

JEE Main April 2024
Question Paper With Text Solution
05 April | Shift-II

MATHEMATICS



JEE Main & Advanced | XI-XII Foundation | VI-X Pre-Foundation

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**JEE MAIN APRIL 2024 | 05TH APRIL SHIFT-II****SECTION - A**

Question ID : 87827055703

1. 60 words can be made using all the letters of the word BHBJO, with or without meaning. If these words are written as in a dictionary, then the 50th word is :

- (1) JBBOH (2) OBBJH (3) OBBHJ (4) HBBJO

Ans. Official answer NTA(2)

Sol.

Question ID : 87827055709

2. The area enclosed between the curves $y = x|x|$ and $y = x - |x|$ is :

- (1) $\frac{2}{3}$ (2) $\frac{4}{3}$ (3) $\frac{8}{3}$ (4) 1

Ans. Official answer NTA(2)

Sol.

Question ID : 87827055716

3. Consider three vectors $\vec{a}, \vec{b}, \vec{c}$. Let $|\vec{a}| = 2, |\vec{b}| = 3$ and $\vec{a} = \vec{b} \times \vec{c}$. If $\alpha \in \left[0, \frac{\pi}{3}\right]$ is the angle between the vectors \vec{b} and \vec{c} , then the minimum value of $27|\vec{c} - \vec{a}|^2$ is equal to :

- (1) 105 (2) 124 (3) 121 (4) 110

Ans. Official answer NTA(2)

Sol.

Question ID : 87827055705

4. For $x \geq 0$, the least value of K, for which $4^{1+x} + 4^{1-x}, \frac{K}{2}, 16^x + 16^{-x}$ are three consecutive terms of an A.P., is equal to :

- (1) 10 (2) 4 (3) 16 (4) 8

Ans. Official answer NTA(1)

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**Sol.**

Question ID : 87827055710

5. The differential equation of the family of circles passing through the origin and having center at the line $y = x$ is :

- (1) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 - 2xy)dy$ (2) $(x^2 - y^2 + 2xy)dx = (x^2 - y^2 + 2xy)dy$
 (3) $(x^2 + y^2 - 2xy)dx = (x^2 + y^2 + 2xy)dy$ (4) $(x^2 + y^2 + 2xy)dx = (x^2 + y^2 - 2xy)dy$

Ans. Official answer NTA(1)**Sol.**

Question ID : 87827055717

6. If $y(\theta) = \frac{2 \cos \theta + \cos 2\theta}{\cos 3\theta + 4 \cos \theta + 5 \cos \theta + 2}$, then at $\theta = \frac{\pi}{2}$, $y'' + y' + y$ is equal to :

- (1) 2 (2) 1/2 (3) 1 (4) 3/2

Ans. Official answer NTA(1)**Sol.**

Question ID : 87827055702

7. Let $\alpha\beta \neq 0$ and $A = \begin{bmatrix} \beta & \alpha & 3 \\ \alpha & \alpha & \beta \\ -\beta & \alpha & 2\alpha \end{bmatrix}$. If $B = \begin{bmatrix} 3\alpha & -9 & 3\alpha \\ -\alpha & 7 & -2\alpha \\ -2\alpha & 5 & -2\beta \end{bmatrix}$ is the matrix of cofactors of the elements of

A, then $\det(AB)$ is equal to :

- (1) 125 (2) 64 (3) 343 (4) 216

Ans. Official answer NTA(4)**Sol.**



Question ID : 87827055713

11. Let A(-1, 1) and B(2, 3) be two points and P be a variable point above the line AB such that the area of ΔPAB is 10. If the locus of P is $ax + by = 15$, then $5a + 2b$ is :

- (1) 6 (2) 4 (3) $-\frac{6}{5}$ (4) $-\frac{12}{5}$

Ans. Official answer NTA(4)

Sol.

