JEE Main April 2023

Question Paper With Text Solution

06 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2023 | 06TH APRIL SHIFT-2

SECTION – A

Question ID : 3666943027

1. If the coefficients of x^7 in $\left(ax^2+\frac{1}{2bx}\right)^{11}$ and x^{-7} in $\left(ax-\frac{1}{3bx^2}\right)^{11}$ are equal, then :

यदि $\left(ax^2+\frac{1}{2bx}\right)^{11}$ में x^7 तथा $\left(ax-\frac{1}{3bx^2}\right)^{11}$ में x^{-7} के गुणांक बराबर है, तो :

(1) $64ab = 243$ (2) $243ab = 64$ (3) $729ab = 32$ (4) $32ab = 729$

Ans. Official Answer NTA (3)

Sol. Coefficient of x^7 in $\left(ax^2+\frac{1}{2bx}\right)^{11}$

$$T_{r+1} = {}^{11}C_r\left(ax^2\right)^{11-r}\left(\frac{1}{2bx}\right)^r$$

$$= {}^{11}C_r(a)^{11-r}\left(\frac{1}{2b}\right)^r x^{22-3r}$$

$$22-3r = 7 \Rightarrow r = 5$$

Coefficient of x^{-7} in $\left(ax-\frac{1}{3bx^2}\right)^{11}$

$$T_{r+1} = {}^{11}C_r(ax)^{11-r}\left(-\frac{1}{3bx^2}\right)^r$$

$$= {}^{11}C_r a^{11-r}\left(-\frac{1}{3b}\right)^r x^{11-3r}$$

$$\therefore \quad {}^{11-3r} = -7 \Rightarrow r = 6$$

$$\Rightarrow \quad {}^{11}C_5(ab)^6\left(\frac{1}{2b}\right)^5 = 32$$

$$\Rightarrow \quad 729ab = 32$$

Question ID : 3666943036

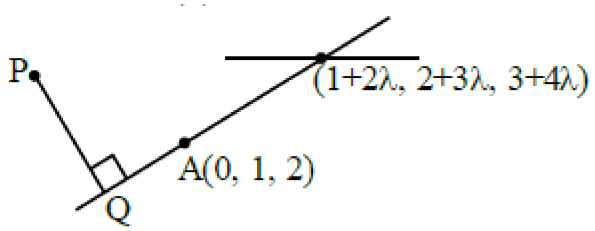
2. Let the line L pass through the point (0,1,2), intersect the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and be parallel to the plane $2x+y-3z=4$. Then the distance of the point P(1, –9, 2) from the line L is :

माना रेखा L, बिंदु (0,1,2) से होकर जाती है, रेखा $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ को काटती है तथा समतल $2x+y-3z=4$ समांतर है। तो बिंदु P(1, –9, 2) की रेखा L से दूरी है :

(1) $\sqrt{74}$ (2) 9 (3) $\sqrt{69}$ (4) $\sqrt{54}$

Ans. Official Answer NTA (1)

Sol.



$\overrightarrow{AB}\cdot\vec{n}$

$\Rightarrow[(1+2\lambda)\hat{i}+(1+3\lambda)\hat{j}+(1+4\lambda)\hat{k}]\cdot(2\hat{i}+\hat{j}-3\hat{k})$

$2+4\lambda+1+3\lambda-3-12\lambda=0$

$5\lambda=0\Rightarrow\lambda=0$

Line $\overrightarrow{AB},\vec{r}=\hat{j}+2\hat{k}+\mu(\hat{i}+\hat{j}+\hat{k})$

General form : $Q(\mu,1+\mu,2+\mu)$

$\therefore\overrightarrow{PQ}\cdot\overrightarrow{AB}=0$

$(\mu-1)+(10+\mu)+\mu=0$

$3\mu=-9\Rightarrow\mu=-3$

$\therefore$ distance $=\sqrt{16+49+9}=\sqrt{74}$

Question ID : 3666943040

3. Among the statements

(S1) : $(p\Rightarrow q)\vee((\sim p)\wedge q)$ is a tautology

(S2) : $(q\Rightarrow p)\Rightarrow((\sim p)\wedge q)$ is a contradiction

(1) only (S1) is True (2) neither (S1) and (S2) is True

(3) both (S1) and (S2) are True (4) only (S2) is True

कथनों

(S1) : $(p \Rightarrow q) \vee ((\sim p) \wedge q)$ पुनरूक्ति

(S2) : $(q \Rightarrow p) \Rightarrow ((\sim p) \wedge q)$ विरोधोक्ति है में से

(1) केवल (S1) सत्य है (2) न तो (S1) न ही (S2) सत्य है

(3) दोनों (S1) तथा (S2) सत्य है (4) केवल (S2) सत्य है

Ans. Official Answer NTA (2)

Sol. S1

P	Q	$\sim p$	$\sim p \wedge q$	$p \Rightarrow q$	$(p \Rightarrow q) \vee (\sim p \wedge q)$
T	T	F	F	T	T
T	F	F	F	F	F
F	T	T	T	T	T
F	F	T	F	T	T

S2

P	Q	$q \Rightarrow p$	$\sim p$	$(\sim p) \wedge q$	$(p \Rightarrow q) \Rightarrow (\sim p \wedge q)$
T	T	T	F	F	F
T	F	T	F	F	F
F	T	F	T	T	T
F	F	T	T	F	F

Question ID : 3666943034

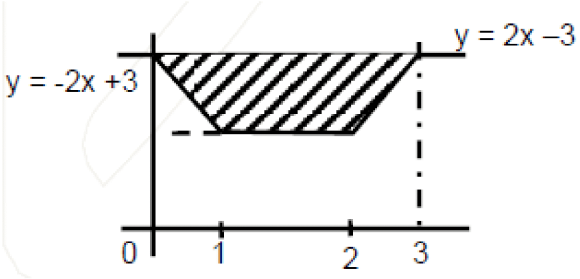
4. The area bounded by the curves $y = |x - 1| + |x - 2|$ and $y = 3$ is equal to :

