JEE Main April 2023

Question Paper With Text Solution

06 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911

Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN APRIL 2023 | 06TH APRIL SHIFT-1

SECTION – A

Question ID : 7155053701

1. Statement $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ is logically equivalent to :

कथन $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$ के तर्क संगत तुल्य कथन है :

(1) $(P \vee R) \Rightarrow Q$ (2) $(P \wedge R) \Rightarrow Q$ (3) $(P \Rightarrow R) \vee (Q \Rightarrow R)$ (4) $(P \Rightarrow R) \wedge (Q \Rightarrow R)$

Ans. Official Answer NTA (1)

Sol. $(P \Rightarrow Q) \wedge (R \Rightarrow Q)$

We know that $P \Rightarrow Q \equiv \sim P \vee Q$

$= (\sim P \vee Q) \wedge (\sim R \vee Q)$

$= \sim (P \vee R) \vee Q$

$= (P \vee R) \Rightarrow Q$

Question ID : 7155053687

2. If the ratio of the fifth term from the beginning to the fifth term from the end in the expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ is $\sqrt{6}:1$, then the third term from the beginning is :

यदि $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ के विस्तार में आरंभ से पाँचवें पद का अंत से पाँचवे पद से अनुपात $\sqrt{6}:1$ है, तब आरंभ से तीसरा पद है :

(1) $30\sqrt{3}$ (2) $60\sqrt{3}$ (3) $60\sqrt{2}$ (4) $30\sqrt{2}$

Ans. Official Answer NTA (2)

Sol.
$$\frac{{}^nC_4 2^{\frac{n-4}{4}}\cdot\left(3^{\frac{-1}{4}}\right)^4}{{}^nC_4 3^{-\left(\frac{n-4}{4}\right)}\cdot\left(2^{\frac{1}{4}}\right)^4}=\frac{\sqrt{6}}{1}$$

$\Rightarrow n=10$

So $T_3={}^{10}C_2 2^{\frac{1}{4}\cdot 8}\cdot 3^{-\frac{1}{4}\cdot 2}=\frac{45.4}{\sqrt{3}}=60\sqrt{3}$

Question ID : 7155053698

3. The mean and variance of a set of 15 numbers are 12 and 14 respectively. The mean and variance of another set of 15 numbers are 14 and σ^2 respectively. If the variance of all the 30 numbers in the two sets is 13 , then σ^2 is equal to :

15 संख्याओं के माध्य व प्रसरण क्रमशः 12 व 14 हैं। 15 अन्य और संख्याओं के माध्य व प्रसरण क्रमशः 14 व σ^2 है। यदि सभी 30 संख्याओं का प्रसरण 13 है, तो σ^2 बराबर है :

(1) 9 (2) 12 (3) 10 (4) 11

Ans. Official Answer NTA (3)

Sol. Combine var. $= \frac{n_1\sigma^2 + n_2\sigma^2}{n_1 + n_2} + \frac{n_1 n_2 (m_1 - m_2)^2}{(n_1 + n_2)^2}$

$$13 = \frac{15\cdot 14 + 15\cdot\sigma^2}{30} + \frac{15\cdot 15(12-14)^2}{30\times 30}$$

$$13 = \frac{14+\sigma^2}{2} + \frac{4}{4}$$

$$\sigma^2 = 10$$

Question ID : 7155053696

4. Let the position vectors of the points A, B, C and D be $5\hat{i}+5\hat{j}+2\lambda\hat{k}, \hat{i}+2\hat{j}+3\hat{k}, -2\hat{i}+\lambda\hat{j}+4\hat{k}$ and $-\hat{i}+5\hat{j}+6\hat{k}$. Let the set $S = \{\lambda \in \mathbb{R}$: the points A, B, C and D are coplanar$\}$. Then $\sum_{\lambda \in S}(\lambda+2)^2$ is equal to :

माना बिंदुओं A, B, C व D के स्थिति सदिश $5\hat{i}+5\hat{j}+2\lambda\hat{k}, \hat{i}+2\hat{j}+3\hat{k}, -2\hat{i}+\lambda\hat{j}+4\hat{k}$ व $-\hat{i}+5\hat{j}+6\hat{k}$ है। माना समुच्चय $S = \{\lambda \in \mathbb{R}$: बिंदु A, B, C व D सहतलीय हैं$\}$ है, तब $\sum_{\lambda \in S}(\lambda+2)^2$ बराबर है :

(1) 25 (2) 41 (3) $\frac{37}{2}$ (4) 13

Ans. Official Answer NTA (2)

Sol. $\vec{a} = 5\hat{i}+5\hat{j}+2\lambda\hat{k}$

$\hat{b} = \hat{i}+2\hat{j}+3\hat{k}$

$\vec{c} = -2\hat{i}+\lambda\hat{j}+4\hat{k}$

$\hat{d} = -\hat{i}+5\hat{j}+6\hat{k}$

$\overrightarrow{BA}\cdot(\overrightarrow{BC}\times\overrightarrow{BD})=0$

$$\begin{vmatrix} 4 & 3 & 2\lambda-3 \\ -3 & \lambda-2 & 1 \\ -2 & 3 & 3 \end{vmatrix}=0\Rightarrow\lambda=2,3$$

