

JEE Main April 2024
Question Paper With Text Solution
06 April | Shift-2

MATHEMATICS



JEE Main & Advanced | XI-XII Foundation | VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

**JEE MAIN APRIL 2024 | 06TH APRIL SHIFT-2****SECTION – A**

Question ID : 87827055893

1. Let ABC be an equilateral triangle. A new triangle is formed by joining the middle points of all sides of the triangle ABC and the same process is repeated infinitely many times. If P is the sum of perimeters and Q is the sum of areas of all the triangles formed in this process, then :

(1) $P = 36\sqrt{3}Q^2$ (2) $P^2 = 36\sqrt{3}Q$ (3) $P^2 = 72\sqrt{3}Q$ (4) $P^2 = 6\sqrt{3}Q$

Ans. Official answer NTA(2)**Sol.**

Question ID : 87827055881

2. If A is a square matrix of order 3 such that $\det(A) = 3$ and $\det\left(\text{adj}\left(-4\text{adj}\left(-3\text{adj}\left(3\text{adj}\left((2A)^{-1}\right)\right)\right)\right)\right) = 2^m 3^n$, then $m + 2n$ is equal to :

(1) 3 (2) 2 (3) 6 (4) 4

Ans. Official answer NTA(4)**Sol.**

Question ID : 87827055884

3. A software company sets up m number of computer systems to finish an assignment in 17 days. If 4 computer systems crashed on the start of the second day, 4 more computer systems crashed on the start of the third day and so on, then it took 8 more days to finish the assignment. The value of m is equal to :

(1) 150 (2) 125 (3) 180 (4) 160

Ans. Official answer NTA(1)**Sol.**

Question ID : 87827055883

4. Let $0 \leq r \leq n$. If ${}^{n+1}C_{r+1} : {}^n C_r : {}^{n-1}C_{r-1} = 55 : 35 : 21$, then $2n + 5r$ is equal to :

(1) 55 (2) 60 (3) 62 (4) 50

MATRIX JEE ACADEMY**Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911****Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in**

**Ans.** Official answer NTA (4)**Sol.**

Question ID : 87827055880

5. If z_1, z_2 are two distinct complex number such that $\left| \frac{z_1 - 2z_2}{\frac{1}{2} - z_1 \bar{z}_2} \right| = 2$, then :

- (1) both z_1 and z_2 lie on the same circle
 (2) z_1 lies on a circle of radius $\frac{1}{2}$ and z_2 lies on a circle of radius 1
 (3) either z_1 lies on a circle of radius 1 or z_2 lies on a circle of radius $\frac{1}{2}$
 (4) either z_1 lies on a circle of radius $\frac{1}{2}$ or z_2 lies on a circle of radius 1

Ans. Official answer NTA (3)**Sol.**

Question ID : 87827055885

6. If the function $f(x) = \left(\frac{1}{x}\right)^{2x}$; $x > 0$ attains the maximum value at $x = \frac{1}{e}$ then :

- (1) $e^{2\pi} < (2\pi)^e$ (2) $e^\pi > \pi^e$ (3) $e^\pi < \pi^e$ (4) $(2e)^\pi > \pi^{(2e)}$

Ans. Official answer NTA (2)**Sol.**

Question ID : 87827055896

7. Let $\vec{a} = 6\hat{i} + \hat{j} - \hat{k}$ and $\vec{b} = \hat{i} + \hat{j}$. If \vec{c} is a vector such that $|\vec{c}| \geq 6, \vec{a} \cdot \vec{c} = 6|\vec{c}|, |\vec{c} - \vec{a}| = 2\sqrt{2}$ and the angle between $\vec{a} \times \vec{b}$ and \vec{c} is 60° , then $\left| (\vec{a} \times \vec{b}) \times \vec{c} \right|$ is equal to :

- (1) $\frac{9}{2}(6 - \sqrt{6})$ (2) $\frac{9}{2}(6 + \sqrt{6})$ (3) $\frac{3}{2}\sqrt{3}$ (4) $\frac{3}{2}\sqrt{6}$

Ans. Official answer NTA (2)

**Sol.**

Question ID : 87827055895

8. Let $\vec{a} = 2\hat{i} + \hat{j} - \hat{k}$, $\vec{b} = \left((\vec{a} \times (\hat{i} + \hat{j})) \times \hat{i} \right) \times \hat{i}$. Then the square of the projection of \vec{a} on \vec{b} is :
- (1) $\frac{2}{3}$ (2) $\frac{1}{5}$ (3) $\frac{1}{3}$ (4) 2

