JEE Main April 2023

Question Paper With Text Solution

08 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2023 | 08TH APRIL SHIFT-2

SECTION – A

Question ID : 7155054228

1. The absolute difference of the coefficients of x^{10} and x^7 in the expansion of $\left(2x^2+\frac{1}{2x}\right)^{11}$ is equal to :

$\left(2x^2+\frac{1}{2x}\right)^{11}$ के प्रसार में x^{10} तथा x^7 के गुणांकों का निरपेक्ष अंतर बराबर है :

(1) 11^3-11 (2) 12^3-12 (3) 13^3-13 (4) 10^3-10

Ans. Official Answer NTA (2)

Sol. $T_{r+1}={}^{11}C_r\left(2x^2\right)^{11-r}(2x)^{-r}$

For coefficient of x^7

$\therefore\quad 22-2r-r=7$

$\therefore\quad r=5$

For coefficient of x^{10}

$22-3r=10$

$\Rightarrow r=4$

Absolute difference $=\left|{}^{11}C_5\cdot\frac{2^6}{2^5}-{}^{11}C_4\cdot\frac{2^7}{2^4}\right|$

$=|924-2640|=1716$

$=12^3-12$

Question ID : 7155054224

2. Let S be the set of all values of $\theta\in[-\pi,\pi]$ for which the system of linear equations

$x+y+\sqrt{3}z=0$

$-x+(\tan\theta)y+\sqrt{7}z=0$

$x+y+(\tan\theta)z=0$

has non-trivial solution. Then $\frac{120}{\pi}\sum_{\theta\in s}\theta$ is equal to :

माना $\theta\in[-\pi,\pi]$ के सभी मानों, जिनके लिए रैखिक समीकरण निकाय

$x+y+\sqrt{3}z=0$

$-x+(\tan\theta)y+\sqrt{7}z=0$

$x+y+(\tan\theta)z=0$

का अतुच्छ हल है, का समुच्च S है। तो $\frac{120}{\pi}\sum_{\theta\in s}\theta$ बराबर है :

(1) 30 (2) 20 (3) 10 (4) 40

Ans. Official Answer NTA (2)

Sol. For non trivial solutions

$D=0$

$$\begin{vmatrix} 1 & 1 & \sqrt{3} \\ -1 & \tan\theta & \sqrt{7} \\ 1 & 1 & \tan\theta \end{vmatrix}=0$$

$$\tan^2\theta-\left(\sqrt{3}-1\right)\tan\theta-\sqrt{3}=0$$

$$\tan\theta=\sqrt{3},-1$$

$$\theta=\left\{\frac{\pi}{3},\frac{-2\pi}{3},\frac{-\pi}{4},\frac{3\pi}{4}\right\}$$

$$\frac{120}{\pi}\left(\sum\theta\right)=\frac{120}{\pi}\times\frac{\pi}{6}=20$$

Question ID : 7155054231

3. The integral $\int\left(\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x\right)\log_2 x\,dx$ is equal to :

समाकलन $\int\left(\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x\right)\log_2 x\,dx$ बराबर है :

(1) $\left(\frac{x}{2}\right)^x\log_2\left(\frac{x}{2}\right)+C$

(2) $\left(\frac{x}{2}\right)^x\log_2\left(\frac{2}{x}\right)+C$

(3) $\left(\frac{x}{2}\right)^x-\left(\frac{2}{x}\right)^x+C$

(4) $\left(\frac{x}{2}\right)^x+\left(\frac{2}{x}\right)^x+C$

Ans. Official Answer NTA (2) **Bonus**

Sol. $\int\left(x^x 2^{-x}+2^x x^{-x}\right)\log_2 x dx$

$\int\left(e^{x\ln x}\cdot e^{-x\ln 2}+e^{x\ln 2}\cdot e^{-x\ln x}\right)dx$

$\int\left(e^{x\ln x-x\ln 2}+e^{x\ln 2-x\ln x}\right)\frac{\ln x}{\ln 2}dx$

let $x\ln x-x\ln 2=t$

$(\ln x+1-\ln 2)dx=dt$

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4. Let a_n be the n^{th} term of the series 5 + 8 + 14 + 23 + 35 + 50 +....... and $S_n=\sum_{k=1}^{n}a_k$. Then $S_{30}-a_{40}$ is equal to :

माना श्रेणी 5 + 8 + 14 + 23 + 35 + 50 +....... का नवाँ पद a_n है तथा $S_n=\sum_{k=1}^{n}a_k$ तो $S_{30}-a_{40}$ बराबर है :

(1) 11260 (2) 11310 (3) 11280 (4) 11290

Ans. Official Answer NTA (4)

Sol. $S_n=5+8+14+23+35+\ldots\ldots+a_{n-1}+a_n$

$S_n=\quad 5+8+14+23\ldots\ldots\ldots\ldots+a_{n-1}+a_n$

$a_n=5+3+6+9+12+\ldots\ldots$

$a_n=5+\frac{n-1}{2}[6+(n-2)3]$

$a_n=5+\left(\frac{n-1}{2}\right)(3n)=\frac{3}{2}n(n-1)+5$

$a_n=\frac{3n^2}{2}-\frac{3n}{2}+5$

$a_{40}=3\times\frac{40^2}{2}-3\times\frac{40}{2}+5$

$=2400-60+5=2345$

$S_{30}=\frac{3}{2}\times\frac{30\times31\times61}{6}-\frac{3}{2}\frac{30\times31}{2}+5\times30$

$=13635$

$S_{30}-a_{40}=11290$

Question ID : 7155054223

5. Let $A=\left\{\theta\in(0,2\pi):\frac{1+2i\sin\theta}{1-i\sin\theta}\text{ is purely imaginary}\right\}$. Then the sum of the elements in A is :

माना $A=\left\{\theta\in(0,2\pi):\frac{1+2i\sin\theta}{1-i\sin\theta}\text{ मात्र काल्पनिक है}\right\}$ । तो A में अवयवों का योग है :

(1) π (2) 4π (3) 2π (4) 3π

Ans. Official Answer NTA (2)