$\Sigma(\lambda+2)^2=16+25=41$

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5. Let $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}, \vec{b}=\hat{i}-2\hat{j}-2\hat{k}$ and $\vec{c}=-\hat{i}+4\hat{j}+3\hat{k}$. If $\vec{d}$ is a vector perpendicular to both $\vec{b}$ and $\vec{c}$, and $\vec{a}\cdot\vec{d}=18$, then $|\vec{a}\times\vec{d}|^2$ is equal to :

माना $\vec{a}=2\hat{i}+3\hat{j}+4\hat{k}, \vec{b}=\hat{i}-2\hat{j}-2\hat{k}$ व $\vec{c}=-\hat{i}+4\hat{j}+3\hat{k}$ है। यदि एक सदिश $\vec{d}$ सदिशों $\vec{b}$ व $\vec{c}$ दोनों के लम्बवत् है और $\vec{a}\cdot\vec{d}=18$ है, तब $|\vec{a}\times\vec{d}|^2$ का मान है :

(1) 640 (2) 760 (3) 720 (4) 680

Ans. Official Answer NTA (3)

Sol. $\vec{b}\times\vec{c}=2\hat{i}-\hat{j}+2\hat{k}$

$\therefore \quad \vec{d}=\lambda(2\hat{i}-\hat{j}+2\hat{k})$

$\vec{a}\cdot\vec{d}=18$

$\Rightarrow\lambda=2$

$\therefore \quad |\vec{a}\times\vec{d}|^2=|\vec{a}|^2|\vec{d}|^2-(\vec{a}\cdot\vec{d})^2$

$=720$

Question ID : 7155053688

6. The sum of the first 20 terms of the series 5 + 11 + 19 + 29 + 41 +_______.

श्रेणी 5 + 11 + 19 + 29 + 41 + ... के प्रथम 20 पदों का योग है :

(1) 3520 (2) 3420 (3) 3250 (4) 3450

Ans. Official Answer NTA (1)

Sol. $S_{20}=5+11+19+29+\ldots\ldots$

Let $T_r=ar^2+br+c$

$T_1 = a + b + c = 5$

$T_2 = 4a + 2b + c = 11$

$T_3 = 9a + 3b + c = 19$

$a = 1, b = 3, c = 1$

Hence $S_{20} = \sum_{r=1}^{20} r^2 + 3\sum_{r=1}^{20} r + \sum_{r=1}^{20} 1 = 3520$

Question ID : 7155053691

7. Let $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$. Then $18\int_1^2 f(x)dx$ is equal to :

माना $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3, x > 0$ है। तब $18\int_1^2 f(x)dx$ का मान है :

(1) $5\log_e 2 - 3$ (2) $5\log_e 2 + 3$ (3) $10\log_e 2 - 6$ (4) $10\log_e 2 + 6$

Ans. Official Answer NTA (3)

Sol. $5f(x) + 4f\left(\frac{1}{x}\right) = \frac{1}{x} + 3$

$x \to \frac{1}{x}$

$5f\left(\frac{1}{x}\right) + 4f(x) = x + 3$

$(1) \times 5 - (2) \times 4$

$\Rightarrow f(x) = \frac{5}{9x} - \frac{4}{9}x + \frac{1}{3}$

$\Rightarrow 18\int_1^2 f(x)dx = 18\left(\frac{5}{9}\ln 2 - \frac{4}{9} \times \frac{3}{2} + \frac{1}{3}\right)$

$= 10\ln 2 - 6$

Question ID : 7155053693

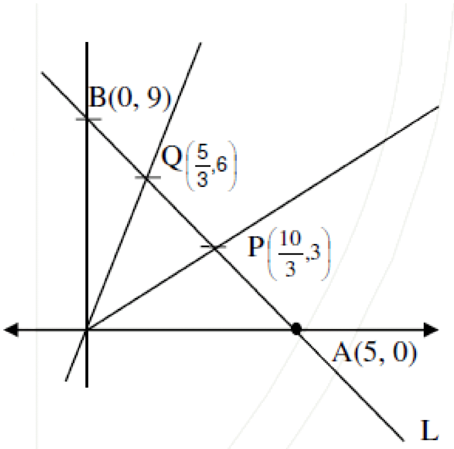
8. The straight lines l_1 and l_2 pass through the origin and trisect the line segment of the line $L : 9x + 5y = 45$ between the axes. If m_1 and m_2 are the slopes of the lines l_1 and l_2 then the point of intersection of the line $y = (m_1 + m_2)x$ with L lies on :

सरल रेखाएँ l_1 व l_2 मूल बिंदु से होकर जाती है और रेखा L : 9x + 5 y = 45 के अक्षों के बीच रेखाखंड को तीन बराबर भागों में बाँटती है। यदि रेखाओं l_1 क l_2 की प्रवणताएँ m_1 व m_2 हैं, तब रेखाओं $y = (m_1 + m_2)x$ और L का प्रतिच्छेदन बिंदु किस रेखा पर है :

(1) 6x + y = 10 (2) y – 2x = 5 (3) y – x = 5 (4) 6x – y = 15

Ans. Official Answer NTA (3)

Sol.