Ans. Official answer NTA(4)**Sol.**

Question ID : 87827055897

9. If three letters can be posted to any one of the 5 different addresses, then the probability that the three letters are posted to exactly two addresses is :
- (1) $\frac{6}{25}$ (2) $\frac{18}{25}$ (3) $\frac{4}{25}$ (4) $\frac{12}{25}$

Ans. Official answer NTA(4)**Sol.**

Question ID : 87827055887

10. Suppose for a differentiable function h , $h(0)=0$, $h(1)=1$ and $h'(0) = h'(1) = 2$. If $g(x) = h(e^x)e^{h(x)}$, then $g'(0)$ is equal to :
- (1) 3 (2) 8 (3) 4 (4) 5

Ans.**Ans.** Official answer NTA(3)

Question ID : 87827055892

11. If $P(6, 1)$ be the orthocentre of the triangle whose vertices are $A(5, -2)$, $B(8, 3)$ and $C(h, k)$, then the point C lies on the circle :
- (1) $x^2 + y^2 - 52 = 0$ (2) $x^2 + y^2 - 61 = 0$ (3) $x^2 + y^2 - 74 = 0$ (4) $x^2 + y^2 - 65 = 0$

Ans. Official answer NTA(4)**MATRIX JEE ACADEMY****Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911****Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in**

**Sol.**

Question ID : 87827055888

12. If the area of the region $\left\{ (x, y) : \frac{a}{x^2} \leq y \leq \frac{1}{x}, 1 \leq x \leq 2, 0 < a < 1 \right\}$ is $(\log_e 2) - \frac{1}{7}$ then the value of $7a - 3$ is equal to :

- (1) 1 (2) 2 (3) 0 (4) -1

Ans. Official answer NTA (4)**Sol.**

Question ID : 87827055882

13. If all the words with or without meaning made using all the letters of the word "NAGPUR" are arranged as in a dictionary, then the word at 315th position in this arrangement is :

- (1) NRAGUP (2) NRAPUG (3) NRAGPU (4) NRAPGU

Ans. Official answer NTA (4)**Sol.**

Question ID : 87827055891

14. If the locus of the point, whose distances from the point (2, 1) and (1, 3) are in the ratio 5 : 4, is $ax^2 + by^2 + cxy + dx + ey + 170 = 0$, then the value of $a^2 + 2b + 3c + 4d + e$ is equal to :

- (1) -27 (2) 37 (3) 5 (4) 437

Ans. Official answer NTA (2)**Sol.**

Question ID : 87827055890

15. Suppose the solution of the differential equation $\frac{dy}{dx} = \frac{(2 + \alpha)x - \beta y + 2}{\beta x - 2\alpha y - (\beta\gamma - 4\alpha)}$ represents a circle passing through origin. Then the radius of this circle is :

- (1) $\frac{\sqrt{17}}{2}$ (2) 2 (3) $\sqrt{17}$ (4) $\frac{1}{2}$

MATRIX JEE ACADEMY**Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911****Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in**



Ans. Official answer NTA(1)

Sol.

Question ID : 87827055886

16. $\lim_{n \rightarrow \infty} \frac{(1^2 - 1)(n - 1) + (2^2 - 2)(n - 2) + \dots + ((n - 1)^2 - (n - 1)) \cdot 1}{(1^3 + 2^3 + \dots + n^3) - (1^2 + 2^2 + \dots + n^2)}$ is equal to :

(1) $\frac{2}{3}$

(2) $\frac{3}{4}$

(3) $\frac{1}{2}$

(4) $\frac{1}{3}$

Ans. Official answer NTA(4)

Sol.

Question ID : 87827055894

17. Let $P(\alpha, \beta, \gamma)$ be the image of the point $Q(3, -3, 1)$ in the line $\frac{x-0}{1} = \frac{y-3}{1} = \frac{z-1}{-1}$ and R be the point $(2, 5, -1)$. If the area of the triangle PQR is λ and $\lambda^2 = 14K$, then K is equal to :

(1) 18

(2) 36

(3) 72

(4) 81

Ans. Official answer NTA(4)

Sol.

Question ID : 87827055878

18. Let $f(x) = \frac{1}{7 - \sin 5x}$ be a function defined on R. Then the range of the function $f(x)$ is equal to :

(1) $\left[\frac{1}{8}, \frac{1}{6}\right]$

(2) $\left[\frac{1}{7}, \frac{1}{5}\right]$

(3) $\left[\frac{1}{8}, \frac{1}{5}\right]$

(4) $\left[\frac{1}{7}, \frac{1}{6}\right]$

Ans. Official answer NTA(1)

Sol.