Sol. $\frac{1+2i\sin\theta}{1-i\sin\theta}$ is purely imaginary

So, $\frac{1-2\sin^2\theta}{1+\sin^2\theta}=0$

$\Rightarrow \sin^2\theta=\frac{1}{2}$

$\therefore\ \theta=\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$, for $\theta\in(0,2\pi)$

$\therefore$ Sum of all values $=\frac{16\pi}{4}=4\pi$

Question ID : 7155054238

6. If the probability that the random variable X takes values x is given by $P(X=x)=k(x+1)3^{-x}$, $x=0,1,2,3,\ldots$, where k is a constant, then $P(X\geq 2)$ is equal to :

माना यादृच्छिक चर X के मान x लेने की प्रायिकता $P(X=x)=k(x+1)3^{-x}$, $x=0,1,2,3,\ldots$, है, जहा k एक अचर है, तो $P(X\geq 2)$ बराबर है :

(1) $\frac{11}{18}$ (2) $\frac{20}{27}$ (3) $\frac{7}{18}$ (4) $\frac{7}{27}$

Ans. Official Answer NTA (1)

Sol. $P(x=0)+P(x=1)+p(x=2)+p(x=3)+\ldots\ldots=1$

$\frac{K}{3^0}+\frac{2K}{3^1}+\frac{3K}{3^2}+\frac{4K}{3^3}+\ldots\ldots..\infty=1$

$K\left(1+\frac{2}{3}+\frac{3}{3^2}+\frac{4}{3^3}+\ldots\ldots\infty\right)=1$

Now

Let, $S = 1 + \frac{2}{3} + \frac{3}{3^2} + \frac{4}{3^3} + \ldots\infty$

$\frac{S}{3} = 0 + \frac{1}{3} + \frac{2}{3^2} + \frac{3}{3^3} + \ldots.\infty$

$\frac{2S}{3} = 1 + \frac{1}{3} + \frac{1}{3^2} + \frac{1}{3^3} + \ldots\infty$

$= \frac{1}{1 - \frac{1}{3}}$

$S = \frac{3}{2} \times \frac{3}{2} = \frac{9}{4}$

Now $K.\frac{9}{4} = 1$

$K = \frac{4}{9}$

Now $P(x \geq 2) = P(2) + P(3) + \ldots\ldots\infty$

$= 1 - P(0) - P(1)$

$= 1 - \left(\frac{K}{1} + \frac{2K}{3}\right) = 1 - \frac{5K}{3}$

$1 - \frac{20}{27} = \frac{7}{27}$

Question ID : 7155054234

7. Let P be the plane passing through the line $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$ and the point(2, 4, –3). If the image of the point (–1, 3, 4) in the plane P is (α, β, γ) then α + β + γ is equal to :

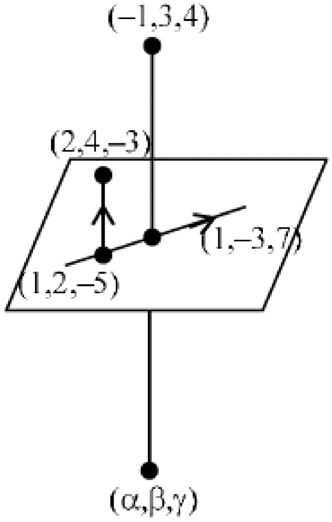
माना बिंदु (2, 4, –3) तथा रेखा $\frac{x-1}{1} = \frac{y-2}{-3} = \frac{z+5}{7}$ से होकर जाने वाला समतल P है। यदि बिंदु (–1, 3, 4) का समतल P में प्रतिबिम्ब (α, β, γ) है, तो α + β + γ बराबर है :

(1) 12 (2) 9 (3) 10 (4) 11

Ans. Official Answer NTA (3)

Sol. Equation of plane is given by

$$\begin{vmatrix} x-1 & y-2 & z+5 \\ 1 & 2 & 2 \\ 1 & -3 & 7 \end{vmatrix} = 0$$



$4x - y - z = 7$

$$\frac{\alpha+1}{4} = \frac{\beta-3}{-1} = \frac{\gamma-4}{-1} = \frac{-2(-4-3-4-7)}{16+1+1} = 2$$

$\alpha = 7, \beta = 1, \gamma = 2$

$\alpha + \beta + \gamma = 10$

Question ID : 7155054233

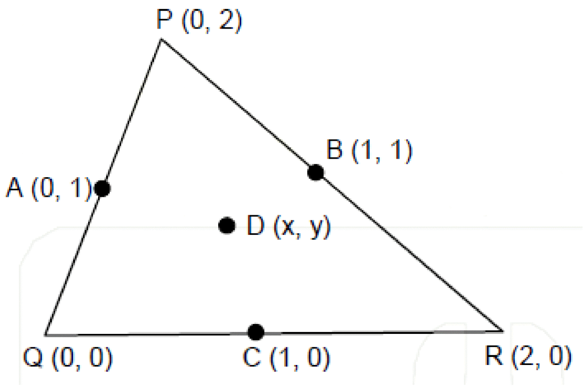
8. Let A(0, 1), B(1, 1) and C(1, 0) be the mid-points of the sides of a triangle with incentre at the point D. If the focus of the parabola $y^2 = 4ax$ passing through D is $(\alpha + \beta\sqrt{2}, 0)$, where α and β are rational numbers, then $\frac{\alpha}{\beta^2}$ is equal to :

माना एक त्रिभुज की भुजाओं के मध्य बिंदु A(0, 1), B(1, 1) तथा C(1, 0) हैं तथा इसका अंतःकेन्द्र बिंदु D पर है। यदि D से होकर जाने वाले परवलय $y^2 = 4ax$ की नाभि $(\alpha + \beta\sqrt{2}, 0)$ है, जहाँ α तथा β परिमेय संख्याऐं हैं, तो $\frac{\alpha}{\beta^2}$ बराबर है :

(1) $\frac{9}{2}$ (2) 12 (3) 6 (4) 8

Ans. Official Answer NTA (4)

Sol.



