JEE Main April 2024

Question Paper With Text Solution

06 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2024 | 06TH APRIL SHIFT-1

SECTION – A

Question ID : 68019114079

1. Let C be the circle of minimum area touching the parabola $y = 6 - x^2$ and the lines $y = \sqrt{3}|x|$. Then, which one of the following points lies on the circle C ?

(1) (1, 2) (2) (2, 2) (3) (1, 1) (4) (2, 4)

Ans. Official answer NTA (4)

Sol.

Question ID : 68019114080

2. The shortest distance between the lines $\frac{x-3}{2} = \frac{y+15}{-7} = \frac{z-9}{5}$ and $\frac{x+1}{2} = \frac{y-1}{1} = \frac{z-9}{-3}$ is :

(1) $8\sqrt{3}$ (2) $6\sqrt{3}$ (3) $4\sqrt{3}$ (4) $5\sqrt{3}$

Ans. Official answer NTA (3)

Sol.

Question ID : 68019114082

3. If $A(3,1,-1), B\left(\frac{5}{3},\frac{7}{3},\frac{1}{3}\right)$, C(2, 2, 1) and $D\left(\frac{10}{3},\frac{2}{3},\frac{-1}{3}\right)$ are the vertices of a quadrilateral ABCD, then its area is :

(1) $\frac{2\sqrt{2}}{3}$ (2) $\frac{5\sqrt{2}}{3}$ (3) $2\sqrt{2}$ (4) $\frac{4\sqrt{2}}{3}$

Ans. Official answer NTA (4)

Sol.

Question ID : 68019114073

4. Let a variable line of slope $m > 0$ passing through the point (4, –9) intersect the coordinate axes at the points A and B. The minimum value of the sum of the distances of A and B from the origin is :

(1) 30 (2) 25 (3) 15 (4) 10

Ans. Official answer NTA (2)

Sol.

Question ID : 68019114072

5. The interval in which the function $f(x) = x^x, x > 0$, is strictly increasing is :

(1) $\left[\frac{1}{e^2}, 1\right)$ (2) $\left(0, \frac{1}{e}\right]$ (3) $\left[\frac{1}{e}, \infty\right)$ (4) $(0, \infty)$

Ans. Official answer NTA (3)

Sol.

Question ID : 68019114077

6. Let $y = y(x)$ be the solution of the differential equation $\left(2x \log_e x\right)\frac{dy}{dx} + 2y = \frac{3}{x}\log_e x, x > 0$ and $y\left(e^{-1}\right) = 0$.

Then, y(e) is equal to :

(1) $-\frac{3}{2e}$ (2) $-\frac{3}{e}$ (3) $-\frac{2}{e}$ (4) $-\frac{2}{3e}$

Ans. Official answer NTA (2)

Sol.

Question ID : 68019114081

7. The mean and standard deviation of 20 observations are found to be 10 and 2, respectively. On rechecking, it was found that an observation by mistake was taken 8 instead of 12. The correct standard deviation is :

(1) 1.8 (2) $\sqrt{3.96}$ (3) $\sqrt{3.86}$ (4) 1.94

Ans. Official answer NTA (2)

Sol.

Question ID : 68019114069

8. Let $f:(-\infty,\infty)-\{0\}\to\mathbb{R}$ be a differentiable function such that $f'(1)=\lim\limits_{a\to\infty}a^2f\left(\frac{1}{a}\right)$. Then $\lim\limits_{a\to\infty}\frac{a(a+1)}{2}\tan^{-1}\left(\frac{1}{a}\right)+a^2-2\log_e a$ is equal to :

(1) $\frac{5}{2}+\frac{\pi}{8}$ (2) $\frac{3}{4}+\frac{\pi}{8}$ (3) $\frac{3}{8}+\frac{\pi}{4}$ (4) $\frac{3}{2}+\frac{\pi}{4}$

Ans. Official answer NTA (1)

Sol.

Question ID : 68019114071

9. If $f(x)=\begin{cases}x^3\sin\left(\frac{1}{x}\right), & X\neq 0\\ 0, & X=0\end{cases}$ then :

(1) $f''\left(\frac{2}{\pi}\right)=\frac{24-\pi^2}{2\pi}$ (2) $f''\left(\frac{2}{\pi}\right)=\frac{12-\pi^2}{2\pi}$ (3) $f''(0)=1$ (4) $f''(0)=0$

Ans. Official answer NTA (1)

Sol.