Question ID : 87827055879

19. Let $A = \{1, 2, 3, 4, 5\}$. Let R be a relation on A defined by xRy if and only if $4x \leq 5y$. Let m be the number of elements in R and n be the minimum number of elements from $A \times A$ that are required to be added to R to make it a symmetric relation. Then $m + n$ is equal to :

- (1) 26 (2) 25 (3) 23 (4) 24

Ans. Official answer NTA (2)**Sol.**

Question ID : 87827055889

20. If $\int \frac{1}{a^2 \sin^2 x + b^2 \cos^2 x} dx = \frac{1}{12} \tan^{-1}(3 \tan x) + \text{constant}$, then the maximum value of $a \sin x + b \cos x$, is :

- (1) $\sqrt{40}$ (2) $\sqrt{42}$ (3) $\sqrt{39}$ (4) $\sqrt{41}$

Ans. Official answer NTA (1)**Sol.****SECTION - B**

Question ID : 87827055899

21. If the system of equations

$$2x + 7y + \lambda z = 3$$

$$3x + 2y + 5z = 4$$

$$x + \mu y + 32z = -1$$

has infinitely many solutions, then $(\lambda - \mu)$ is equal to _____.

Ans. Official answer NTA (38)**Sol.**

Question ID : 87827055902

22. Let $[t]$ denote the largest integer less than or equal to t . If

$$\int_0^3 \left([x^2] + \left[\frac{x^2}{2} \right] \right) dx = a + b\sqrt{2} - \sqrt{3} - \sqrt{5} + c\sqrt{6} - \sqrt{7}, \text{ where } a, b, c \in \mathbb{Z}, \text{ then } a + b + c \text{ is equal to}$$

_____.

Ans. Official answer NTA (23)**MATRIX JEE ACADEMY**

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911

Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

**Sol.**

Question ID : 87827055901

23. Let $[t]$ denote the greatest integer less than or equal to t . Let $f : [0, \infty) \rightarrow \mathbb{R}$ be a function defined by $f(x) = \left[\frac{x}{2} + 3 \right] - [\sqrt{x}]$. Let S be the set of all points in the interval $[0, 8]$ at which f is not continuous. Then $\sum_{a \in S} a$ is equal to _____.**Ans.** Official answer NTA(17)**Sol.**

Question ID : 87827055903

24. If the solution $y(x)$ of the given differential equation $(e^y + 1) \cos x dx + e^y \sin x dy = 0$ passes through thepoint $\left(\frac{\pi}{2}, 0\right)$, then the value of $e^{y\left(\frac{\pi}{6}\right)}$ is equal to _____.**Ans.** Official answer NTA(3)**Sol.**

Question ID : 87827055907

25. In a triangle ABC , $BC = 7$, $AC = 8$, $AB = \alpha \in \mathbb{N}$ and $\cos A = \frac{2}{3}$. If $49 \cos(3C) + 42 = \frac{m}{n}$, where $\gcd(m, n) = 1$, then $m + n$ is equal to _____.**Ans.** Official answer NTA(39)**Sol.**

Question ID : 87827055900

26. If $S(x) = (1+x) + 2(1+x)^2 + 3(1+x)^3 + \dots + 60(1+x)^{60}$, $x \neq 0$, and $(60)^2 S(60) = a(b)^b + b$, where $a, b \in \mathbb{N}$, then $(a+b)$ equal to _____.**Ans.** Official answer NTA(3660)**Sol.**



Question ID : 87827055906

27. From a lot of 12 items containing 3 defectives, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Let items in the sample be drawn one by one without replacement. If variance of X is $\frac{m}{n}$, where $\gcd(m, n) = 1$, then $n - m$ is equal to _____.

Ans. Official answer NTA (71)**Sol.**

Question ID : 87827055905

28. If the shortest distance between the lines $\frac{x-\lambda}{3} = \frac{y-2}{-1} = \frac{z-1}{1}$ and $\frac{x+2}{-3} = \frac{y+5}{2} = \frac{z-4}{4}$ is $\frac{44}{\sqrt{30}}$, then the largest possible value of $|\lambda|$ is equal to _____.

Ans. Official answer NTA (43)**Sol.**

Question ID : 87827055898

29. Let α, β be roots of $x^2 + \sqrt{2}x - 8 = 0$. If $U_n = \alpha^n + \beta^n$, then $\frac{U_{10} + \sqrt{2}U_9}{2U_8}$ is equal to _____.

Ans. Official answer NTA (4)**Sol.**

Question ID : 87827055904

30. The length of the latus rectum and directrices of a hyperbola with eccentricity e are 9 and $x = \pm \frac{4}{\sqrt{3}}$, respectively. Let the line $y - \sqrt{3}x + \sqrt{3} = 0$ touch this hyperbola at (x_0, y_0) . If m is the product of the focal distances of the point (x_0, y_0) , then $4e^2 + m$ is equal to _____.

Ans. Official answer NTA (61)**Sol.**