Question ID : 87827055714

12. Let (α, β, γ) be the image of the point (8, 5, 7) in the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-2}{5}$. Then $\alpha + \beta + \gamma$ is

equal to :

- (1) 14 (2) 20 (3) 18 (4) 16

Ans. Official answer NTA(1)

Sol.

Question ID : 87827055707

13. Let $f: [-1, 2] \rightarrow \mathbb{R}$ be given by $f(x) = 2x^2 + x + [x^2] - [x]$, where $[t]$ denotes the greatest integer less than or equal to t . The number of points, where f is not continuous is :

- (1) 3 (2) 5 (3) 6 (4) 4

Ans. Official answer NTA(4)

Sol.

Question ID : 87827055712

14. Let the circle $C_1: x^2 + y^2 - 2(x + y) + 1 = 0$ and C_2 be a circle having centre at $(-1, 0)$ and radius 2. If the line of common chord of C_1 and C_2 intersects the y-axis at the point P, then the square of the distance of P from the centre of C_1 is :

- (1) 6 (2) 4 (3) 2 (4) 1

Ans. Official answer NTA(3)

Sol.

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Question ID : 87827055701

15. The values of m, n , for which the system of equations

$$x + y + z = 4,$$

$$2x + 5y + 5z = 17,$$

$$x + 2y + mz = n$$

has infinitely many solutions, satisfy the equation :

(1) $m^2 + n^2 - m - n = 46$

(2) $m^2 + n^2 - mn = 39$

(3) $m^2 + n^2 + m + n = 64$

(4) $m^2 + n^2 - m n = 68$

Ans. Official answer NTA(2)**Sol.**

Question ID : 87827055704

16. The coefficients a, b, c in the quadratic equation $ax^2 + bx + c = 0$ are from the set $\{1, 2, 3, 4, 5, 6\}$. If the probability of this equation having one real root bigger than the other is p , then $216p$ equals :

(1) 76

(2) 57

(3) 19

(4) 38

Ans. Official answer NTA(4)**Sol.**

Question ID : 87827055698

17. Let the set $S = \{2, 4, 8, 16, \dots, 512\}$ be partitioned into 3 sets A, B, C with equal number of elements such that $A \cup B \cup C = S$ and $A \cap B = B \cap C = A \cap C = \phi$. The maximum number of such possible partitions of S is equal to :

(1) 1640

(2) 1680

(3) 1520

(4) 1710

Ans. Official answer NTA(2)**Sol.**



Question ID : 87827055700

18. Let $S_1 = \{z \in \mathbb{C} : |z| \leq 5\}$, $S_2 = \left\{z \in \mathbb{C} : \operatorname{Im} \left(\frac{z+1-\sqrt{3}i}{1-\sqrt{3}i} \right) \geq 0 \right\}$ and $S_3 = \{z \in \mathbb{C} : \operatorname{Re}(z) \geq 0\}$. Then the area of

the region $S_1 \cap S_2 \cap S_3$ is :

- (1) $\frac{125\pi}{24}$ (2) $\frac{125\pi}{4}$ (3) $\frac{125\pi}{12}$ (4) $\frac{125\pi}{6}$

Ans. Official answer NTA(3)

Sol.

Question ID : 87827055706

19. If the constant term in the expansion of $\left(\frac{\sqrt[5]{3}}{x} + \frac{2x}{\sqrt[3]{5}} \right)^{12}$, $x \neq 0$ is $\alpha \times 2^8 \times \sqrt[5]{3}$, then 25α is equal to :

- (1) 724 (2) 742 (3) 693 (4) 639

Ans. Official answer NTA(3)

Sol.

Question ID : 87827055708

20. Let $\beta(m, n) = \int_0^1 x^{m-1} (1-x)^{n-1} dx$, $m, n > 0$. If $\int_0^1 (1-x^{10})^{20} dx = a \times \beta(b, c)$, then $100(a + b + c)$ equals :

- (1) 1021 (2) 2120 (3) 2012 (4) 1120

Ans. Official answer NTA(2)

Sol.