वक्रों $y = |x - 1| + |x - 2|$ तथा $y = 3$ से परिबद्ध क्षेत्र का क्षेत्रफल है :

(1) 4 (2) 3 (3) 6 (4) 5

Ans. Official Answer NTA (1)

Sol.



$$\text{area} = \frac{1}{2}\times(1+3)\times 2 = 4$$

Question ID : 3666943028

5. All the letters of the word PUBLIC are written in all possible orders and these words are written as in a dictionary with serial numbers. Then the serial number of the word PUBLIC is :

PUBLIC शब्द के सभी अक्षरों को सभी संभव क्रम में लिखा जाता है तथा इन शब्दों को शब्दकोष के अनुसार क्रम संख्या के साथ लिखा जाता है। तो शब्द PUBLIC की क्रम संख्या है :

(1) 576 (2) 582 (3) 578 (4) 580

Ans. Official Answer NTA (2)

Sol.

5 6 1 4 3 2

P U B L I C

4 4 0 2 1 0

5! 4! 3! 2! 1! 0!

Rank = (1 × 1! + 2 × 2! + 4 × 4! + 4 × 5!) + 1

= (1 + 4 + 96 + 480) + 1

= 582

Question ID : 3666943032

6. If the solution curve f(x, y)=0 of the differential equation $\left(1+\log_e x\right)\frac{dx}{dy} - x\log_e x = e^y, x>0$, passes through the points (1, 0) and (α, 2) then α^α is equal to :

यदि अवकल समीकरण $\left(1+\log_e x\right)\frac{dx}{dy} - x\log_e x = e^y, x>0$ का हल वक्र f(x, y) = 0 बिंदुओं (1, 0) तथा (α, 2) से होकर जाता है, तो α^α बराबर है :

(1) e^{2e^2} (2) e^{e^2} (3) $e^{2e^{\sqrt{2}}}$ (4) $e^{\sqrt{2}e^2}$

Ans. Official Answer NTA (1)

Sol. $(1+\ln x)\dfrac{dx}{dy}-x\ln x=e^y$

Let $x\ln x=t$

$(1+\ln x)\dfrac{dx}{dy}=\dfrac{dt}{dy}$

$\dfrac{dt}{dy}-t=e^y$

If $=e^{\int -dy}=e^{-y}$

$t\cdot e^{-y}=\int e^y e^{-y}dy+c$

$te^{-y}=y+c$

$x\ln xe^{-y}=y+c$

$x\ln x=ye^y+ce^y$

$(1,0)0=C$

$\Rightarrow x\ln x=ye^y$

$\Rightarrow \alpha\ln\alpha=2e^2$

$\alpha^\alpha=e^{2e^2}$

Question ID : 3666943022

7. In a group of 100 persons 75 speak English and 40 speak Hindi. Each person speaks at least one of the two languages. If the number of persons, who speak only English is α and the number of persons who speak only Hindi is β, then the eccentricity of the ellipse $25\left(\beta^2x^2+\alpha^2y^2\right)=\alpha^2\beta^2$ is :

100 व्यक्तियों के एक समूह में 75 अंग्रेजी बोलते है तथा 40 हिंदी बोलते हैं। प्रत्येक व्यक्ति इन दो भाषाओं में से कम से कम एक बोलता है। यदि केवल अंग्रेजी बोलने वाले व्यक्तियों की संख्या α तथा केवल हिंदी बोलने वाले व्यक्तियों की संख्या β है, तो दीर्घवृत्त $25\left(\beta^2x^2+\alpha^2y^2\right)=\alpha^2\beta^2$ की उत्केन्द्रता है :

(1) $\dfrac{3\sqrt{15}}{12}$ (2) $\dfrac{\sqrt{117}}{12}$ (3) $\dfrac{\sqrt{129}}{12}$ (4) $\dfrac{\sqrt{119}}{12}$

Ans. Official Answer NTA (4)

Sol. $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$n(A \cap B) = 75 + 40 - 100$

$n(A \cap B) = 15 \qquad \alpha = 60$

Only E $\rightarrow 60 \qquad \beta = 25$

Only H $\rightarrow 25$

Both $= 15$

$$\frac{25x^2}{\alpha^2} + \frac{25y^2}{\beta^2} = 1$$

$$\frac{25x^2}{(60)^2} + \frac{\left(25y^2\right)}{(25)^2} = 1$$

$$e^2 = 1 - \left[\frac{25 \times 25}{(60)^2}\right]$$

$$e^2 = \frac{(60)^2 - (25)^2}{(60)^2}$$

$$e^2 = \frac{(60-25)(60+25)}{60 \times 60}$$

$$e^2 = \frac{(35)(85)}{60 \times 60} = \frac{119}{144}$$

$$e = \frac{\sqrt{119}}{12}$$

Question ID : 3666943035

8. A plane P contains the line of intersection of the plane $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ and $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$. If P passes through the point (0, 2, –2), then the square of distance of the point (12, 12, 18) from the plane P is :

एक समतल P में समतलों $\vec{r} \cdot (\hat{i} + \hat{j} + \hat{k}) = 6$ तथा $\vec{r} \cdot (2\hat{i} + 3\hat{j} + 4\hat{k}) = -5$ की प्रतिच्छेदन रेखा स्थित है। यदि P, बिंदु (0, 2, –2) से होकर जाता है, तो बिंदु (12, 12, 18) की समतल P से दूरी का वर्ग है :

(1) 310 (2) 1240 (3) 620 (4) 155

Ans. Official Answer NTA (3)

Sol. Given planes

$P_1 : x + y + z - 6 = 0$

$P_2 : 2x + 3y + 4z + 5 = 0$

Equation of plane passing through line of intersection of

$P_1 = 0 \quad \& \quad P_2 = 0 \quad \text{is} \quad P_1 + \lambda P_2 = 0$

$(1+2\lambda)x + (1+3\lambda)y + (1+4\lambda)z + (5\lambda - 6) = 0$

It passes through (0, 2,–2)