$$m_1 = \frac{3}{(10/3)} = \frac{9}{10}$$

$$m_2 = \frac{6}{(5/3)} = \frac{18}{5}$$

$$m_1 + m_2 = \frac{9+36}{10} = \frac{45}{10} = \frac{9}{2}$$

So, $y = (m_1 + m_2)x$ is $y = \frac{9}{2}x$

Now solving $y = \frac{9}{2}x$ and $9x + 5y = 45$

$$\Rightarrow 2y + 5y = 45 \Rightarrow y = \frac{45}{7}, x = \frac{10}{7}$$

$$y - x = \frac{35}{7} = 5$$

Question ID : 7155053686

9. If ${}^{2n}C_3 : {}^{n}C_3 = 10 : 1$, then the ratio $(n^2 + 3n) : (n^2 - 3n + 4)$ is :

यदि $^{2n}C_3 : {}^nC_3 = 10:1$ है, तब अनुपात $\left(n^2+3n\right):\left(n^2-3n+4\right)$ है :

(1) 35:16 (2) 27:11 (3) 2:1 (4) 65:37

Ans. Official Answer NTA (3)

Sol. $^{2n}C_3 : {}^nC_3 = 10:1$

$$\frac{(2n)!}{3!(2n-3)!}\times\frac{3!(n-3)!}{n!}=10$$

$\Rightarrow 4(2n-1)=10n-20$

$\Rightarrow n=8$

$$\text{Now } \frac{\left(n^2+3n\right)}{\left(n^2-3n+4\right)}=\frac{64+24}{64-24+4}=\frac{88}{44}=2$$

Question ID : 7155053682

10. Let $A=\{x\in\mathbb{R}:[x+3]+[x+4]\le 3\}$, $B=\left\{x\in\mathbb{R}:3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3}<3^{-3x}\right\}$, where [t] denotes greatest integer function. Then :

माना $A=\{x\in\mathbb{R}:[x+3]+[x+4]\le 3\}$, $B=\left\{x\in\mathbb{R}:3^x\left(\sum_{r=1}^{\infty}\frac{3}{10^r}\right)^{x-3}<3^{-3x}\right\}$ है, जहाँ [t] महत्तम पूर्णांक फलन है।

तब :

(1) $A=B$ (2) $B\subset C, A\neq B$ (3) $A\cap B=\phi$ (4) $A\subset B,\quad A\neq B$

Ans. Official Answer NTA (1)

Sol. $[x]+3+[x]+4\le 3$

$2[x]\le -4$

$[x]\le -2\Rightarrow x\in(-\infty,-1).$ __________A

$$3^x\left(\frac{3\cdot\frac{1}{10}}{1-\frac{1}{10}}\right)^{x-3}<3^{-3x}$$

$27<3^{-3x}$

$-3x>+3$

$x < -1$ ____________B
$A = B$

Question ID : 7155053685

11. If the system of equations
$x + y + az = b$
$2x + 5y + 2z = 6$
$x + 2y + 3z = 3$
has infinitely many solutions, then $2a + 3b$ is equal to :

यदि समीकरण निकाय
$x + y + az = b$
$2x + 5y + 2z = 6$
$x + 2y + 3z = 3$
के अनंत हल है। तब $2a + 3b$ बराबर है।

(1) 20 (2) 25 (3) 28 (4) 23

Ans. Official Answer NTA (4)

Sol. $x + y + az = b$

$2x + 5y + 2z = 6$

$x + 2y + 3z = 3$

For ∞ solution

$\Delta = 0, \Delta_x = 0, \Delta_y = 0, \Delta_z = 0$

$$\Delta = \begin{vmatrix} 1 & 1 & a \\ 2 & 5 & 2 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 11 - 4 - a = 0 \Rightarrow a = 7$$

$$\Delta_z = \begin{vmatrix} 1 & 1 & b \\ 2 & 5 & 6 \\ 1 & 2 & 3 \end{vmatrix} = 0 \Rightarrow 3 - 0 - b = 0 \Rightarrow b = 3$$

Hence $2a + 3b = 23$

Question ID : 7155053700

12. From the top A of a vertical wall AB of height 30m, the angles of depression of the top P and bottom Q of a vertical tower PQ are 15° and 60° respectively, B and Q are on the same horizontal level. If C is a point on AB

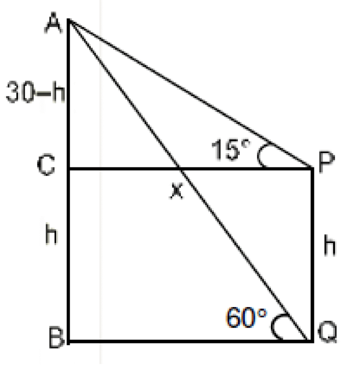
such that CB = PQ, then the area (in m^2) of the quadrilateral BCPQ is equal to :

30 मीटर ऊँची ऊर्ध्वाधर दीवार AB के शिखर A से एक ऊर्ध्वाधर टॉवर PQ के शिखर P व आधार Q के अवनमन कोण क्रमश: 15° व 60° है। B व Q एक ही क्षैतिज तल पर है। यदि AB पर एक बिंदु C इस प्रकार है कि CB = PQ है, तब चतुर्भुज BCPQ का क्षेत्रफल (वर्ग मीटर में) बराबर है :

(1) $300(\sqrt{3}-1)$ (2) $300(\sqrt{3}+1)$ (3) $600(\sqrt{3}-1)$ (4) $200(3-\sqrt{3})$

Ans. Official Answer NTA (3)

Sol.