$D \equiv \left(\frac{0+4+0}{4+2\sqrt{2}}, \frac{4}{4+2\sqrt{2}}\right)$

$D \equiv \left(\frac{2}{2+\sqrt{2}}, \frac{2}{2+\sqrt{2}}\right)$

$D \equiv (2-\sqrt{2}, 2-\sqrt{2},)$

$y^2 = 4ax$

$(2-\sqrt{2})^2 = 4a(2-\sqrt{2})$

$a = \frac{2-\sqrt{2}}{4}$

focus (a, 0)

$\frac{1}{2} - \frac{1}{4}\sqrt{2} = \alpha + \beta\sqrt{2}$

$\alpha = \frac{1}{2}, \beta = -\frac{1}{4}$

$\frac{\alpha}{\beta^2} = \frac{1/2}{1/16} = 8$

Question ID : 7155054236

9. The area of the quadrilateral ABCD with vertices A(2, 1, 1), B(1, 2, 5), C(–2, –3, 5) and D(1, –6, –7) is equal to :

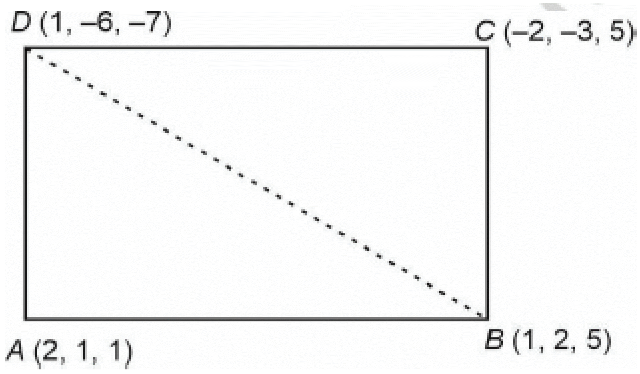
शीर्षों A(2, 1, 1), B(1, 2, 5), C(–2, –3, 5) तथा D(1, –6, –7) के चतुर्भुज ABCD का क्षेत्रफल है :

(1) $8\sqrt{38}$ (2) 54 (3) $9\sqrt{38}$ (4) 48

Ans. Official Answer NTA (1)

Sol. $\overrightarrow{AB} = -\hat{i} + \hat{j} + 4\hat{k}$

$\overrightarrow{AD} = -\hat{i} - 7\hat{j} - 8\hat{k}$



$\therefore$ Area of ΔABD

$$=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 1 & 4\\ -1 & -7 & -8\end{vmatrix}$$

$=|10\hat{i}-6\hat{j}+4\hat{k}|=2\sqrt{38}$

and $\overrightarrow{CB}=3\hat{i}+5\hat{j}$ and $\overrightarrow{CD}=3\hat{i}-3\hat{j}-12\hat{k}$

$$\therefore \text{ Area of } \Delta CBD=\frac{1}{2}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & -3 & -12\\ 3 & 5 & 0\end{vmatrix}$$

$=|6(5\hat{i}-3\hat{j}-2\hat{k})|$

$=6\sqrt{38}$

$\therefore$ Area of $\square ABCD=8\sqrt{38}$ square units

Question ID : 7155054232

10. Let O be the origin and OP and OQ be the tangents to the circle $x^2+y^2-6x+4y+8=0$ at the points P and Q on it. If the circumcircle of the triangle OPQ passes through the point $\left(\alpha,\frac{1}{2}\right)$, then a value of α is :

माना O मूल बिंदु है तथा OP और OQ वृत्त $x^2+y^2-6x+4y+8=0$ के बिंदुओं P तथा Q पर स्पर्श रेखाऐं है। यदि त्रिभुज OPQ का परिवृत्त, बिंदु $\left(\alpha,\frac{1}{2}\right)$ से होकर जाता है, तो α का एक मान है :

(1) $-\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) 1

Ans. Official Answer NTA (3)

Sol. Equation of circumcircle whose diametric end point is (3, –2) and (0, 0)

$x(x-3)+y(y+2)=0$

$x^2+y^2-3x+2y=0$

pt $\left(\alpha, \frac{1}{2}\right)$ is on circle

$\alpha^2+\frac{1}{4}-3\alpha+1=0$

$\alpha^2-3\alpha+\frac{5}{4}=0$

$4\alpha^2-12\alpha+5=0$

$\alpha=\frac{10}{4}, \alpha=\frac{2}{4}$

$\alpha=\frac{5}{2}, \frac{1}{2}$

Question ID : 7155054239

11. Let the mean and variance of 12 observations be $\frac{9}{2}$ and 4 respectively. Later on, it was observed that two observations were considered as 9 and 10 instead of 7 and 14 respectively. If the correct variance is $\frac{m}{n}$, where m and n are coprime, then m + n is equal to :

माना 12 प्रेक्षणों के माध्य तथा प्रसरण क्रमशः $\frac{9}{2}$ तथा 4 हैं। बाद में यह पाया गया कि दो प्रेक्षणों 7 तथा 14 के स्थान पर क्रमशः 9 तथा 10 ले लिए गए थे। यदि सही प्रसरण $\frac{m}{n}$ है, जहाँ m तथा n असहभाज्य है, तो m + n बराबर है :

(1) 315 (2) 314 (3) 317 (4) 316

Ans. Official Answer NTA (3)

Sol. $\frac{\Sigma x}{12}=\frac{9}{2}$

$\sum x=54$

$$\frac{\Sigma x^2}{12}-\left(\frac{9}{2}\right)^2=4$$

$$\sum x^2=291$$

$$\sum x_{new}=54-(9+10)+7+14=56$$

$$\sum x_{new}^2=291-(81+100)+49+196=355$$

$$\sigma_{new}^2=\frac{355}{12}-\left(\frac{56}{12}\right)^2$$

$$\sigma_{new}^2=\frac{281}{36}=\frac{m}{n}$$

$$m+n=317$$

Question ID : 7155054226

12. If the number of words, with or without meaning, which can be made using all the letters of the word MATHEMATICS in which C and S do not come together, is (6!)k, then k is equal to :