$2(1+3\lambda) + 6(1+4\lambda)(-2) + 5\lambda - 6 = 0$

$2 + 6\lambda - 2 - 8\lambda + 5\lambda - 6 = 0$

$3\lambda - 6 = 0, \lambda = 2$

So plane : $5x + 7y + 9z + 4 = 0$

$$\text{distance of } (12,12,18) = \frac{5\times12 + 7\times12 + 9\times18 + 4}{\sqrt{25+49+81}}$$

$$d = \frac{310}{\sqrt{155}} = 2\sqrt{155}$$

$d^2 = 4 \times 155 = 620$

Question ID : 3666943031

9. Let f(x) be a function satisfying $f(x) + f(\pi - x) = \pi^2, \forall x \in \mathbb{R}$. Then $\int_0^{\pi} f(x)\sin x dx$ is equal to :

माना एक फलन f(x) के लिए $f(x) + f(\pi - x) = \pi^2, \forall x \in \mathbb{R}$ है। तो $\int_0^{\pi} f(x)\sin x dx$ बराबर है :

(1) $\frac{\pi^2}{2}$ (2) π^2 (3) $\frac{\pi^2}{4}$ (4) $2\pi^2$

Ans. Official Answer NTA (2)

Sol. $I = \int_0^{\pi} f(x)\sin x dx$

$I = \int_0^{\pi} f(\pi - x)\sin x dx$

$2I = \int_0^{\pi} \sin x(f(x) + f(\pi - x))dx$

$2I = \pi^2 \int_0^{\pi} \sin x dx$

$2I = 2\pi^2 \int_0^{\frac{\pi}{2}} \sin x dx$

$I = \pi^2$

Question ID : 3666943023

10. Let $a \neq b$ be two non-zero real numbers. Then the number of elements in the set $X = \left\{z \in \mathbb{C} : \text{Re}\left(az^2 + bz\right) = a \text{ and } \text{Re}\left(bz^2 + az\right) = b\right\}$ is equal to :

माना $a \neq b$ दो शून्येतर वास्तविक संख्याएँ है। तो समुच्चय $X = \left\{z \in \mathbb{C} : \text{Re}\left(az^2 + bz\right) = a \text{ तथा } \text{Re}\left(bz^2 + az\right) = b\right\}$ में अवयवों की संख्या है :

(1) 1 (2) 3 (3) 2 (4) 0

Ans. Official Answer NTA (4)

Sol. $\text{Re}\left(az^2 + bz\right) = a$

$az^2 + bz + a\bar{z}^2 + b\bar{z} = 2a$

$a\left(z^2 + \bar{z}^2\right) + b(z + \bar{z}) = 2a$ .....(1)

$\text{Re}\left(bz^2 + az\right) = b$

$bz^2 + az + b\bar{z}^2 + a\bar{z} = 2b$

$b\left(z^2 + \bar{z}^2\right) + a(z + \bar{z}) = 2b$ .....(2)

$(1) \times b - (2) \times (a)$

$\Rightarrow \left(b^2 - a^2\right)(z + \bar{z}) = 0$

$\Rightarrow \quad (z + \bar{z}) = 0 \quad \left(a^2 \neq b^2\right)$

$(1) \times a - (2) \times (b)$

$\Rightarrow \quad \left(a^2 - b^2\right)(z + \bar{z}) = 2\left(a^2 - b^2\right) \quad \left(a^2 \neq b^2\right)$

$z^2 + \bar{z}^2 = 2$

$\Rightarrow (z + \bar{z})^2 - 2z\bar{z} = 2$

$z\bar{z} = -1$

$\Rightarrow 1 + 1^2 = -1$

$\Rightarrow$ No solution

But when $a = -b$,

$\text{Re}(az - az) = a$

$\Rightarrow \quad \text{Re}\left(a\left(x^2 - y^2 + i2xy\right) - a(x + iy)\right) = a$

$\Rightarrow a\left(x^2-y^2\right)-ax=a$

$\Rightarrow x^2-y^2-x=1$

$\Rightarrow x^2-x-1=y^2$

For any real values of y there two values of x, hence infinite complex numbers are possible.

Question ID : 3666943038

11. Let the vectors $\vec{a},\vec{b},\vec{c}$ represent three coterminous edges of a parallelopiped of volume V. Then the volume of the parallelopiped, whose coterminous edges are represented by $\vec{a},\vec{b}+\vec{c}$ and $\vec{a}+2\vec{b}+3\vec{c}$ is equal to :

माना सदिश $\vec{a},\vec{b},\vec{c}$ आपतन V के एक समांतर षट्फलक के सहावसानी किनारों को निरूपित करते है। तो उस समांतर षटफक, जिसके सहावसानी किनारें सदिशों $\vec{a},\vec{b}+\vec{c}$ तथा $\vec{a}+2\vec{b}+3\vec{c}$ से निरूपित होते हैं, का आयतन बराबर है :

(1) V (2) 3V (3) 6V (4) 2V

Ans. Official Answer NTA (1)

Sol. $=\left[\vec{a}\ \ \vec{b}\ \ \vec{c}\right]v_1$

$=\left[\vec{a}\ \ \vec{b}+\vec{c}\ \ \vec{a}+2\vec{b}+3\vec{c}\right]v_1$

$=\begin{bmatrix}1&0&0\\0&1&1\\1&2&3\end{bmatrix}\left[\vec{a}\,\vec{b}\,\vec{c}\right]v_1$

$=(3-2)v$

$=v$

Question ID : 3666943026

12. Among the statements :

(S1) : $2023^{2022}-1999^{2022}$ is divisible by 8

(S2) : $13(13)^n-11n-13$ is divisible by 144 for infinitely many $n\in\mathbb{N}$