$$\tan 15^\circ = \frac{30-h}{x}$$

$$\tan 60^\circ = \frac{30}{x} \Rightarrow x = \frac{30}{\sqrt{3}}$$

$$\frac{2-\sqrt{3}}{\sqrt{3}} = \frac{30-h}{30} \Rightarrow 30\left(\frac{2-\sqrt{3}}{\sqrt{3}}\right) = 30-h$$

$$h = 30 - 30\times\frac{2}{\sqrt{3}} + 30 = 60 - \frac{60}{\sqrt{3}}$$

$$\text{Area} = \frac{30(60)(\sqrt{3}-1)}{3} = 600(\sqrt{3}-1)$$

Question ID : 7155053684

13. Let $A = [a_{ij}]_{2\times2}$, where $a_{ij} \neq 0$ for all i, j and $A^2 = I$. Let a be the sum of all diagonal elements of A and b = |A|. Then $3a^2 + 4b^2$ is equal to :

माना $A = [a_{ij}]_{2\times2}$, जहाँ सभी i, j के लिए $a_{ij} \neq 0$ एवं $A^2 = I$ है। माना A के विकर्ण के सभी अवयवों का योग a है और b = |A| है। तब $3a^2 + 4b^2$ बराबर है :

(1) 7 (2) 3 (3) 14 (4) 4

Ans. Official Answer NTA (4)

Sol. Let $A = \begin{bmatrix} p & q \\ r & s \end{bmatrix}$

$A^2 = \begin{bmatrix} p & q \\ r & s \end{bmatrix}\begin{bmatrix} p & q \\ r & s \end{bmatrix} = \begin{bmatrix} p^2+qr & pq+qs \\ rp+rs & qr+s^2 \end{bmatrix}$

$A^2 = I$

$\Rightarrow p^2 + qr = 1 \quad q(p+s) = 0$

$r(p+s) = 0 \qquad qr + s^2 = 1$

$q \neq 0 \Rightarrow p + s = 0 \Rightarrow a = 0$

$b = |A| = ps - qr = -p^2 - qr = -1 (\because s = -p)$

$\therefore 3a^2 + 4b^2 = 4$

Question ID : 7155053694

14. If the equation of the plane passing through the line of intersection of the planes $2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ and parallel to the line $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ is $ax + by + cz + 6 = 0$, then $a + b + c$ is equal to :

यदि समतलों $2x - y + z = 3$, $4x - 3y + 5z + 9 = 0$ की प्रतिच्छेदन रेखा से होकर जाने वाले तथा रेखा $\frac{x+1}{-2} = \frac{y+3}{4} = \frac{z-2}{5}$ के समांतर समतल का समीकरण $ax + by + cz + 6 = 0$ है, तब $a + b + c$ बराबर है :

(1) 13 (2) 12 (3) 15 (4) 14

Ans. Official Answer NTA (4)

Sol. Equation of family of plane

$(2x - y + z - 3) + \lambda(4x - 3y + 5z + 9) = 0$

$x(2 + 4\lambda) - y(1 + 3\lambda) + z(1 + 5\lambda) - 3 + 9\lambda = 0$

Parallel to the line

$-2(2 + 4\lambda) - (1 + 3\lambda)4 + (1 + 5\lambda)5 = 0$

$5\lambda = 3$

$\lambda = \frac{3}{5}$

equation of plane

$11x - 7y + 10z + 6 = 0$

$a+b+c=14$

Question ID : 7155053695

15. One vertex of a rectangular parallelopiped is at the origin O and the lengths of its edges along x, y and z axes are 3, 4 and 5 units respectively. Let P be the vertex (3,4,5). Then the shortest distance between the diagonal OP and an edge parallel to z axis, not passing through O or P is:

एक आयताकार समांतर षट्फलक का एक शीर्ष मूल बिंदु O पर है और x, y तथा z अक्षों के अनुदिश इसके किनारों (edges) की लम्बाईयाँ क्रमशः 3, 4 तथा 5 इकाई है। माना इसका शीर्ष P बिंदु (3,4,5) पर है। तब विकर्ण OP तथा z अक्ष के समांतर इसके एक किनारे, जो O या P से होकर नहीं जाता है, के बीच न्यूनतम दूरी है :

(1) $\frac{12}{5\sqrt{5}}$ (s2) $\frac{12}{\sqrt{5}}$ (3) $12\sqrt{5}$ (4) $\frac{12}{5}$

Ans. Official Answer NTA (4)

Sol. Equation of OP is $\frac{x}{3}=\frac{y}{4}=\frac{z}{5}$

$a_1=(0,0,0)$ $a_2=(3,0,5)$

$b_1=(3,4,5)$ $b_2=(0,0,1)$

Equation of edge parallel to z axis

$$\frac{x-3}{0}=\frac{y-0}{0}=\frac{z-5}{1}$$

$$S.D=\frac{\left(\vec{a}_2.\vec{a}_1\right).\left(\vec{b}_1\times\vec{b}_2\right)}{\left|\vec{b}_1\times\vec{b}_2\right|}$$

$$\frac{\begin{vmatrix}3&0&5\\3&4&5\\0&0&1\end{vmatrix}}{\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&4&5\\0&0&1\end{vmatrix}}=\frac{3(4)}{\left|4\hat{i}-3\hat{j}\right|}=\frac{12}{5}$$

