : 3666942935

20. Let $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ and $Q = PAP^T$. If $P^TQ^{2007}\ P=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, then $2a + b - 3c - 4d$ equal to :

माना $P=\begin{bmatrix}\frac{\sqrt{3}}{2} & \frac{1}{2}\\ -\frac{1}{2} & \frac{\sqrt{3}}{2}\end{bmatrix}$, $A=\begin{bmatrix}1 & 1\\ 0 & 1\end{bmatrix}$ तथा $Q = PAP^T$ है। यदि $P^TQ^{2007}\ P=\begin{bmatrix}a & b\\ c & d\end{bmatrix}$, तो $2a + b - 3c - 4d$ बराबर है :

(1) 2004 (2) 2007 (3) 2006 (4) 2005

Ans. Official Answer NTA (4)

Sol. $PP^T = \begin{bmatrix} \frac{\sqrt{3}}{2} & \frac{1}{2} \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix}\begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \frac{\sqrt{3}}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I = P^TP$

$P^TQ^{2007}P^T = P^T\left[\left(PAP^T\right)\left(PAP^T\right)........\left(PAP^T\right)\right]P$

$= P^T\left[PA^{2007}P^T\right]P$

$= A^{2007}$

$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

$A^2 = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

$A^{2007} = \begin{bmatrix} 1 & 2007 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$a = 1, b = 2007, c = 0, d = 1$

$2a + b - 3c - 4d = 2005$

SECTION – B

Question ID : 3666942955

21. If the solution curve of the differential equation $\left(y - 2\log_e x\right)dx + \left(x\log_e x^2\right)dy = 0, x > 1$ passes through the points $\left(e, \frac{4}{3}\right)$ and (e^4, α), then α is equal to ________.

यदि अवकल समीकरण $\left(y - 2\log_e x\right)dx + \left(x\log_e x^2\right)dy = 0, x > 1$ का हल वक्र बिंदुओं $\left(e, \frac{4}{3}\right)$ तथा (e^4, α) से होकर जाता है, तो α बराबर है ________

Ans. Official Answer NTA (3.00)

Sol. $\because \quad (y - 2\ln x)dx + (2x\ln x)dy = 0.$

$2x\ln x\frac{dy}{dx} + y = 2\ln x$

$$\frac{dy}{dx}+\frac{y}{2x\ln x}=\frac{1}{x}$$

$\therefore$ I.F. $=e^{\int\frac{1}{2x\ln x}dx}=\sqrt{\ln x}$

$\therefore$ Solution of the equation is :

$$y\cdot\sqrt{\ln x}=\int\frac{\sqrt{\ln x}}{x}dx$$

$$\therefore\ y\cdot\sqrt{\ln x}=\frac{2}{3}(\ln x)^{\frac{3}{2}}+C \quad \ldots \text{(i)}$$

$\therefore$ eq. (i) passes through point $\left(e,\frac{4}{3}\right)$.

$$\therefore C=\frac{2}{3}$$

$$\therefore y\sqrt{\ln x}=\frac{2}{3}(\ln x)^{\frac{3}{2}}+\frac{2}{3} \quad \ldots \text{(ii)}$$

This equation passes through point (e^4, α)

$\therefore\ \alpha = 3.$

Question ID : 3666942959

22. Let $\vec{a}=6\hat{i}+9\hat{j}+12\hat{k}$, $\vec{b}=\alpha\hat{i}+11\hat{j}-2\hat{k}$ and $\vec{c}$ be vectors such that $\vec{a}\times\vec{c}=\vec{a}\times\vec{b}$. If $\vec{a}\cdot\vec{c}=-12, \vec{c}\cdot(\hat{i}-2\hat{j}+\hat{k})=5$, then $\vec{c}\cdot(\hat{i}+\hat{j}+\hat{k})$ is equal to ________.

माना सदिश $\vec{a}=6\hat{i}+9\hat{j}+12\hat{k}$, $\vec{b}=\alpha\hat{i}+11\hat{j}-2\hat{k}$ तथा $\vec{c}$ इस प्रकार हैं कि $\vec{a}\times\vec{c}=\vec{a}\times\vec{b}$ है। यदि $\vec{a}\cdot\vec{c}=-12, \vec{c}\cdot(\hat{i}-2\hat{j}+\hat{k})=5$ हैं, तो $\vec{c}\cdot(\hat{i}+\hat{j}+\hat{k})$ बराबर है ________

Ans. Official Answer NTA (11)

Sol.