(1) both (S1) and (S2) are correct (2) both (S1) and (S2) are incorrect

(3) only (S2) is correct (4) only (S1) is correct

कथनों :

(S1) : $2023^{2022} - 1999^{2022}$, 8 से विभाज्य है

(S2) : $13(13)^n - 11n - 13$ अनंत $n \in \mathbb{N}$ के लिए 144 से विभाज्य है

में से

(1) दोनों (S1) तथा (S2) सही है (2) दोनों (S1) तथा (S2) गलत है

(3) केवल (S2) सही है (4) केवल (S1) सही है

Ans. Official Answer NTA (4)

Sol. Statement – 1 : $x^{2022} - y^{2022}$ is divisible by $(x - y)$

Hence $(2023)^{2022} - (1999)^{2022}$ is divisible by $2023 - 1999 = 24$

$\Rightarrow$ statement : 1 is correct

Statement – 2 : $13\,(13)^n - 11n - 13$

$= 13(12+1)^n - 11n - 13$

$= 13\left[12^n + n(12)^{n-1} + \frac{n(n-1)}{2}(12)^{n-2} + \ldots\ldots\ldots\ldots + {}^nC_n\right] - 11n - 13$

$= 13\left[12^n + n(12)^{n-1} + {}^nC_2(12)^{n-2} + \ldots\ldots\ldots + {}^nC_{n-2}(12)^2\right] - 11n - 13$

$= 13\left[12^n + n(12)^{n-1} + {}^nC_2(12)^{n-2} + \ldots\ldots\ldots + {}^nC_{n-2}(12) + 1\right] + 156n - 11n$

$= 13\left[12^n + n\cdot 12^{n-1} + {}^nC_2(12)^{n-2} + \ldots\ldots\ldots + {}^nC_{n-2}(12)^2\right] + 145n$

Which is not divisible by 144 for all $n \in N$ but divisible by 144 for infinitely many values of n if $n = 144m$ where $m \in N$.

Question ID : 3666943037

13. The sum of all values of α, for which the points whose position vectors are $\hat{i} - 2\hat{j} + 3\hat{k}, 2\hat{i} - 3\hat{j} + 4\hat{k}$, $(\alpha+1)\hat{i} + 2\hat{k}$ and $9\hat{i} + (\alpha - 8)\hat{j} + 6\hat{k}$ are coplanar, is equal to :

a के सभी मानों, जिनके लिए स्थिति सदिशों $\hat{i} - 2\hat{j} + 3\hat{k}, 2\hat{i} - 3\hat{j} + 4\hat{k}$, $(\alpha+1)\hat{i} + 2\hat{k}$ तथा $9\hat{i} + (\alpha - 8)\hat{j} + 6\hat{k}$ के बिंदु सहतलीय है, का योग बराबर है :

(1) 4 (2) –2 (3) 6 (4) 2

Ans. Official Answer NTA (4)

Sol. Let the points be A, B, C, D

$\overrightarrow{AB} = \hat{i} - \hat{j} + \hat{k}$

$\overrightarrow{AC} = \alpha\hat{i} + 2\hat{j} - \hat{k}$

$\overrightarrow{AD} = 8\hat{i} + (\alpha - 6)\hat{j} + 3\hat{k}$

So, vectors $\overrightarrow{AB}, \overrightarrow{AC}, \overrightarrow{AD}$ are coplanar.

$$\begin{vmatrix} 1 & -1 & 1 \\ \alpha & 2 & -1 \\ 8 & (\alpha-6) & 3 \end{vmatrix} = 0$$

$$(6+\alpha-6)+(3\alpha+8)+\left(\alpha^2-6\alpha-16\right)=0$$

$$\alpha^2-2\alpha-14=0$$

Sum of values of $\alpha = 2$

Question ID : 3666943033

14. If the tangents at the points P and Q on the circle $x^2+y^2-2x+y=5$ meet at the point $R\left(\frac{9}{4},2\right)$, then the area of the triangle PQR is :

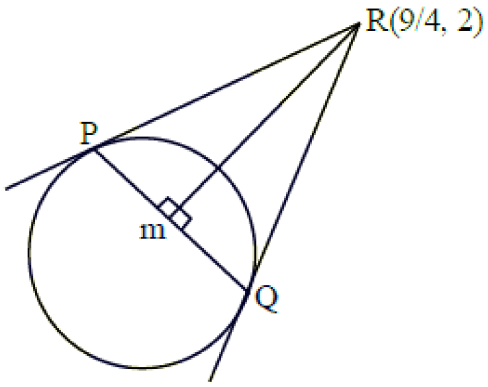
यदि वृत $x^2 + y^2 - 2x + y = 5$ के बिंदुओं P तथा Q पर स्पर्श रेखाएँ बिंदु $R\left(\frac{9}{4},2\right)$ पर मिलती है, तो त्रिभुज PQR का क्षेत्रफल है :

(1) $\frac{5}{4}$ (2) $\frac{5}{8}$ (3) $\frac{13}{8}$ (4) $\frac{13}{4}$

Ans. Official Answer NTA (2)

Sol. Equation of circle is $x^2+y^2-2x+y-5=0$

$$R=\frac{5}{2}$$



Length of PR = QR = $\sqrt{S_1}$

$$=\sqrt{\frac{81}{16}+4-\frac{2\times9}{4}+2-5}=\frac{5}{4}$$

$$\text{Area of triangle PQR} = \frac{RL^3}{R^2+L^2} = \frac{\frac{5}{2}\cdot\frac{125}{64}}{\frac{25}{4}+\frac{25}{16}} = \frac{5}{8}$$

Question ID : 3666943029

15. $\lim\limits_{n\to\infty}\left\{\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right).....\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)\right\}$ is equal to :

$\lim\limits_{n\to\infty}\left\{\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right).....\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)\right\}$ बराबर है :