Question ID : 7155053683

16. The sum of all the roots of the equation $\left|x^2-8x+15\right|-2x+7=0$ is :

समीकरण $\left|x^2-8x+15\right|-2x+7=0$ के सभी मूलों का योग है :

(1) $11+\sqrt{3}$ (2) $11-\sqrt{3}$ (3) $9+\sqrt{3}$ (4) $9-\sqrt{3}$

Ans. Official Answer NTA (3)

Sol. $\left|x^2-8x+15\right|-2x+7=0$

$|(x-3)(x-5)|-2x+7=0$

$\text{Case}-\text{I}: \text{If } x\in(-\infty,3]\cup[5,\infty)$

$x^2-8x+15-2x+7=0$

$x^2-10x+22=0$

$x=\dfrac{10\pm\sqrt{12}}{2}=5\pm\sqrt{3}$

$\therefore x=5+\sqrt{3}$ is accepted

$\text{Case}-\text{II}: \text{If } x\in(3,5)$

$-x^2+8x-15-2x+7=0$

$x^2-6x+8=0$

$x=2,4\Rightarrow x=4$ is only accepted

$\therefore$ Sum of roots $5+\sqrt{3}+4=9+\sqrt{3}$

Question ID : 7155053692

17. Let $I(x)=\int\dfrac{x^2\left(x\sec^2x+\tan x\right)}{(x\tan x+1)^2}dx$. If $I(0)=0$, then $I\left(\dfrac{\pi}{4}\right)$ is equal to :

माना $I(x)=\int\dfrac{x^2\left(x\sec^2x+\tan x\right)}{(x\tan x+1)^2}dx$ है। यदि $I(0)=0$ है, तब $I\left(\dfrac{\pi}{4}\right)$ का मान है :

(1) $\log_e\dfrac{(\pi+4)^2}{16}+\dfrac{\pi^2}{4(\pi+4)}$ (2) $\log_e\dfrac{(\pi+4)^2}{32}-\dfrac{\pi^2}{4(\pi+4)}$

(3) $\log_e\dfrac{(\pi+4)^2}{32}+\dfrac{\pi^2}{4(\pi+4)}$ (4) $\log_e\dfrac{(\pi+4)^2}{16}-\dfrac{\pi^2}{4(\pi+4)}$

Ans. Official Answer NTA (2)

Sol. $\int x^2\left(\dfrac{x\sec^2x+\tan x}{(x\tan x+1)^2}\right)dx$

$$=\frac{x^2}{(x\tan x+1)}+\int\frac{2x}{x\tan x+1}dx$$

$$I=2\int\frac{x}{x\tan x+1}dx$$

$$=2\int\frac{x\cos x}{x\sin x+\cos x}dx$$

Let $x\sin x+\cos x=t$

$(x\cos x+\sin x-\sin x)dx=dt$

$$=2\int\frac{dt}{t}=2\log t+c$$

$=2\log|x\sin x+\cos x|+c$

$$\therefore\int\frac{x^2\left(x\sec^2x+\tan x\right)}{(x\tan+1)^2}dx\quad=\frac{-x^2}{x\tan x+1}+2\log\left|x\sin x+\cos x\right|+c$$

$I(0)=0$ $\qquad\Rightarrow c=0$

$$I\left(\frac{\pi}{4}\right)=\frac{-\pi}{\frac{\pi}{4}\times1+1}+2\log\left|\frac{1}{\sqrt{2}}\left(\frac{\pi}{4}\right)+1\right|$$

$$=\log_e\frac{(\pi+4)^2}{32}-\frac{\pi^2}{4(\pi+4)}$$

Question ID : 7155053690

18. If $2x^y+3y^x=20$, then $\frac{dy}{dx}$ at (2, 2) is equal to :

यदि $2x^y+3y^x=20$ है, तब (2, 2) पर $\frac{dy}{dx}$ का मान है :

(1) $-\left(\frac{3+\log_e16}{4+\log_e8}\right)$ (2) $-\left(\frac{3+\log_e4}{2+\log_e8}\right)$ (3) $-\left(\frac{3+\log_e8}{2+\log_e4}\right)$ (4) $-\left(\frac{2+\log_e8}{3+\log_e4}\right)$

Ans. Official Answer NTA (4)

Sol. $2x^y+3y^x=20$

$$2x^y\left[\frac{y}{x}+(\ln x)y'\right]+3y^x\left[\frac{xy'}{y}+\ln y\right]=0$$

$$y' = \frac{-(12\ln 2+8)}{12+8\ln 2} = -\left(\frac{2+\log_e 8}{3+\log_e 4}\right)$$

Question ID : 7155053699

19. A pair of dice is thrown 5 times. For each throw, a total of 5 is considered a success. If the probability of at least 4 successes is $\frac{k}{3^{11}}$, then k is equal to :

दो पासों को 5 बार फेंका जाता है तथा हर बार प्राप्त संख्याओं का योग 5 होना एक सफलता मानी जाती है। यदि कम से कम 4 सफलताओं की प्रायिकता $\frac{k}{3^{11}}$ है, तब k बराबर है :

(1) 164 (2) 75 (3) 82 (4) 123

Ans. Official Answer NTA (4)