यदि शब्द MATHEMATICS के सभी अक्षरों के प्रयोग से बनाए जा सकने वाले, अर्थपूर्ण या अर्थहीन शब्दों, जिनमें C तथा S एक साथ न हो, की संख्या (6!)k है, तो k बराबर है :

(1) 5670 (2) 1890 (3) 945 (4) 2835

Ans. Official Answer NTA (1)

Sol. Required no. of ways $=\frac{11!}{2!2!2!}-\frac{10!}{2!2!2!}\times 2!$

$$=\frac{9\times 10!}{8}$$

Now $\frac{9\times 10!}{8}=k\times 6!$

$k = 9 \times 9 \times 10 \times 7$

$= 5670$

Question ID : 7155054241

13. The negation of $(p\wedge(\sim q))\vee(\sim p)$ is equivalent to :

$(p\wedge(\sim q))\vee(\sim p)$ का निषेधन किसके तुल्य है :

(1) $p \wedge (\sim q)$ (2) $p \wedge (q \wedge (\sim p))$ (3) $p \wedge q$ (4) $p \vee (q \vee (\sim p))$

Ans. Official Answer NTA (3)

Sol. $[(p \wedge (\sim q)) \vee (\sim p)]$

$\sim [(p \wedge \sim q) \wedge (p)]$

$(\sim p \vee q) \wedge (p)$

$= (\sim p \wedge p) \vee (q \wedge p)$

$= C \vee (q \wedge p)$

$= (p \wedge q)$

Question ID : 7155054240

14. The value of $36\left(4\cos^2 9^\circ - 1\right)\left(4\cos^2 27^\circ - 1\right)\left(4\cos^2 81^\circ - 1\right)\left(4\cos^2 243^\circ - 1\right)$ is :

$36\left(4\cos^2 9^\circ - 1\right)\left(4\cos^2 27^\circ - 1\right)\left(4\cos^2 81^\circ - 1\right)\left(4\cos^2 243^\circ - 1\right)$ का मान है :

(1) 54 (2) 27 (3) 18 (4) 36

Ans. Official Answer NTA (4)

Sol. $4\cos^2\theta - 1 = 4\left(1 - \sin^2\theta\right) - 1 = 3 - 4\sin^2\theta = \frac{\sin 3\theta}{\sin\theta}$

so given expression can be written as

$$36 \times \frac{\sin 27^\circ}{\sin 9^\circ} \times \frac{\sin 81^\circ}{\sin 27^\circ} \times \frac{\sin 243^\circ}{\sin 81^\circ} \times \frac{\sin 729^\circ}{\sin 243^\circ}$$

$$36 \times \frac{\sin 729^\circ}{\sin 9^\circ} = 36$$

Question ID : 7155054230

15. If $\alpha > \beta > 0$ are the roots of the equation $ax^2 + bx + 1 = 0$, and $\lim\limits_{x \to \frac{1}{\alpha}} \left(\frac{1 - \cos\left(x^2 + bx + a\right)}{2(1 - \alpha x)^2}\right)^{\frac{1}{2}} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$,

then k is equal to :

यदि समीकरण $ax^2 + bx + 1 = 0$ के मूल $\alpha > \beta > 0$ हैं तथा $\lim\limits_{x \to \frac{1}{\alpha}} \left(\frac{1 - \cos\left(x^2 + bx + a\right)}{2(1 - \alpha x)^2}\right)^{\frac{1}{2}} = \frac{1}{k}\left(\frac{1}{\beta} - \frac{1}{\alpha}\right)$ है, तो k बराबर

है :

(1) α (2) 2α (3) β (4) 2β

Ans. Official Answer NTA (2)

Sol. $\therefore ax^2+bx+1=a(x-\alpha)(x-\beta)\therefore \alpha\beta=\frac{1}{a}$

$\therefore x^2+bx+a=a(1-\alpha x)(1-\beta x)$

$$\therefore \lim_{x\to\frac{1}{\alpha}}\left\{\frac{1-\cos\left(x^2+bx+a\right)}{2(1-\alpha x)^2}\right\}^{\frac{1}{2}}=\lim_{x\to\frac{1}{\alpha}}\left\{\frac{1-\cos\left(a(1-\alpha x)(1-\beta x)\right)}{2\{a(1-\alpha x)(1-\beta x)\}^2}\cdot a^2(1-\beta x)^2\right\}^{\frac{1}{2}}$$

$$=\left[\frac{1}{2}\cdot\frac{1}{2}a^2\left(1-\frac{\beta}{\alpha}\right)^2\right]^{\frac{1}{2}}$$

$$=\frac{1}{2}\frac{1}{\alpha\beta}\left(1-\frac{\beta}{\alpha}\right)=\frac{1}{2}\left(\frac{1}{\alpha\beta}-\frac{1}{\alpha^2}\right)$$

$$=\frac{1}{2\alpha}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)=\frac{1}{k}\left(\frac{1}{\beta}-\frac{1}{\alpha}\right)$$

$\therefore k=2\alpha$

Question ID : 7155054227

16. $25^{190}-19^{190}-8^{190}+2^{190}$ is divisible by :

(1) 14 but not by 34 (2) both 14 and 34 (3) 34 but not by 14 (4) neither 14 nor 34

$25^{190}-19^{190}-8^{190}+2^{190}$

(1) 14 से विभाज्य है परन्तु 34 से नहीं (2) 14 तथा 34 दोनों से विभाज्य है

(3) 34 वे विभाज्य है परन्तु 14 से नहीं (4) न तो 14 से नही 34 से विभाज्य है

Ans. Official Answer NTA (3)

Sol. $(25^{190}-19^{190})-(8^{190}-2^{190})$ is divisible by 6.

$(25^{190}-8^{190})-(19^{190}-2^{190})$ is divisible by 17.