Question ID : 68019114066

10. The number of triangles whose vertices are at the vertices of a regular octagon but none of whose sides is a side of the octagon is :

(1) 56 (2) 16 (3) 48 (4) 24

Ans.

Ans. Official answer NTA (2)

Question ID : 68019114075

11. Let the area of the region enclosed by the curves $y=3x$, $2y=27-3x$ and $y=3x-x\sqrt{x}$ be A. Then 10A is equal to :

(1) 184 (2) 172 (3) 154 (4) 162

Ans. Official answer NTA (4)

Sol.

Question ID : 68019114075

12. Let α, β be the distinct roots of the equation $x^2-\left(t^2-5t+6\right)x+1=0, t \in \mathbb{R}$ and $a_n=\alpha^n+\beta^n$. Then the minimum value of $\frac{a_{2023}+a_{2025}}{a_{2024}}$ is :

(1) $-1/2$ (2) $1/2$ (3) $1/4$ (4) $-1/4$

Ans. Official answer NTA (4)

Sol.

Question ID : 68019114064

13. Let the relations R_1 and R_2 on the set $X=\{1,2,3,\ldots,20\}$ be given by $R_1=\{(x, y): 2x-3y=2\}$ and $R_2=\{(x, y): -5x+4y=0\}$. If M and N be the minimum number of elements required to be added in R_1 and R_2, respectively, in order to make the relations symmetric, then M + N equals :

(1) 8 (2) 16 (3) 10 (4) 12

Ans. Official answer NTA (3)

Sol.

Question ID : 68019114083

14. A company has two plants A and B to manufacture motorcycles. 60% motorcycles are manufactured at plant A and the remaining are manufactured at plant B .80% of the motorcycles manufactured at plant A are rated of the standard quality, while 90% of the motorcycles manufactured at plant B are rated of the standard quality. A motorcycle picked up randomly from the total production is found to be of the standard quality. If p is the probability that it was manufactured at plant B, then 126p is :

(1) 66 (2) 54 (3) 56 (4) 64

Ans. Official answer NTA (2)

Sol.

Question ID : 68019114074

15. $\int_0^{\pi/4} \frac{\cos^2 x \sin^2 x}{\left(\cos^3 x + \sin^3 x\right)^2} dx$ is equal to :

(1) 1/9 (2) 1/6 (3) 1/3 (4) 1/12

Ans. Official answer NTA (2)

Sol.

Question ID : 68019114070

16. Let $A = \{n \in [100, 700] \cap \mathbb{N} : n$ is neither a multiple of 3 nor a multiple of 4$\}$. Then the number of elements in A is :

(1) 310 (2) 300 (3) 280 (4) 290

Ans. Official answer NTA (2)

Sol.

Question ID : 68019114078

17. A circle is inscribed in an equilateral triangle of side of length 12. If the area and perimeter of any square inscribed in this circle are m and n, respectively, then $m + n^2$ is equal to :

(1) 396 (2) 408 (3) 312 (4) 414

Ans. Official answer NTA (2)

Sol.

Question ID : 68019114065

18. The function $f(x) = \frac{x^2 + 2x - 15}{x^2 - 4x + 9}, x \in \mathbb{R}$ is :

(1) both one-one and onto (2) onto but not one-one

(3) neither one-one nor onto (4) one-one but not onto

Ans. Official answer NTA (3)

Sol.

Question ID : 68019114068

19. For $\alpha, \beta \in R$ and a natural number n, let $A_r = \begin{vmatrix} r & 1 & \frac{n^2}{2}+\alpha \\ 2r & 2 & n^2-\beta \\ 3r-2 & 3 & \frac{n(3n-1)}{2} \end{vmatrix}$. Then $2A_{10}-A_8$ is :

(1) $4\alpha + 2\beta$ (2) $2n$ (3) 0 (4) $2\alpha + 4\beta$

Ans. Official answer NTA (1)

Sol.