Sol. $n(\text{total } 5) = \{(1,4),(2,3),(3,2),(4,1)\} = 4$

$$P(\text{success}) = \frac{4}{36} = \frac{1}{9}$$

$$P(\text{atleast } 4 \text{ success}) = P(4 \text{ success}) + P(5 \text{ success})$$

$$= {}^5C_4 \cdot \left(\frac{1}{9}\right)^4 \cdot \frac{8}{9} + {}^5C_5\left(\frac{1}{9}\right)^5 = \frac{41}{9^5} = \frac{41}{3^{10}} = \frac{123}{3^{11}} = \frac{k}{3^{11}}$$

K = 123

Question ID : 7155053689

20. Let $a_1, a_2, a_3, \ldots, a_n$ be n positive consecutive terms of an arithmetic progression. if $d > 0$ is its common difference, then $\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots.+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ is :

माना $a_1, a_2, a_3, \ldots, a_n$ समांतर श्रेणी के धनात्मक क्रमागत n पद हैं। यदि सार्वन्तर $d > 0$ है, तब

$\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+\ldots\ldots.+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$ का मान है :

(1) 1 (2) 0 (3) $\sqrt{d}$ s (4) $\frac{1}{\sqrt{d}}$

Ans. Official Answer NTA (1)

Sol. $$\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+----+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$$

$$=\left(\frac{\sqrt{a_2}-\sqrt{a_1}}{a_2-a_1}+\frac{\sqrt{a_3}-\sqrt{a_2}}{a_3-a_2}+\frac{\sqrt{a_4}-\sqrt{a_3}}{a_4-a_3}+----+\frac{\sqrt{a_n}-\sqrt{a_{n-1}}}{a_n-a_{n-1}}\right)$$

$$=\frac{1}{d}\left(\sqrt{a_n}-\sqrt{a_1}\right)$$

$$\therefore \lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{1}{\sqrt{a_1}+\sqrt{a_2}}+\frac{1}{\sqrt{a_2}+\sqrt{a_3}}+----+\frac{1}{\sqrt{a_{n-1}}+\sqrt{a_n}}\right)$$

$$=\lim_{n\to\infty}\sqrt{\frac{d}{n}}\left(\frac{\sqrt{a_n}-\sqrt{a_1}}{d}\right)$$

$$=\lim_{n\to\infty}\left(\frac{\sqrt{a_1+(n-1)d}-\sqrt{a_1}}{\sqrt{n}\sqrt{d}}\right)=\lim_{n\to\infty}\left(\sqrt{\frac{a_1}{nd}+\left(1-\frac{1}{n}\right)}-\sqrt{\frac{a_1}{nd}}\right)=1$$

SECTION – B

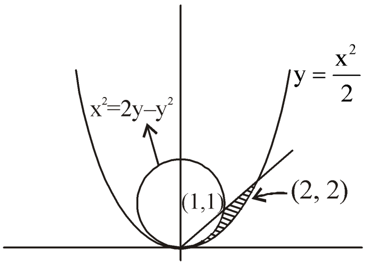
Question ID : 7155053707

21. If the area of the region $S=\{(x,y):2y-y^2\le x^2\le 2y, x\ge y\}$ is equal to $\frac{n+2}{n+1}-\frac{\pi}{n-1}$, then the natural number n is equal to ________.

यदि क्षेत्र $S=\{(x,y):2y-y^2\le x^2\le 2y, x\ge y\}$ का क्षेत्रफल $\frac{n+2}{n+1}-\frac{\pi}{n-1}$ के बराबर है, तब प्राकृतिक संख्या n बराबर है ________

Ans. Official Answer NTA (5.00)

Sol. $2y-y^2\le x^2\le 2y, x\ge y$



$$\text{Area} = \int_1^2 (\sqrt{2y}-y)dy + \int_0^1 \left(\sqrt{2y}-\sqrt{2y-y^2}\right)dy$$

$$= \frac{7-2^{\frac{5}{2}}}{6} + \frac{2^{\frac{7}{2}}-3\pi}{12}$$

$$= \frac{14-2^{\frac{7}{2}}+2^{\frac{7}{2}}-3\pi}{12} = \frac{7}{6}-\frac{\pi}{4}$$

n = 5

Question ID : 7155053703

22. Let the point (p, p + 1) lie inside the region $E=\left\{(x,y): 3-x \le y \le \sqrt{9-x^2}, 0 \le x \le 3\right\}$. If the set of all values of p is the interval (a, b), then b^2+b-a^2 is equal to ________.

माना बिंदु (p, p + 1) क्षेत्र $E=\left\{(x,y): 3-x \le y \le \sqrt{9-x^2}, 0 \le x \le 3\right\}$ के अन्दर स्थित है। यदि p के सभी मानों का समुच्चय अन्तराल (a, b) है, तब b^2+b-a^2 बराबर है ________

Ans. Official Answer NTA (3)

Sol. $3-x \le y \le \sqrt{9-x^2}$

Points (p, p + 1) lies on y = x + 1

So point of intersection between

y = x + 1 & y = 3 – x is x = 1, y = 2

and point of intersection between

$x+1=\sqrt{9-x^2}$ is $x=\frac{-1+\sqrt{17}}{2}$

Hence $p \in \left(1, \frac{-1+\sqrt{17}}{2}\right)$

Hence $b^2+b-a^2=3$

Question ID : 7155053702

23. Let A = {1, 2, 3, 4,___,10} and B = {0, 1, 2, 3, 4}. The number of elements in the relation $R = \{(a, b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$ is ________.