$\vec{a}=6\hat{i}+9\hat{j}+12\hat{k}$ $\qquad$ $\vec{a}\cdot\vec{b}=6\alpha+99-24=6\alpha+75$

$\vec{b}=\alpha\hat{i}+11\hat{j}-2\hat{k}$ $\qquad$ $|\vec{a}|=\sqrt{36+81+144}=\sqrt{261}$

$\vec{a}\times\vec{c}=\vec{a}\times\vec{b}$, $\qquad$ $\vec{a}\cdot\vec{c}=-12, \quad \vec{c}.(\hat{i}-2\hat{j}+\hat{k})=5$

$\vec{a}\times\vec{c}=\vec{a}\times\vec{b}.$ $\qquad$ $\Rightarrow\ \vec{a}\times\vec{c}-\vec{a}\times\vec{b}.=0$

$\Rightarrow\ \vec{a}\times(\vec{c}-\vec{b})=0$

$\Rightarrow\ \vec{c}-\vec{b}\ \|\vec{a}$

$\Rightarrow\ \vec{c}=\vec{b}+\lambda\vec{a}$

Now $\qquad$ $\Rightarrow\vec{c}\cdot\vec{a}=\vec{b}\cdot\vec{a}+\lambda(\vec{a}\cdot\vec{a})=-12$

$$6\alpha + 75 + \lambda(261) = -12$$

$$6\alpha + 261\lambda = -87 \quad \_\_\_\_\_\_\_\_(i)$$

$$\vec{c}.(\hat{i} - 2\hat{j} + \hat{k}) = 5, \Rightarrow (\alpha - 22 - 2) + \lambda(6 - 18 + 12) = 5$$

$$\Rightarrow \alpha = 29$$

From equation (i) $\lambda = -1$

Hence $= \vec{c} \cdot (\hat{i} + \hat{j} + \hat{k}) = (\vec{b} - \vec{a}) \cdot (\hat{i} + \hat{j} + \hat{k}) = 11$

Question ID : 3666942956

23. The largest natural number n such that 3^n divides 66! is ________.

अधिकतम धनपूर्णांक n, जिसके लिए 66!, 3^n से विभाज्य है, है ________

Ans. Official Answer NTA (31)

Sol. $\left[\frac{66}{3}\right] + \left[\frac{66}{9}\right] + \left[\frac{66}{27}\right]$

22 + 7 + 2 = 31

Question ID : 3666942953

24. If a_α is the greatest term in the sequence $a_n = \frac{n^3}{n^4 + 147}, n = 1, 2, 3, \ldots$ then α is equal to ________.

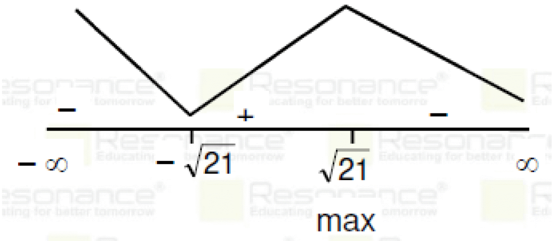
यदि अनुक्रम $a_n = \frac{n^3}{n^4 + 147}, n = 1, 2, 3, \ldots$ का अधिकतम पद a_α है, तो α बराबर है ________

Ans. Official Answer NTA (5)

Sol. $$f'(n) = \frac{3n^2\left(n^4 + 147\right) - 4n^3\left(n^3\right)}{\left(n^4 + 147\right)^2}$$

$$f'(n) = \frac{n^2\left[3n^4 + 441 - 4n^4\right]}{\left(n^4 + 147\right)^2}$$

$$f'(n) = \frac{-n^2\left(n^4 - 441\right)}{\left(n^4 + 147\right)^2} = \frac{-n^2\left(n^2 + 21\right)\left(n^2 - 21\right)}{\left(n^4 + 147\right)^2} = \frac{-n^2\left(n^2 + 21\right)}{\left(n^4 + 147\right)^2}(n + \sqrt{21})(n - \sqrt{21})$$



When $n = 4$ $a_4 = \frac{4^3}{4^4 + 147} = \frac{64}{256+147} = \frac{64}{403}$

When $n = 5$ $a_5 = \frac{125}{625+147} = \frac{125}{772}$

a_5 is $\max = \frac{125}{772}$ hence $n = 5$

Question ID : 3666942951

25. Let $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ and R be the relation defined on A such that $R = \{(x, y) \in A \times A : x - y$ is odd positive integer or $x - y = 2\}$. The minimum number of elements that must be added to the relation R, so that it is a symmetric relation, is equal to ________.