Question ID : 68019114076

20. Let $y = y(x)$ be the solution of the differential equation $\left(1+x^2\right)\frac{dy}{dx}+y = e^{\tan^{-1}x}$, $y(1)=0$. Then $y(0)$ is :

(1) $\frac{1}{2}\left(1-e^{\pi/2}\right)$ (2) $\frac{1}{4}\left(e^{\pi/2}-1\right)$ (3) $\frac{1}{4}\left(1-e^{\pi/2}\right)$ (4) $\frac{1}{2}\left(e^{\pi/2}-1\right)$

Ans. Official answer NTA (1)

Sol.

SECTION – B

Question ID : 68019114093

21. For $n \in N$, if $\cot^{-1}3+\cot^{-1}4+\cot^{-1}5+\cot^{-1}n = \frac{\pi}{4}$, then n is equal to _____.

Ans. Official answer NTA (47)

Sol.

Question ID : 68019114084

22. Let x_1, x_2, x_3, x_4 be the solution of the equation $4x^4+8x^3-17x^2-12x+9=0$ and $\left(4+x_1^2\right)\left(4+x_2^2\right)\left(4+x_3^2\right)\left(4+x_4^2\right) = \frac{125}{16}m$. Then the value of m is _____.

Ans. Official answer NTA (221)

Sol.

Question ID : 68019114092

23. Let $\vec{a}=2\hat{i}-3\hat{j}+4\hat{k}, \vec{b}=3\hat{i}+4\hat{j}-5\hat{k}$ and a vector $\vec{c}$ be such that $\vec{a}\times(\vec{b}+\vec{c})+\vec{b}\times\vec{c}=\hat{i}+8\hat{j}+13\hat{k}$. If $\vec{a}\cdot\vec{c}=13$, then $\left(24-\vec{b}\cdot\vec{c}\right)$ is equal to _____.

Ans. Official answer NTA (46)

Sol.

Question ID : 68019114086

24. If the second, third and fourth terms in the expansion of $(x+y)^n$ are 135, 30 and $\frac{10}{3}$, respectively, then $6\left(n^3+x^2+y\right)$ is equal to _____.

Ans. Official answer NTA (806)

Sol.

Question ID : 68019114087

25. Let the first term of a series be $T_1=6$ and its r^{th} term $T_r=3T_{r-1}+6^r, r=2,3,.....,n$. If the sum of the first n terms of this series is $\frac{1}{5}\left(n^2-12n+39\right)\left(4\cdot6^n-5\cdot3^n+1\right)$, then n is equal to _____.

Ans. Official answer NTA (6)

Sol.

Question ID : 68019114091

26. Let P be the point (10, –2, –1) and Q be the foot of the perpendicular drawn from the point R(1, 7, 6) on the line passing through the points (2, –5, 11) and (–6, 7, –5). Then the length of the line segment PQ is equal to _____.

Ans. Official answer NTA (13)

Sol.

Question ID : 68019114090

27. Let a conic C pass through the point (4, –2) and P(x, y), $x \geq 3$, be any point on C. Let the slope of the line touching the conic C only at a single point P be half the slope of the line joining the points P and (3, –5). If the focal distance of the point (7, 1) on C is d, then 12d equals _____.

Ans. Official answer NTA (75)

Sol.

Question ID : 68019114085

28. Let $\alpha\beta\gamma = 45$; $\alpha, \beta, \gamma \in R$. If $x(\alpha, 1, 2) + y(1, \alpha, 2) + z(2, 3, \gamma) = (0, 0, 0)$ for some $x, y, z \in R$, $x\,y\,z \neq 0$, then $6\alpha + 4\beta + \gamma$ is equal to _____.

Ans. Official answer NTA (55)

Sol.

Question ID : 68019114088

29. Let $r_k = \dfrac{\int_0^1 \left(1-x^7\right)^k dx}{\int_0^1 \left(1-x^7\right)^{k+1} dx}$, $k \in \mathbb{N}$. Then the value of $\sum_{k=1}^{10} \dfrac{1}{7\left(r_k - 1\right)}$ is equal to _____.

Ans. Official answer NTA (65)

Sol.

Question ID : 68019114089

30. Let L_1, L_2 be the lines passing through the point P(0, 1) and touching the parabola $9x^2 + 12x + 18y - 14 = 0$. Let Q and R be the points on the lines L_1 and L_2 such that the ΔPQR is an isosceles triangle with base QR. If the slopes of the lines QR are m_1 and m_2, then $16\left(m_1^2 + m_2^2\right)$ is equal to _____.

Ans. Official answer NTA (68)

Sol.