(1) 1 (2) $\sqrt{2}$ (3) 0 (4) $\frac{1}{\sqrt{2}}$

Ans. Official Answer NTA (3)

Sol. $P = \lim\limits_{n\to\infty}\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)\left(2^{\frac{1}{2}}-2^{\frac{1}{5}}\right).....\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)$

Let

$2^{\frac{1}{2}}-2^{\frac{1}{3}} \rightarrow$ Smallest

$2^{\frac{1}{2}}-2^{\frac{1}{2n+1}} \rightarrow$ Largest

Sandwich th.

$$\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)^n \le P \le \left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)^n$$

$$\begin{pmatrix}\text{lie b/w}\\ 0 \text{ and } 1\end{pmatrix}^n$$

$$\lim_{n\to\infty}\left(2^{\frac{1}{2}}-2^{\frac{1}{3}}\right)^n = 0$$

$$\lim_{n\to\infty}\left(2^{\frac{1}{2}}-2^{\frac{1}{2n+1}}\right)^n = 0$$

$\therefore P = 0$

Question ID : 3666943039

16. Three dice are rolled. If the probability of getting different numbers on the threse dice is $\frac{p}{q}$, where p and q are co-prime, then q – p is equal to :

तीन पासे फेंके जाते है। यदि तीनों पासों पर भिन्न संख्याएँ प्राप्त करने की प्रायिकता $\frac{p}{q}$ है, जहाँ p तथा q असहभाज्य है, तो q – p बराबर है :

(1) 1 (2) 4 (3) 2 (4) 3

Ans. Official Answer NTA (2)

Sol. Total number of outcomes = $6 \times 6 \times 6$

Total number of favourable out comes = $6 \times 5 \times 4$

So probability $= \frac{5}{9} = \frac{p}{q}$

Question ID : 3666943025

17. For the system of equations

$x + y + z = 6$

$x + 2y + \alpha z = 10$

$x + 3y + 5z = \beta$, which one of the following is NOT true?

(1) System has a unique solution for $\alpha = -3, \beta = 14$

(2) System has a unique solution for $\alpha = -3, \beta \neq 14$

(3) System has no solution for $\alpha = 3, \beta = 24$

(4) System has infinitely many solutions for $\alpha = 3, \beta = 14$

समीकरण निकाय

$x + y + z = 6$

$x + 2y + \alpha z = 10$

$x + 3y + 5z = \beta$, के लिए निम्न में से कौन सा सत्य नहीं है ?

(1) $\alpha = -3, \beta = 14$ के लिए समीकरण निकाय का अद्वितीय हल नहीं

(2) $\alpha = -3, \beta \neq 14$ के लिए समीकरण निकाय का अद्वितीय हल है

(3) $\alpha = 3, \beta = 24$ के लिए समीकरण निकाय का कोई हल नहीं है

(4) $\alpha = 3, \beta = 14$ के लिए समीकरण निकाय के अनंत हल है

Ans. Official Answer NTA (4)

Sol. $D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & \alpha \\ 1 & 3 & 5 \end{vmatrix}$

$= 1(10-3\alpha) - (5-\alpha) + (3-2)$

$= 6 - 2\alpha$

$D \neq 0 \Rightarrow \alpha \neq 3$

Unique solution $\Rightarrow \alpha \neq 3$

Question ID : 3666943030

18. If gcd (m, n) = 1 and $1^2 - 2^2 + 3^2 - 4^2 + \ldots + (2021)^2 - (2022)^2 + (2023)^2 = 1012m^2n$ then $m^2 - n^2$ is equal to :

यदि gcd (m, n) = 1 है तथा $1^2 - 2^2 + 3^2 - 4^2 + \ldots + (2021)^2 - (2022)^2 + (2023)^2 = 1012m^2n$ है, तो $m^2 - n^2$ बराबर है :

(1) 220 (2) 200 (3) 240 (4) 180

Ans. Official Answer NTA (3)

Sol. $1^2 - 2^2 + 3^2 - 4^2 + \ldots (2021)^2 - (2022)^2 + (2023)^2 = 1012$

$m^2n = (1-2)(1+2) + (3-4)(3+4) + \ldots + (2021-2022)(2021+2022) + (2023)^2$

$= (-1)(1+2+3+4+\ldots+2022) + (2023)^2$

$= (-1) \cdot \frac{(2022)(2023)}{2} + (2023)^2$

$= 2023(2023-1011) = 2023 \times 1012$

$m^2n = 2023 = 17^2 \cdot 7$

$m = 17, n = 7$

$m^2 - n^2 = 17^2 - 7^2 = 240$

Question ID : 3666943024

19. Let P be a square matrix such that $P^2 = I - P$. For $\alpha, \beta, \gamma, \delta \in \mathbb{N}$, if $P^\alpha + P^\beta = \gamma I - 29P$ and $P^\alpha - P^\beta = \delta I - 13P$, then $\alpha + \beta + \gamma - \delta$ is equal to :

माना P एक वर्ग आव्यूह है जिसके लिए $P^2 = I - P$ है। तो $\alpha,\beta,\gamma,\delta \in \mathbb{N}$ के लिए यदि $P^\alpha + P^\beta = \gamma I - 29P$ तथा $P^\alpha - P^\beta = \delta I - 13P$ है, तो $\alpha+\beta+\gamma-\delta$ बराबर है :

(1) 18 (2) 22 (3) 40 (4) 24

Ans. Official Answer NTA (4)