माना $A = \{0, 3, 4, 6, 7, 8, 9, 10\}$ है तथा A पर एक संबंध R, $R = \{(x, y) \in A \times A : x - y$ विषम धनात्मक पूर्णांक है या $x - y = 2\}$. द्वारा परिभाषित है। संबंध R के सममित संबंध होने के लिए इसमें कम से कम कितने अवयव जोड़े जाएँ________

Ans. Official Answer NTA (19)

Sol. $A = \{10, 9, 8, 7, 6, 4, 3, 0\}$

$R = \{(10, 9), (10, 8), (10, 7), (10, 3), (9, 8), (9, 7), (9, 6), (9, 4), (9, 0), (8, 7), (8, 6), (8, 3), (7, 6), (7, 4), (7, 0), (6, 4), (6, 3), (4, 3), (3, 0)\}$

All the elements of R, (a, b) are of type $a > b$.

Hence we need to add total of 19 more elements to R to make in symmetric.

Question ID : 3666942957

26. Consider a circle $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$. Let its mirror image in the line $y = 2x + 1$ be another circle $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$. Let r be the radius of C_2. Then $\alpha + r$ is equal to ________.

एक वृत $C_1 : x^2 + y^2 - 4x - 2y = \alpha - 5$ का विचार कीजिए। माना रेखा $y = 2x + 1$ में इसका दर्पण प्रतिबिंब वृत $C_2 : 5x^2 + 5y^2 - 10fx - 10gy + 36 = 0$ है। माना वृत C_2 की त्रिज्या r है। तो $\alpha + r$ बराबर है ________

Ans. Official Answer NTA (2)

Sol. $x^2 + y^2 - 4x - 2y + 5 - \alpha = 0$

$C_1(2,1) r_1 = \sqrt{\alpha}$

$2x - y + 1 = 0$

Image of (2, 1)

$$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-2(4-1+1)}{5}$$

$$\frac{x-2}{2} = \frac{y-1}{-1} = \frac{-8}{5}$$

$$x = 2 - \frac{16}{5} = \frac{-6}{5}, y = 1 + \frac{8}{5} = \frac{13}{5}$$

$$x^2 + y^2 - 2fx - 2gy + \frac{36}{5} = 0$$

$$C_2(f, g)$$

$$r_2 = \sqrt{f^2 + g^2 - \frac{36}{5}}$$

$$\alpha = f^2 + g^2 - \frac{36}{5}$$

$$\therefore f = -\frac{6}{5}, g = \frac{13}{5}$$

$$\alpha = \frac{36}{25} + \frac{169}{25} - \frac{36}{5}$$

$$= \frac{36 + 169 - 180}{25} \Rightarrow \alpha = 1 \Rightarrow r = 1$$

$$\therefore \alpha + r = 2$$

Question ID : 3666942960

27. Let the mean and variance of 8 numbers x, y, 10, 12, 6, 12, 4, 8 be 9 and 9.25 respectively. If $x > y$, then $3x - 2y$ is equal to ________.

माना 8 संख्याओं x, y, 10, 12, 6, 12, 4, 8 के माध्य तथा प्रसरण क्रमश: 9 तथा 9.25 है। यदि $x > y$ है, तो $3x - 2y$ बराबर है ________

Ans. Official Answer NTA (25)

Sol. $$\frac{x + y + 52}{8} = 9 \Rightarrow x + y = 20$$

For variance

$$x-9, y-9, 3, 3, 1, -5, -1, -3$$

$$\bar{x} = 0$$

$\therefore \frac{(x-9)^2+(y-9)^2+54}{8}-\overline{0}^2=9.25$

$(x-9)^2+(11-x)^2=20$

$x=7 \text{ or } 13 \therefore y=13,7$

$3x-2y=3\times 13-2\times 7=25$

Question ID : 3666942952

28. Let [t] denote the greatest integer $\leq t$. If the constant term in the expansion of $\left(3x^2-\frac{1}{2x^5}\right)^7$ is α, then $[\alpha]$ is equal to ________.