$25^{190}-8^{190}$ is not divisible by 7

but $19^{190}-2^{190}$ is divisible by 7

So, $25^{190}-19^{190}-8^{190}+2^{190}$ is divisible by 34 but not 14

Question ID : 7155054237

17. Let the vectors $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}$, $\vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ and $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ be coplanar. If the vectors $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}$, $\vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ and $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$ are also coplanar, then $6(a+b+c)$ is equal to :

माना सदिश $\vec{u}_1 = \hat{i} + \hat{j} + a\hat{k}$, $\vec{u}_2 = \hat{i} + b\hat{j} + \hat{k}$ तथा $\vec{u}_3 = c\hat{i} + \hat{j} + \hat{k}$ सह–तलीय है। यदि सदिश $\vec{v}_1 = (a+b)\hat{i} + c\hat{j} + c\hat{k}$, $\vec{v}_2 = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ तथा $\vec{v}_3 = b\hat{i} + b\hat{j} + (c+a)\hat{k}$ भी सह–तलीय हैं, तो $6(a + b + c)$ बराबर है :

(1) 6 (2) 4 (3) 12 (4) 0

Ans. Official Answer NTA (3)

Sol. Given : $\overrightarrow{u_1} = \hat{i} + \hat{j} + a\hat{k}, \overrightarrow{u_2} = \hat{i} + b\hat{j} + \hat{k}$ and $\overrightarrow{u_3} = c\hat{i} + \hat{j} + \hat{k}$

and $\overrightarrow{v_1} = (a+b)\hat{i} + c\hat{j} + c\hat{k}$, $\overrightarrow{v_2} = a\hat{i} + (b+c)\hat{j} + a\hat{k}$ and $\overrightarrow{v_3} = b\hat{i} + b\hat{j} + (c+a)\hat{k}$

$$\text{Now} \begin{vmatrix} 1 & 1 & a \\ 1 & b & 1 \\ c & 1 & 1 \end{vmatrix} = 0 \Rightarrow (b-1) - (1-c) + a(1-bc) = 0$$

$a + b + c = 2 + abc$ …(i)

$$\text{and} \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ b & b & c+a \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} a+b & c & c \\ a & b+c & a \\ -2a & -2c & 0 \end{vmatrix} = 0 \quad \left(R_3 \to R_3 - (R_1 + R_2)\right)$$

$\Rightarrow abc = 0$

$\therefore a + b + c = 2$

$\therefore \quad 6(a+b+c) = 12$

Question ID : 7155054225

18. If $A = \begin{bmatrix} 1 & 5 \\ \lambda & 10 \end{bmatrix}$, $A^{-1} = \alpha A + \beta I$ and $\alpha + \beta = -2$, then $4\alpha^2 + \beta^2 + \lambda^2$ is equal to :

यदि $A=\begin{bmatrix}1 & 5\\ \lambda & 10\end{bmatrix}$, $A^{-1}=\alpha A+\beta I$ तथा $\alpha+\beta=-2$ हैं, तो $4\alpha^2+\beta^2+\lambda^2$ बराबर है :

(1) 12 (2) 14 (3) 10 (4) 19

Ans. Official Answer NTA (2)

Sol. $|A-xI|=0\Rightarrow\begin{vmatrix}1-x & 5\\ \lambda & 10-x\end{vmatrix}=0\Rightarrow x^2-11x+10-5\lambda=0$

$\Rightarrow(10-5\lambda)A^{-1}=-A+11I$

$\therefore\alpha=\dfrac{-1}{10-5\lambda}$ and $\beta=\dfrac{+11}{10-5\lambda}$

$\alpha+\beta=-2\Rightarrow\dfrac{10}{10-5\lambda}=-2\Rightarrow10-5\lambda=-5\Rightarrow\lambda=3$

$\therefore\alpha=\dfrac{1}{5}$ and $\beta=\dfrac{-11}{5}$

$\therefore 4a^2+\beta^2+\lambda^2=\dfrac{4}{25}+\dfrac{121}{25}+3^2=14$ Ans.

Question ID : 7155054222

19. Let $A=\{1,2,3,4,5,6,7\}$. Then the relation $R=\{(x,y)\in A\times A : x+y=7\}$ is :

(1) transitive but neither symmetric nor reflexive

(2) symmetric but neither reflexive nor transitive

(3) reflexive but neither symmetric nor transitive

(4) an equivalence relation

माना $A=\{1,2,3,4,5,6,7\}$ है। तो संबंध $R=\{(x,y)\in A\times A : x+y=7\}$:

(1) संक्रामक है परन्तु न तो स्वतुल्य है न ही सममित है

(2) स्वतुल्य है परन्तु न तो सममित है नही संक्रामक है

(3) एक सममित है परन्तु न तो स्वतुल्य है न ही संक्रामक है

(4) एक तुल्यता संबंध है

Ans. Official Answer NTA (2)

Sol. $R=\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}$

Question ID : 7155054235

20. For a, $b \in \mathbb{Z}$ and $|a-b| \leq 10$, let the angle between the plane $P: ax+y-z=b$ and the line $l: x-1=a-y=z+1$ be $\cos^{-1}\left(\frac{1}{3}\right)$. If the distance of the point (6, –6, 4) from the plane P is $3\sqrt{6}$, then a^4+b^2 is equal to :

a, $b \in \mathbb{Z}$ तथा $|a-b| \leq 10$ के लिए, माना समतल $P: ax+y-z=b$ तथा रेखा $l: x-1=a-y=z+1$ के बीच का कोण $\cos^{-1}\left(\frac{1}{3}\right)$ है। यदि बिंदु (6, –6, 4) की समतल P से दूरी $3\sqrt{6}$ है, तो a^4+b^2 बराबर है :

(1) 25 (2) 85 (3) 48 (4) 32

Ans. Official Answer NTA (4)

Sol. $ax+y-z=b$

$\frac{x-1}{1}=\frac{y-a}{-1}=\frac{z+1}{1}$

$\vec{n}=a\hat{i}+\hat{j}-\hat{k}$

$\vec{p}=\hat{i}-\hat{j}+\hat{k}$

$\sin\theta=\left|\frac{\vec{n}\cdot\vec{p}}{|\vec{n}|\vec{p}|}\right|$

$\left|\frac{a-1-1}{\sqrt{a^2+2}\sqrt{3}}\right|=\frac{2\sqrt{2}}{3}$

$3(a-2)^2=8\times3\left(a^2+2\right)$

$5a^2+12a+4=0 \Rightarrow (5a+2)(a+2)=0$

$a=-2 \quad \& \quad a=-2/5$ (reject)

$a=-2, \quad a-b\leq10$

$b\geq-8n$

Now distance of given point from the plane

$-2x+y-z=b$

$\frac{12+6+4+b}{\sqrt{6}}=\pm3\sqrt{6}$

$b+22=\pm18$

$b=-4$ and $b=-40$ (Reject)

$a^4+b^2=16+16=32$

SECTION – B

Question ID : 7155054245

21. Let $0 < z < y < x$ be three real numbers such that $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ are in an arithmetic progression and $x, \sqrt{2}y, z$ are in a geometric progression. If $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$, then $3(x+y+z)^2$ is equal to ________.