माना A = {1, 2, 3, 4,___,10} और B = {0, 1, 2, 3, 4} है। सम्बन्ध

$R = \{(a, b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$ में अवयवों की संख्या है ________

Ans. Official Answer NTA (18)

Sol. A = {1,2,3,......10}

B = {0,1, 2,3, 4}

$R = \{(a, b) \in A \times A : 2(a-b)^2 + 3(a-b) \in B\}$

Now $2(a-b)^2 + 3(a-b) = (a-b)(2(a-b)+3)$

$\Rightarrow a = b$ or $a - b = -2$

When $a = b \Rightarrow$ 10 order pairs

When $a - b = -2 \Rightarrow$ 8 order pairs

Total = 18

Question ID : 7155053706

24. Let $a \in \mathbb{Z}$ and [t] be the greatest integer $\le$ t. Then the number of points, where the function $f(x)=[a + 13 \sin x]$, $x \in (0, \pi)$ is not differentiable, is ________.

माना $a \in \mathbb{Z}$ है तथा [t] महत्तम पूर्णांक $\le t$ है। तब उन बिंदुओं, जहाँ $f(x)=[a + 13 \sin x]$, $x \in (0, \pi)$ अवकलनीय नहीं है, की संख्या है ________

Ans. Official Answer NTA (25)

Sol. $f(x) = a + [13 \sin x]$ $\because a \in I$ and $x \in (0, \pi)$

$\because$ total number of points of non-differentiability of $[p \sin x] = 2p - 1$ here $p = 13$

$\therefore$ total number of points of non-differentiability of $[13 \sin x] = 25$

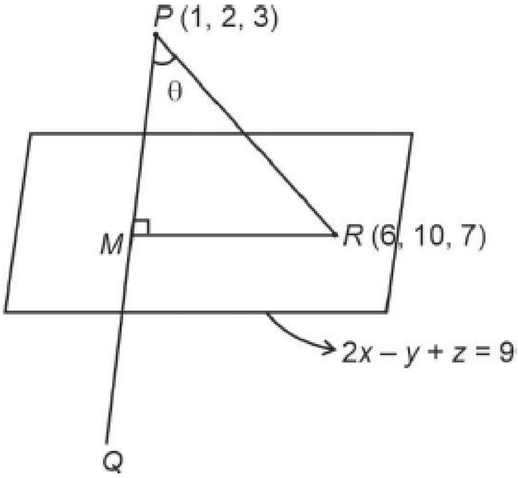
Question ID : 7155053711

25. Let the image of the point P(1, 2, 3) in the plane $2x - y + z = 9$ be Q. If the coordinates of the point R are (6, 10, 7), then the square of the area of the triangle PQR is ________.

माना समतल $2x - y + z = 9$ में बिंदु P(1, 2, 3) का प्रतिबिम्ब Q है। यदि बिंदु R के निर्देशांक (6, 10, 7) है, तब त्रिभुज PQR के क्षेत्रफल का वर्ग है ________

Ans. Official Answer NTA (594.00)

Sol.



R lies on plane

$PR = \sqrt{5^2 + 8^2 + 4^2} = \sqrt{105}$

$\cos\theta = \dfrac{(5\hat{i} + 8\hat{j} + 4\hat{k})(2\hat{i} - \hat{j} + \hat{k})}{\sqrt{105}\sqrt{6}}$

$= \dfrac{6}{\sqrt{630}}$

Area $(\Delta PQR) = 2\,\text{area}(\Delta PMR)$

$= 2 \cdot \dfrac{1}{2}(PR)^2 \sin\theta\cos\theta$

$= 105 \cdot \dfrac{6}{\sqrt{630}} \cdot \dfrac{\sqrt{594}}{\sqrt{630}}$

$= \sqrt{594}$

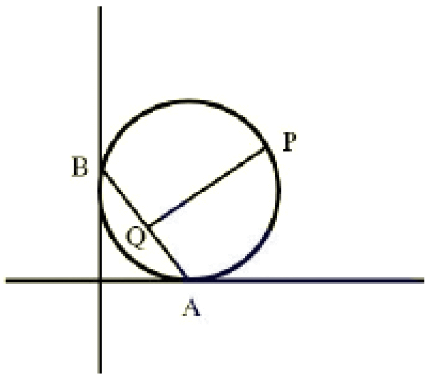
Question ID : 7155053708

26. A circle passing through the point P(α, β) in the first quadrant touches the two coordinate axes at the points A and B. The point P is above the line AB. The point Q on the line segment AB is the foot of perpendicular from P on AB. If PQ is equal to 11 units, then the value of αβ is ________.

प्रथम चतुर्थांश में बिंदु P(α, β) से होकर जाने वाला एक वृत्त, निर्देशांक अक्षों को बिंदुओं A तथा B पर स्पर्श करता है। बिंदु P, रेखा AB से ऊपर है। बिंदु P से AB पर डाले गए लंब पाद रेखाखंड AB पर बिंदु Q है। यदि PQ = 11 इकाई है, तो αβ का मान है ________

Ans. Official Answer NTA (121)

Sol.