माना [t] महत्तम पूर्णांक $\leq t$ है। यदि $\left(3x^2-\frac{1}{2x^5}\right)^7$ के प्रसार में अचर पद α है, तो $[\alpha]$ बराबर है ________

Ans. Official Answer NTA (1275)

Sol. $T_{r+1}={}^7C_r\left(3x^2\right)^{7-r}\left(-\frac{1}{2x^5}\right)^r$

$T_{r+1}={}^7C_r 3^{7-r}\left(-\frac{1}{2}\right)^r x^{14-7r}$

For term independent of x : $14-7r=0 \Rightarrow r=2$

Required coefficient $={}^7C_2 3^{7-2}\left(-\frac{1}{2}\right)^2=\frac{7.6}{2}\cdot 3^5\cdot\frac{1}{2^2}$

$=\frac{7\cdot 3^6}{4}=1275.85$

$[1275.85]=1275$

Question ID : 3666942958

29. Let λ_1, λ_2 be the values of λ for which the points $\left(\frac{5}{2},1,\lambda\right)$ and (–2, 0, 1) are at equal distance from the plane $2x+3y-6z+7=0$. If $\lambda_1>\lambda_2$, then the distance of the point $(\lambda_1-\lambda_2, \lambda_2, \lambda_1)$ from the line $\frac{x-5}{1}=\frac{y-1}{2}=\frac{z+7}{2}$ is ________.

माना λ के मान, जिनके लिए बिंदु $\left(\frac{5}{2},1,\lambda\right)$ तथा (–2, 0, 1) समतल $2x+3y-6z+7=0$ से समान दूरी पर है, λ_1, λ_2 है।

यदि $\lambda_1 > \lambda_2$ है, तो बिंदु $(\lambda_1 - \lambda_2, \lambda_2, \lambda_1)$ की रेखा $\frac{x-5}{1} = \frac{y-1}{2} = \frac{z+7}{2}$ से दूरी है ________

Ans. Official Answer NTA (9.00)

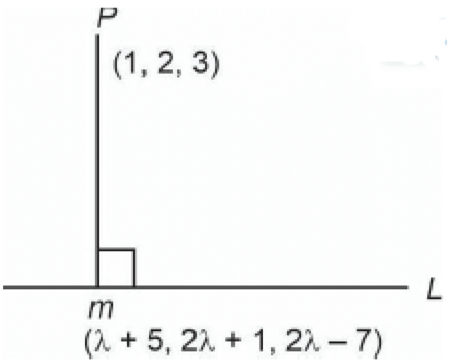
Sol. $\left|\frac{-4+0-6+7}{7}\right| = \left|\frac{15-6\lambda}{7}\right|$

$\frac{3}{7} = \left|\frac{15-6\lambda}{7}\right|$

$\lambda = 2 \text{ or } 3$

$\lambda_1 = 3, \lambda_2 = 2$

$(\lambda_1 - \lambda_2, \lambda_2, \lambda_1) = (1,2,3)$



$\overrightarrow{PM} \cdot (\hat{i} + 2\hat{j} + 2k) = 0$

$(\lambda+4) + 2(2\lambda-1) + 2(2\lambda-10) = 0$

$\Rightarrow 9\lambda = 18 \text{ or } \lambda = 2$

Distance $= \sqrt{6^2+3^2+6^2} = 9$

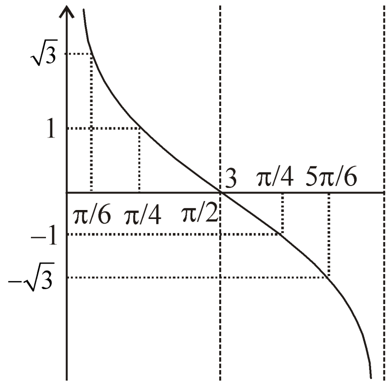
Question ID : 3666942954

30. Let [t] denote the greatest integer $\le$ t. Then $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6}(8[\operatorname{cosec} x] - 5[\cot x])dx$ is equal to ________.

माना [t] महत्तम पूर्णांक $\le$ t है। तो $\frac{2}{\pi}\int_{\pi/6}^{5\pi/6}(8[\operatorname{cosec} x] - 5[\cot x])dx$ बराबर है ________

Ans. Official Answer NTA (14)

Sol.



$$I = \frac{2}{\pi}\int_{\pi/6}^{5\pi/6}\left(8[\operatorname{cosec} x] - 5[\cot x]\right)dx$$

For $x \in (\pi/6, 5\pi/6)$

$\operatorname{cosec} x \in [1, 2)$

$[\operatorname{cosec} x] = 1$

$$I = \frac{2}{\pi}\left[\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} 8(1)dx - \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} 5(1)dx - \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} 5(0)dx - \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} 5(-1)dx - \int_{\frac{3\pi}{4}}^{\frac{5\pi}{6}} 5(-2)dx\right] = 14$$