माना तीन वास्तविक संख्याएँ $0 < z < y < x$ इस प्रकार हैं कि $\frac{1}{x}, \frac{1}{y}, \frac{1}{z}$ एक समांतर श्रेढ़ी में हैं तथा $x, \sqrt{2}y, z$ एक गुणोत्तर श्रेढ़ी में हैं। यदि $xy + yz + zx = \frac{3}{\sqrt{2}}xyz$ है, तो $3(x+y+z)^2$ बराबर है ________

Ans. Official Answer NTA (150)

Sol. As given, $\frac{2}{y} = \frac{1}{x} + \frac{1}{z}$ ....(i)

$2y^2 = xz$ ....(ii)

and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{3}{\sqrt{2}}$ ....(iii)

From (i) and (iii) $y = \sqrt{2}$ ....(iv)

Now from (ii) $xz = 4$ ....(v)

Now using (ii), (iv) and (v)

$x + z = 4\sqrt{2}$

Required, $3(x+y+z)^2 = 3(\sqrt{2} + 4\sqrt{2})^2$

$= 150$

Question ID : 7155054244

22. Let $R = \{a, b, c, d, e\}$ and $S = \{1, 2, 3, 4\}$. Total number of onto functions $f : R \to S$ such that $f(a) \neq 1$, is equal to ________.

माना $R = \{a, b, c, d, e\}$ तथा $S = \{1, 2, 3, 4\}$ हैं। आच्छादक फलनों $f : R \to S$ जिनके लिए $f(a) \neq 1$ है, की कुल संख्या है :

Ans. Official Answer NTA (384)

Ans. Matrix (180)

Sol. Total onto function

$\frac{\lfloor 5}{\lfloor 3 \lfloor 2} \times \lfloor 4 = 240$

Now when f(a) = 1

$\lfloor 4 + \frac{\lfloor 4}{\lfloor 2 \lfloor 2} \times \lfloor 3 = 24 + 36 = 60$

so required $f^n = 240 - 60 = 180$

Question ID : 7155054249

23. Let the solution curve $x = x(y), 0 < y < \frac{\pi}{2}$, of the differential equation

$(\log_e(\cos y))^2 \cos y\, dx - (1 + 3x \log_e(\cos y)) \sin y dy = 0$ satisfy $x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$. If

$x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$, where m and n are coprime, then mn is equal to ________.

माना अवकल समीकरण $(\log_e(\cos y))^2 \cos y\, dx - (1 + 3x \log_e(\cos y)) \sin y dy = 0$, का हल वक्र $x = x(y), 0 < y < \frac{\pi}{2}$;

$x\left(\frac{\pi}{3}\right) = \frac{1}{2\log_e 2}$ को संतुष्ट करता है। यदि $x\left(\frac{\pi}{6}\right) = \frac{1}{\log_e m - \log_e n}$ है, जहाँ m तथा n असहभाज्य है, तो mn बराबर है

:

Ans. Official Answer NTA (12)

Sol. $\cos y \ln^2 \cos y dx = (1 + 3x \ln \cos y) \sin y dy$

$\frac{dx}{dy} = \tan y \left(\frac{3x}{\ln \cos y} + \frac{1}{\ln^2 \cos y} \right)$

$\frac{dx}{dy} - \left(\frac{3\tan y}{\ln \cos y} \right) x = \frac{\tan y}{\ln^2 \cos y}$

$IF = e^{\int \frac{-3\sin y}{\ln \cos y \cos y} dy}$

$\ln \cos y = t$

$\frac{1}{\cos y}(-\sin y) dy = dt$

$If = e^{\int \frac{3}{t} dt} = e^{3\ln t} = t^3 = \ln^3 \cos y$

solution is $x \cdot \ln^3 \cos y = \frac{\sin y}{\cos y} \cdot \ln \cos y dy + C$

$x \ln^3 \cos y = \frac{-\ln^2 \cos y}{2} + C$

$x\left(\frac{\pi}{3}\right) = \frac{1}{2\ln 2}$ so $\frac{1}{2\ln 2} \times \ln^3\left(\frac{1}{2}\right) = -\frac{\ln^2\left(\frac{1}{2}\right)}{2} + C$

$C = 0$

$y = \frac{\pi}{6} \quad \Rightarrow x \ln^3 \frac{\sqrt{3}}{2} = -\frac{1}{2}\ln^2 \frac{\sqrt{3}}{2} + 0$

$x = -\frac{1}{2\ln\left(\frac{\sqrt{3}}{2}\right)}$

$x = \frac{1}{\ln \frac{4}{3}} = \frac{1}{\ln 4 - \ln 3}$

$mn = 12$

Question ID : 7155054251

24. Let P_1 be the plane $3x - y - 7z = 11$ and P_2 be the plane passing through the points $(2, -1, 0), (2, 0, -1)$, and $(5, 1, 1)$. If the foot of the perpendicular drawn from the point $(7, 4, -1)$ on the line of intersection of the planes P_1 and P_2 is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to ________.