**SECTION - B**

Question ID : 87827055722

21. If $f(t) = \int_0^{\pi} \frac{2x \, dx}{1 - \cos^2 t \sin^2 x}$, $0 < t < \pi$, then the value of $\int_0^{\pi/2} \frac{\pi^2 dt}{f(t)}$ equals _____.

Ans. Official answer NTA(1)**Sol.**

Question ID : 87827055725

22. Let the point $(-1, \alpha, \beta)$ lie on the line of the shortest distance between the lines $\frac{x+2}{-3} = \frac{y-2}{4} = \frac{z-5}{4} = \frac{z-5}{2}$ and $\frac{x+2}{-1} = \frac{y+6}{2} = \frac{z-1}{0}$. Then $(\alpha - \beta)^2$ is equal to _____.

Ans. Official answer NTA(25)**Sol.**

Question ID : 87827055719

23. If $1 + \frac{\sqrt{3} - \sqrt{2}}{2\sqrt{3}} + \frac{5 - 2\sqrt{6}}{18} + \frac{9\sqrt{3} - 11\sqrt{2}}{36\sqrt{3}} + \frac{49 - 20\sqrt{6}}{180} + \dots$ upto $\infty = 2 + \left(\sqrt{\frac{b}{a}} + 1\right) \log_e \left(\frac{a}{b}\right)$, where a and b are integers with $\gcd(a, b) = 1$, then $11a + 18b$ is equal to _____.

Ans. Official answer NTA(76)**Sol.**

Question ID : 87827055724

24. Let a line perpendicular to the line $2x - y = 10$ touch the parabola $y^2 = 4(x - 9)$ at the point P. The distance of the point P from the centre of the circle $x^2 + y^2 - 14x - 8y + 56 = 0$ is _____.

Ans. Official answer NTA(10)**Sol.**



Question ID : 87827055727

25. The number of real solutions of the equation $x|x+5|+2|x+7|-2=0$ is _____.**Ans.** Official answer NTA (3)**Sol.**

Question ID : 87827055721

26. Let maximum and minimum values of $(\sqrt{8x-x^2-12}-4)^2+(x-7)^2$, $x \in \mathbb{R}$ be M and m , respectively. Then M^2-m^2 is equal to _____.**Ans.** Official answer NTA (1600)**Sol.**

Question ID : 87827055720

27. Let $a > 0$ be a root of the equation $2x^2+x-2=0$. If $\lim_{x \rightarrow \frac{1}{a}} \frac{16(1-\cos(2+x-2x^2))}{(1-ax)^2} = \alpha + \beta\sqrt{17}$ where $\alpha, \beta \in \mathbb{Z}$, then $\alpha + \beta$ is equal to _____.**Ans.** Official answer NTA (170)**Sol.**

Question ID : 87827055726

28. Let the mean and the standard deviation of the probability distribution

X	α	1	0	-3
P(X)	$\frac{1}{3}$	K	$\frac{1}{6}$	$\frac{1}{4}$

be μ and σ , respectively. If $\sigma - \mu = 2$, then $\sigma + \mu$ is equal to _____.**Ans.** Official answer NTA (5)**Sol.**



Question ID : 87827055718

29. The number of solutions of $\sin^2 x + (2 + 2x - x^2) \sin x - 3(x - 1)^2 = 0$, where $-\pi \leq x \leq \pi$, is _____.**Ans.** Official answer NTA (2)**Sol.**

Question ID : 87827055723

30. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx} + \frac{2x}{(1+x^2)^2} y = x e^{\frac{1}{1+x^2}}$; $y(0) = 0$.Then the area enclosed by the curve $f(x) = y(x) e^{-\frac{1}{1+x^2}}$ and the line $y - x = 4$ is _____.**Ans.** Official answer NTA (18)**Sol.**