Sol. $P^2 = I - P$

$P^\alpha + P^\beta = \gamma I - 29P$

$P^\alpha - P^\beta = \delta I - 13P$

$P^4 = (I-P)^2 = I + P^2 - 2P$

$P^4 = I + I - P - 2P = 2I - 3P$

$P^8 = \left(P^4\right)^2 = (2I-3P)^2 = 4I + 9P^2 - 12P$

$= 4I + 9(I-P) - 12P$

$P^8 = 13I - 21P$

$P^6 = P^4 \cdot P^2 = (2I-3P)(I-P)$

$= 2I - 5P + 3P^2$

$= 2I - 5P + 3(I-P)$

$= 5I - 8P$

(1) + (2) $\quad P^8 + P^6 = 18I - 29P$

(1) − (2) $\quad P^8 - P^6 = 8I - 13P$

From (A) $\quad \alpha = 8, \quad \beta = 6$

$\gamma = 18$

$\delta = 8$

$\alpha+\beta+\gamma-\delta = 32 - 8 = 24$

Question ID : 3666943021

20. Let the sets A and B denote the domain and range respectively of the function $f(x) = \frac{1}{\sqrt{[x]-x}}$, where [x] denotes the smallest integer greater than or equal to x. Then among the statements :

(S1) : $A \cap B = (1,\infty) - \mathbb{N}$ and

(S2) : $A \cup B = (1,\infty)$

(1) only (S2) is true (2) neither (S1) nor (S2) is true

(3) only (S1) is true (4) both (S1) and (S2) are true

माना फलन $f(x)=\frac{1}{\sqrt{[x]-x}}$, जहाँ $[x]$ न्यूनतम पूर्णांक $\geq x$ है, के प्रांत तथा परिसर क्रमशः समुच्चय A तथा B है। तो कथनों

(S1) : $A \cap B = (1,\infty) - \mathbb{N}$ तथा

(S2) : $A \cup B = (1,\infty)$

(1) केवल (S2) सही है (2) न तो (S1) न ही (S2) सही है

(3) केवल (S1) सही है (4) दोनों (S1) तथा (S2) सही है

Ans. Official Answer NTA (3)

Sol. $[x]-x>0 \Rightarrow [x]>x$

$\Rightarrow x \notin I$ So $A = R - I$

Also $0<[x]-x<1$ when $x \in I$

So $f(x) \in (1,\infty) \Rightarrow B = (1,\infty)$

$A \cap B = (1,\infty) - N$

$A \cup B = (1,\infty) \cup \{R - I\}$

So S_1 is true & S_2 is false

SECTION – B

Question ID : 3666943042

21. The number of 4-letter words, with or without meaning, each consisting of 2 vowels and 2 consonants, which can be formed from the letters of the word UNIVERSE without repetition is _____.

बिना पुनरावृति के UNIVERSE शब्द के अक्षरों से बनाए जा सकने वाले 4 अक्षरों, जिनमें 2 स्वर तथा 2 व्यंजक हो, के अर्थपूर्ण या अर्थहीन शब्दों की संख्या है _____

Ans. Official Answer NTA (432)

Sol. UNIVERSE

E, E, I, U, (Vowels) + N, R, S, V (Consonants)

Two different vowels + 2 consonants

$= {}^3C_2 . {}^4C_2 . \underline{|4} = 432$

Question ID : 3666943048

22. If the lines $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$ and $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$ intersect, then the magnitude of the minimum value of $8\alpha\beta$ is _____.

यदि रेखाएँ $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$ तथा $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$ एक दूसरे को काटती है, तो $8\alpha\beta$ के न्यूनतम मान का परिमाण है _____

Ans. Official Answer NTA (18)

Sol. If the lines $\frac{x-1}{2}=\frac{2-y}{-3}=\frac{z-3}{\alpha}$

And $\frac{x-4}{5}=\frac{y-1}{2}=\frac{z}{\beta}$ intersect

Point on first line (1, 2, 3) and point on second line (4, 1, 0).

Vector joining both points is $-3\hat{i}+\hat{j}+3\hat{k}$

Now vector along first line is $2\hat{i}+3\hat{j}+\alpha\hat{k}$

Also vector along second line is $5\hat{i}+2\hat{j}+\beta\hat{k}$

Now these three vectors must be coplanar

$$\Rightarrow \begin{vmatrix} 2 & 3 & \alpha \\ 5 & 2 & \beta \\ -3 & 1 & 3 \end{vmatrix} = 0$$

$\Rightarrow 2(6-\beta)-3(15+3\beta)+\alpha(11)=0$

$\Rightarrow \alpha-\beta=3$

Now $\alpha=3+\beta$

Given expression $8(3+\beta)\cdot\beta=8\left(\beta^2+3\beta\right)$

$=8\left(\beta^2+3\beta+\frac{9}{4}-\frac{9}{4}\right)=8\left(\beta+\frac{3}{2}\right)^2-18$

So magnitude of minimum value = 18

Question ID : 3666943046

23. Let $f(x)=\frac{x}{\left(1+x^n\right)^{\frac{1}{n}}}, x\in\mathbb{R}-\{-1\}, n\in\mathbb{N}, n>2$.

If $f^n(x) = (\text{fofof .....upto n times})(x)$, then $\lim\limits_{n\to\infty}\int_0^1 x^{n-2}\left(f^n(x)\right)dx$ is equal to _____.

माना $f(x) = \dfrac{x}{\left(1+x^n\right)^{\frac{1}{n}}}, x \in \mathbb{R}-\{-1\}, n \in \mathbb{N}, n > 2$ है।

यदि $f^n(x) = (\text{fofof .....upto n times})(x)$ है तो $\lim\limits_{n\to\infty}\int_0^1 x^{n-2}\left(f^n(x)\right)dx$ बराबर है _____

Ans. Official Answer NTA (0)

Sol. Let $f(x) = \dfrac{x}{1+x^n\frac{1}{n}}, x \in \mathbb{R}-\{-1\}, n \in \mathbb{N}, n > 2.$