माना P_1 समतल $3x - y - 7z = 11$ है तथा बिंदुओं $(2, -1, 0), (2, 0, -1)$ तथा $(5, 1, 1)$ से होकर जाने वाला समतल P_2 है। यदि बिंदु $(7, 4, -1)$ से समतलों P_1 तथा P_2 की प्रतिच्छेदन रेखा पर डाले गए लंब का पाद (α, β, γ) है, तो $\alpha + \beta + \gamma$ बराबर है :

Ans. Official Answer NTA (11)

Sol. $P_1 : 3x - y - 7z = 11$

$$P_2 : \begin{vmatrix} x-2 & y+1 & z \\ 0 & 1 & -1 \\ 3 & +2 & 1 \end{vmatrix} = 0$$

$3(x-2) - (y+1)(+3) - z(+3) = 0$

$P_2 : x - y - z = 3$

$\vec{P} = \overrightarrow{n_1} \times \overrightarrow{n_2}$

$\overrightarrow{n_1} = 3\hat{i} - \hat{j} - 7\hat{k}$

$\overrightarrow{n_2} = \hat{i} - \hat{j} - \hat{k}$

$\overrightarrow{n_1} \times \overrightarrow{n_2} = -6\hat{i} - 4\hat{j} - 2\hat{k}$

$\vec{P} = 3\hat{i} + 2\hat{j} + \hat{k}$ common point on the planes

Put z = 0

3x – y = 11

x – y = 3

x = 4, y = 1, z = 0

Line is $\frac{x-4}{3} = \frac{y-1}{2} = \frac{z-0}{1}$

Let point on this line $\equiv (3\lambda + 4, 2\lambda + 1, \lambda) \equiv N$

given point is $A \equiv (7, 4, -1)$

$\overrightarrow{AN} = (3\lambda - 3)\hat{i} + (2\lambda - 3)\hat{j} + (\lambda + 1) = 0$

$\overrightarrow{AN} \cdot \vec{P} = 0$

$3(3\lambda - 3) + 2(2\lambda - 3) + 1(\lambda + 1) = 0$

$14\lambda - 14 = 0$ $\quad N \equiv (7, 3, 1)$

$\lambda = 1$ $\quad (\alpha, \beta, \gamma)$

$\alpha + \beta + \gamma = 11$

Question ID : 7155054243

25. Let m and n be the numbers of real roots of the quadratic equations $x^2 - 12x + [x] + 31 = 0$ and $x^2 - 5|x + 2| - 4 = 0$ respectively, where [x] denotes the greatest integer ≤ x. Then $m^2 + mn + n^2$ is equal to ________.

माना समीकरणों $x^2 - 12x + [x] + 31 = 0$ तथा $x^2 - 5|x + 2| - 4 = 0$ के वास्तविक मूलों की संख्या क्रमश: m तथा n है, जहाँ [x] महत्तम पूर्णांक ≤ x है। तो $m^2 + mn + n^2$ बराबर है :

Ans. Official Answer NTA (9.00)

Sol. $x^2 - 12x + [x] + 31 = 0$

$\underbrace{x^2 - 12x + 31}_{\geq -5} + [x] = 0$

It could have its solution in [5, 6] but it does not exist as at x = 5 and 6

LHS = 1

m = 0

$x^2 - 5|x + 2| - 4 = 0$

Case 1

$x \geq -2$

$x^2 - 5(x+2) - 4 = 0$

$x^2 - 5x - 14 = 0 \Rightarrow x = 7, -2$

Case 2

$x < -2$

$x^2 + 5x + 10 - 4 = 0$

$x = -2, -3$

3 solution i.e., $x = -3, -2, 7$

$n = 3$

Question ID : 7155054246

26. Let k and m be positive real numbers such that the function $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2, & x \geq 1 \end{cases}$ is differentiable for all $x > 0$. Then $\dfrac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ is equal to ________.

माना धनात्मक वास्तविक संख्याएँ k तथा m इस प्रकार हैं कि फलन $f(x) = \begin{cases} 3x^2 + k\sqrt{x+1}, & 0 < x < 1 \\ mx^2 + k^2, & x \geq 1 \end{cases}$ सभी $x > 0$ के लिए अवकलनीय है। तो $\dfrac{8f'(8)}{f'\left(\frac{1}{8}\right)}$ बराबर है :

Ans. Official Answer NTA (309)

Sol. $f\left(1^-\right) = f(1) = f\left(1^+\right)$

$3 + k\sqrt{2} = m + k^2$ __________(1)

$f'\left(1^-\right) = f^1\left(1^+\right)$

$6 + \dfrac{k}{2\sqrt{2}} = 2m$ __________(2)

from (1) and (2)

$3 + k\sqrt{2} = 3 + \dfrac{k}{4\sqrt{2}} + k^2 \quad \Rightarrow \quad k^2 + k\left(\dfrac{1}{4\sqrt{2}} - \sqrt{2}\right) = 0$

$k = 0, k = \dfrac{7}{4\sqrt{2}} \Rightarrow \quad \text{If } k = \dfrac{7}{4\sqrt{2}}, m = 3 + \dfrac{7}{32} = \dfrac{103}{32}$

$$f'(x)=\begin{cases}6x+\dfrac{k}{2\sqrt{x+1}}\\ 2mx\end{cases}$$

$$f'(8)=\frac{103}{2}$$

$$f'\left(\frac{1}{8}\right)=\frac{6}{8}+\frac{k\cdot 2\sqrt{2}}{2\cdot 3}=\frac{3}{4}+\frac{\sqrt{2}\cdot k}{3}=\frac{4}{3}\qquad \text{now}\quad \frac{8\cdot f'(8)}{f'\left(\frac{1}{8}\right)}=309$$

Question ID : 7155054242

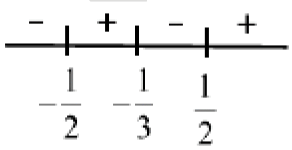
27. If domain of the function $\log_e\left(\frac{6x^2+5x+1}{2x-1}\right)+\cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ is $(\alpha,\ \beta)\cup(\gamma,\ \delta]$, then $18(\alpha^2+\beta^2+\gamma^2+\delta^2)$ is equal to ________.