Let equation of circle is $(x-a)^2+(y-a)^2=a^2$

which is passing through P (α, β)

then $(\alpha-a)^2+(\beta-a)^2=a^2$

$\alpha^2+\beta^2-2\alpha a-2\beta a+a^2=0$

Here equation of AB is $x+y=a$

Let Q (α', β') be foot of perpendicular of P on AB

$$\frac{\alpha'-\alpha}{1}=\frac{\beta'-\beta}{1}=\frac{-(\alpha+\beta-a)}{2}$$

$$PQ^2=\left(\alpha'-\alpha\right)^2+\left(\beta'-\beta\right)^2=\frac{1}{4}(\alpha+\beta-a)^2+\frac{1}{4}(\alpha+\beta-a)^2$$

$$121=\frac{1}{2}(\alpha+\beta-a)^2$$

$$242=\alpha^2+\beta^2-2\alpha a-2\beta a+a^2+2\alpha\beta$$

$\Rightarrow \alpha\beta = 121$

Question ID : 7155053710

27. Let $y=y(x)$ be a solution of the differential equation $(x\cos x)dy+(xy\sin x+y\cos x-1)dx=0, 0<x<\frac{\pi}{2}$.

If $\frac{\pi}{3}y\left(\frac{\pi}{3}\right)=\sqrt{3}$, then $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right)+2y'\left(\frac{\pi}{6}\right)\right|$ is equal to ________.

माना अवकल समीकरण $(x\cos x)dy+(xy\sin x+y\cos x-1)dx=0, 0<x<\frac{\pi}{2}$ का हल $y=y(x)$ है। यदि

$\frac{\pi}{3}y\left(\frac{\pi}{3}\right)=\sqrt{3}$ है, तब $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right)+2y'\left(\frac{\pi}{6}\right)\right|$ का मान है ________

Ans. Official Answer NTA (2)

Sol. $(x\cos x)dy + (xy\sin x + y\cos x - 1)dx = 0, 0 < x < \frac{\pi}{2}$

$$\frac{dy}{dx} + \left(\frac{x\sin x + \cos x}{x\cos x}\right)y = \frac{1}{x\cos x}$$

IF = x secx

$$y \cdot x\sec x = \int \frac{x\sec x}{x\cos x}dx = \tan x + c$$

Since $y\left(\frac{\pi}{3}\right) = \frac{3\sqrt{3}}{\pi}$ Hence $c = \sqrt{3}$

Hence $\left|\frac{\pi}{6}y''\left(\frac{\pi}{6}\right) + y'\left(\frac{\pi}{6}\right)\right| = |-2| = 2$

Question ID : 7155053704

28. The number of ways of giving 20 distinct oranges to 3 children such that each child gets at least one orange is ________.

20 भिन्न संतरों को 3 बच्चों में बाँटने के तरीकों, ताकि प्रत्येक बच्चे को कम से कम एक संतरा मिले, की संख्या है ________

Ans. Official Answer NTA (171)

Sol. $x_1 + x_2 + x_3 = 20$

let $x_1 = 1 + t_1, x_2 = 1 + t_2, x_3 = 1 + t_3$

when $t_1, t_2, t_3 \in \{0, 1, 2, ...17\}$

$t_1 + t_2 + t_3 = 17$--------(1)

So number of such distribution is equal to number of non-negative integral solutions of equation (1)

So required solution = ${}^{17+3-1}C_{3-1} = {}^{19}C_2$

Question ID : 7155053709

29. Let the tangent to the curve $x^2 + 2x - 4y + 9 = 0$ at the point P(1, 3) on it meet the y-axis at A. Let the line passing through P and parallel to the line $x - 3y = 6$ meet the parabola $y^2 = 4x$ at B. If B lies on the line $2x - 3y = 8$, then $(AB)^2$ is equal to ________.

माना वक्र $x^2 + 2x - 4y + 9 = 0$ के बिंदु P(1, 3) पर स्पर्श रेखा y-अक्ष को बिंदु A पर मिलती है। माना P से होकर जाने वाली तथा $x - 3y = 6$ के समांतर रेखा, परवलय $y^2 = 4x$ को बिंदु B पर मिलती है। यदि बिंदु B, रेखा $2x - 3y = 8$ पर है, तो $(AB)^2$ बराबर है ________

Ans. Official Answer NTA (292)

Sol. Equation of tangent at P(1, 3) to the curve
$x^2 + 2x - 4y + 9 = 0$ is $y - x = 2$
Then the point A is (0, 2)
Equation of line passing through P and parallel to the line $x - 3y = 6$
The possible co-ordinate of B are (4, 4) or (16, 8)
But (4, 4) does not satisfy $2x - 3y = 8$
Thus the point B is (16, 8)
Then $(AB)^2 = 292$

Question ID : 7155053705

30. The coefficient of x^{18} in the expansion of $\left(x^4 - \frac{1}{x^3}\right)^{15}$ is ________.

$\left(x^4 - \frac{1}{x^3}\right)^{15}$ के प्रसार में x^{18} का गुणांक है ________

Ans. Official Answer NTA (5005)

Sol. $\left(x^4 - \frac{1}{x^3}\right)^{15}$

$T_{r+1} = {}^{15}C_r \left(x^4\right)^{15-r} \left(\frac{-1}{x^3}\right)^r$

$60 - 7r = 18$
$r = 6$
Hence coeff. of $x^{18} = {}^{15}C_6 = 5005$