If $f^n(x) = n\ (\text{fofof....upto } n \text{ times})\ (x)$

then $\lim\limits_{n\to\infty}\int_0^1 x^{n-2}\left(f^n(x)\right)dx$

$f(f(x)) = \dfrac{x}{\left(1+2x^n\right)^{1/n}}$

$f(f(f(x))) = \dfrac{x}{\left(1+3x^n\right)^{1/n}}$

Similarly $f^n(x) = \dfrac{x}{\left(1+n\cdot x^n\right)^{1/n}}$

Now $\lim\limits_{n\to\infty}\int\dfrac{x^{n-2}\cdot x dx}{\left(1+n\cdot x^n\right)^{1/n}} = \lim\limits_{n\to\infty}\int\dfrac{x^{n-1}\cdot dx}{\left(1+n\cdot x^n\right)^{1/n}}$

Now $1+nx^n = t$

$n^2\cdot x^{n-1}dx = dt$

$x^{n-1}dx = \dfrac{dt}{n^2}$

$\Rightarrow \lim\limits_{n\to\infty}\dfrac{1}{n^2}\int_1^{1+n}\dfrac{dt}{t^{1/n}}$

$\Rightarrow \lim\limits_{n\to\infty}\dfrac{1}{n^2}\left[\dfrac{t^{1-\frac{1}{n}}}{1-\frac{1}{n}}\right]_1^{1+n}$

$$\Rightarrow \lim_{n\to\infty} \frac{1}{n(n-1)}\left((1+n)^{\frac{n-1}{n}}-1\right) \text{ Now let } n=\frac{1}{h}$$

$$\Rightarrow \lim_{h\to 0} \frac{\left(1+\frac{1}{h}\right)^{1-h}-1}{\frac{1}{h}\frac{(1-h)}{h}}$$

Using series expansion

$\Rightarrow 0$

Question ID : 3666943049

24. If the mean and variance of the frequency distribution

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

are 9 and 15.08 respectively, then the value of $\alpha^2+\beta^2-\alpha\beta$ is _____.

यदि बारंबारता बंटन

x_i	2	4	6	8	10	12	14	16
f_i	4	4	α	15	8	β	4	5

के माध्य तथा प्रसरण क्रमशः 9 तथा 15.08 है, तो $\alpha^2+\beta^2-\alpha\beta$ का मान है _____

Ans. Official Answer NTA (25.00)

Sol. $$\text{Mean} = \frac{8+16+120+80+56+80+6\alpha+12\beta}{40+\alpha+\beta}$$

$\Rightarrow 360+9\alpha+9\beta = 360+6\alpha+12\beta$

OR

$3\alpha-3\beta=0$

$16+64+36\alpha+960+800$

$$15.08+81=\frac{+144\alpha+784+1280}{40+2\alpha}$$

$(40+2\alpha)(96.08)=3904+180\alpha$

$3843.20+(192.16)\alpha=3904+180\alpha$

$(12.16)\,\alpha=60.80$

$\alpha=5=\beta$

Question ID : 3666943044

25. The number of points, where the curve $y = x^5 - 20x^3 + 50x + 2$ crosses the x-axis, is _____.

उन बिंदुओं, जहाँ वक्र $y = x^5 - 20x^3 + 50x + 2$, x-अक्ष को काटता है, की संख्या है _____

Ans. Official Answer NTA (5)

Sol. $y = x^5 - 20x^3 + 50x + 2$

$$\frac{dy}{dx} = 5x^4 - 60x^2 + 50 = 5\left(x^4 - 12x^2 + 10\right)$$

$$\frac{dy}{dx} = 0 \Rightarrow x^4 - 12x^2 + 10 = 0$$

$$\Rightarrow x^2 = \frac{12 \pm \sqrt{144 - 40}}{2}$$

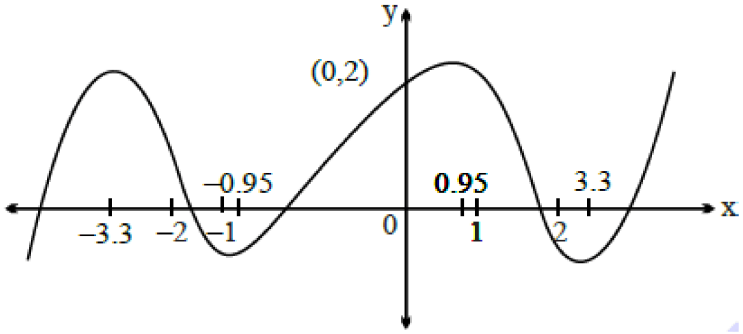
$$\Rightarrow x^2 = 6 \pm \sqrt{26} \Rightarrow x^2 \approx 6 \pm 5.1$$

$$\Rightarrow x^2 \approx 11.1, 0.9$$

$$\Rightarrow x \approx \pm 3.3, \pm 0.95$$

$f(0) = 2, f(1) = +\text{ ve}, f(2) = -\text{ ve}$

$f(-1) = -\text{ve}, f(-2) = +\text{ve}$



Question ID : 3666943050

26. The value of $\tan 9° - \tan 27° - \tan 63° + \tan 81°$ is _____.

$\tan 9° - \tan 27° - \tan 63° + \tan 81°$ का मान है _____

Ans. Official Answer NTA (4)

Sol. $\left(\tan 9^\circ + \cot 9^\circ\right) - \left(\tan 27^\circ + \cot 27^\circ\right)$

$$\frac{1}{\sin 9^\circ \cos 9^\circ} - \frac{1}{\sin 27^\circ \cos 27^\circ}$$

$$\frac{2}{\sin 18^\circ} - \frac{2}{\sin 54^\circ}$$

$$\frac{2(4)}{\sqrt{5}-1}-\frac{2(4)}{(\sqrt{5}+1)}$$

$$\frac{8(\sqrt{5}+1)}{4}-\frac{8(\sqrt{5}-1)}{4}$$

$$2[(\sqrt{5}+1)-(\sqrt{5}-1)]$$

$$=4$$

Question ID : 3666943043

27. If $(20)^{19}+2(21)(20)^{18}+3(21)^2(20)^{17}+\ldots+20(21)^{19}=k(20)^{19}$, then k is equal to _____.

यदि $(20)^{19}+2(21)(20)^{18}+3(21)^2(20)^{17}+\ldots+20(21)^{19}=k(20)^{19}$ है, तो k बराबर है _____