यदि फलन $\log_e\left(\frac{6x^2+5x+1}{2x-1}\right)+\cos^{-1}\left(\frac{2x^2-3x+4}{3x-5}\right)$ का प्राँत $(\alpha,\ \beta)\cup(\gamma,\ \delta]$ है, तो $18(\alpha^2+\beta^2+\gamma^2+\delta^2)$ बराबर है :

Ans. Official Answer NTA (20)

Sol. $\frac{6x^2+5x+1}{2x-1}>0$

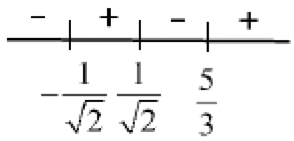
$\frac{(3x+1)(2x+1)}{2x-1}>0$



$x\in\left(\frac{-1}{2},\frac{-1}{3}\right)\cup\left(\frac{1}{2},\infty\right)$ ....(A)

and $-1\le\frac{2x^2-3x+4}{3x-5}\le 1$

$\frac{2x^2-1}{3x-5}\ge 0$ and $\frac{2x^2-6x+9}{3x-5}\le 0$



and $3x - 5 < 0$

$$x \in \left[\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right] \cup \left(\frac{5}{3}, \infty\right) \quad ....(B)$$

$$x < \frac{5}{3} \quad ....(C)$$

$$A \cap B \cap C \equiv \left(\frac{-1}{2}, \frac{-1}{3}\right) \cup \left(\frac{1}{2}, \frac{1}{\sqrt{2}}\right]$$

So $18\left(\alpha^2 + \beta^2 + \gamma^2 + \delta^2\right) = 18\left(\frac{1}{4} + \frac{1}{9} + \frac{1}{4} + \frac{1}{2}\right)$

$= 18 + 2 = 20$

Question ID : 7155054250

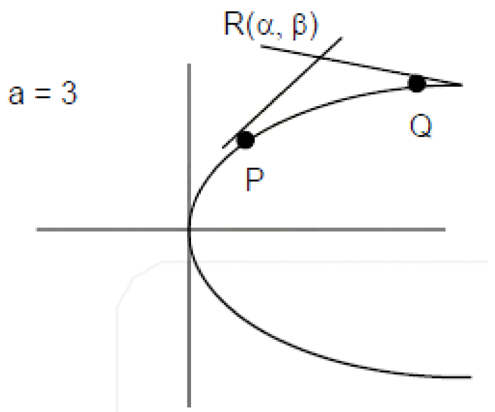
28. The ordinates of the points P and Q on the parabola with focus (3, 0) and directrix $x = -3$ are in the ratio 3 : 1. If $R(\alpha, \beta)$ is the point of intersection of the tangents to the parabola at P and Q, then $\frac{\beta^2}{\alpha}$ is equal to ________.

परवलय जिसकी नाभि (3, 0) तथा नियता $x = -3$ हैं, कि बिंदुओं P तथा Q की कोटियाँ 3 : 1 के अनुपात में हैं। यदि P तथा Q पर परवलय की स्पर्श रेखाओं का प्रतिच्छेदन बिंदु $R(\alpha, \beta)$ है, तो $\frac{\beta^2}{\alpha}$ बराबर है :

Ans. Official Answer NTA (16)

Sol. Parabola is

$y^2 = 12x$



$P\left(at_1^2 \cdot 2at_1\right)$, $Q\left(at_2^2 \cdot 2at_2\right)$ $\alpha = at_1t_2$ and $\beta = a(t_1 + t_2)$

$2at_2 = 3.2at_1$

$t_2 = 3t_1$

$$\frac{\beta^2}{\alpha} = \frac{a^2(t_1+t_2)}{att_1t_2} = \frac{a(16t_1^2)}{3t_1^2} = 16$$

Question ID : 7155054247

29. Let [t] denote the greatest integer function. If $\int_0^{2.4}[x^2]dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$, then $\alpha + \beta + \gamma + \delta$ is equal to ________.

माना [t] महत्तम पूर्णांक फलन है। यदि $\int_0^{2.4}[x^2]dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3} + \delta\sqrt{5}$ है, तो $\alpha + \beta + \gamma + \delta$ बराबर है :

Ans. Official Answer NTA (6.00)

Sol. $\int_0^{2.4}[x^2]dx = \int_0^1 0dx + \int_1^{\sqrt{2}} 1dx + \int_{\sqrt{2}}^{\sqrt{3}} 2dx$

$+ \int_{\sqrt{3}}^{2} 3dx + \int_2^{\sqrt{5}} 4dx + \int_{\sqrt{5}}^{2.4} 5dx$

$= 0 + (\sqrt{2}-1) + 2(\sqrt{3}-\sqrt{2}) + 3(2-\sqrt{3}) + 4(\sqrt{5}-2) + 5(2.4-\sqrt{5})$

$= 9 - \sqrt{2} - \sqrt{3} - \sqrt{5}$

$\alpha = 9, \beta = -1, \gamma = -1, \delta = -1$

Question ID : 7155054248

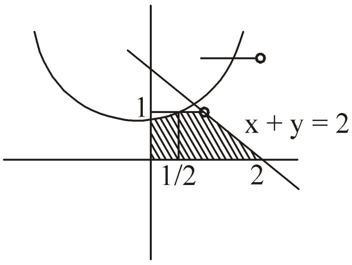
30. Let the area enclosed by the lines $x + y = 2$, $y = 0$, $x = 0$ and the curve $f(x) = \min\left\{x^2 + \frac{3}{4}, 1 + [x]\right\}$ where [x] denotes the greatest integer $\le x$, be A. Then the value of 12A is ________.

माना [x] महत्तम पूर्णांक $\le x$ है। माना रेखाओं $x + y = 2$, $y = 0$, $x = 0$ तथा वक्र $f(x) = \min\left\{x^2 + \frac{3}{4}, 1 + [x]\right\}$ से घिरे क्षेत्र का क्षेत्रफल A है। तो 12 A का मान है :

Ans. Official Answer NTA (17)

Sol. $A = \int_0^{\frac{1}{2}}\left(x^2 + \frac{3}{4}\right)dx + \frac{1}{2}\left(\frac{3}{2} + \frac{1}{2}\right) \times 2$

$\frac{1}{3}.\frac{1}{8} + \frac{3}{4}.\frac{1}{2} + 1$



$$12A = \frac{1}{2} + \frac{9}{2} + 12 = 17$$