Ans. Official Answer NTA (400)

Sol. $k=1+2\left(\frac{21}{20}\right)+3\left(\frac{21}{20}\right)^2+\ldots\ldots..+20\left(\frac{21}{20}\right)^{19}$

$k=1+2r+3r^2+4r^3+\ldots\ldots\ldots 20r^{19}$ where $r=\frac{21}{20}$

$kr=r+2r^2+3r^3+\ldots\ldots+20r^{20}\ldots\ldots\ldots(2)$

$by(1)-(2)$

$(1-r)k=1+r+r^2+\ldots\ldots.r^{19}-20r^{20}$

$$-\frac{1}{20}k=1\frac{\left(r^{20}-1\right)}{r-1}-20r^{20}$$

$$-\frac{1}{20}k=\frac{\left(\frac{21}{20}\right)^{20}-1}{\left(\frac{21}{20}-1\right)}-20\left(\frac{21}{20}\right)^{20}$$

$$-\frac{1}{20}k=20\left(\frac{21}{20}\right)^{20}-20-20\left(\frac{21}{20}\right)^{20}$$

$k=400$

Question ID : 3666943047

28. Let the eccentricity of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is reciprocal to that of the hyperbola $2x^2-2y^2=1$. If the ellipse intersects the hyperbola at right angles, then square of length of the latus-rectum of the ellipse is _____.

माना एक दीर्घवृत्त $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ की उत्केन्द्रता अतिपरवलय $2x^2-2y^2=1$ की उत्केन्द्रता की व्युत्क्रम (reciprocal) है। यदि दीर्घवृत्त अतिपरवलय को लंबवत् काटता है, तो दीर्घवृत्त की नाभिलंब जीवा की लंबाई का वर्ग है _____

Ans. Official Answer NTA (2.00)

Sol. $e_H=\sqrt{2}, e_e=\frac{1}{\sqrt{2}}$

Focus of hyperbola $=(\pm 1,0)$

Both curves are confocal

$ae_e=1\Rightarrow a=\sqrt{2}$

$\frac{2b^2}{a}=2a\left(1-e_e^2\right)$

$=2\sqrt{2}.\frac{1}{2}=\sqrt{2}$

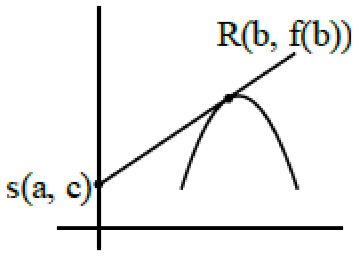
Question ID : 3666943045

29. Let a curve $y=f(x), x\in(0,\infty)$ pass through the points $P\left(1,\frac{3}{2}\right)$ and $Q\left(a,\frac{1}{2}\right)$. If the tangent at any point R(b, f(b)) to the given curve cuts the y-axis at the point S(0, c) such that bc = 3, then $(PQ)^2$ is equal to _____.

माना एक वक्र $y=f(x), x\in(0,\infty)$ बिंदुओं $P\left(1,\frac{3}{2}\right)$ तथा $Q\left(a,\frac{1}{2}\right)$ से होकर जाता है। यदि दिए गए वक्र के किसी भी बिंदु R(b, f(b)) पर स्पर्श रेखा y-अक्ष को बिंदु S(0, c) पर काटती है जबकि bc = 3 तो $(PQ)^2$ बराबर है _____

Ans. Official Answer NTA (5)

Sol.



Equation of tangent at R(b, f(2)) is

$y-f(b)=f'(b).(x-b)$

which passes through (0, c)

$\Rightarrow c-f(b)=f'(b)\cdot(-b)$

$$\Rightarrow \frac{3}{b} - f(b) = f'(b)\cdot(-b)$$

$$\Rightarrow bf'(b) - f(b) = -\frac{3}{b}$$

$$\Rightarrow \frac{bf'(b) - f(b)}{b^2} = -\frac{3}{b^3}$$

$$\Rightarrow d\left(\frac{f(b)}{b}\right) = -\frac{3}{b^3} \Rightarrow \frac{f(b)}{b} = \frac{3}{2b^2} + \lambda$$

Which passes through (1, 3/2)

$$\Rightarrow \frac{3}{2} = \frac{3}{2} + \lambda \Rightarrow \lambda = 0$$

$$\Rightarrow f(b) = \frac{3}{2b}$$

$$f(a) = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{3}{2b} \Rightarrow b = 3$$

$$\Rightarrow c = 1 \Rightarrow Q(3, 1/2)$$

$$\Rightarrow PQ^2 = 2^2 + (1)^2 = 5$$

Question ID : 3666943041

30. For $\alpha, \beta, z \in \mathbb{C}$ and $\lambda > 1$, If $\sqrt{\lambda - 1}$ is the radius of the circle $|z-\alpha|^2 + |z-\beta|^2 = 2\lambda$, then $|\alpha - \beta|$ is equal to _____.

$\alpha, \beta, z \in \mathbb{C}$ तथा $\lambda > 1$ के लिए, यदि वृत्त $|z-\alpha|^2 + |z-\beta|^2 = 2\lambda$ की त्रिज्या $\sqrt{\lambda - 1}$ है, तो $|\alpha - \beta|$ बराबर है _____

Ans. Official Answer NTA (2)

Sol. $|z - z_1|^2 + |z - z_2|^2 = |z_1 - z_2|^2$

$z_1 = \alpha, z_2 = \beta$

$|\alpha - \beta|^2 = 2\lambda$

$|\alpha - \beta| = \sqrt{2\lambda}$

$2r = \sqrt{2\lambda}$

$2\sqrt{\lambda - 1} = \sqrt{2\lambda}$

$\Rightarrow 4(\lambda - 1) = 2\lambda$

$\lambda = 2$

$|\alpha - \beta = 2$