JEE Main April 2024

Question Paper With Text Solution

05 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2024 | 05TH APRIL SHIFT-1

SECTION – A

Question ID: 87827055610

1. Let A and B be two square matrices of order 3 such that |A| = 3 and |B| = 2. Then $\left|A^TA\left(\text{adj}(2A)\right)^{-1}\left(\text{adj}(4B)\right)\left(\text{adj}(AB)\right)^{-1}AA^T\right|$ is equal to :

(1) 81 (2) 108 (3) 32 (4) 64

Ans. Official answer NTA (4)

Sol.

Question ID : 87827055623

2. If the line $\frac{2-x}{3}=\frac{3y-2}{4\lambda+1}=4-z$ makes a right angle with the line $\frac{x+3}{3\mu}=\frac{1-2y}{6}=\frac{5-z}{7}$ then $4\lambda+9\mu$ is equal to :

(1) 6 (2) 12 (3) 4 (4) 5

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055618

3. The integral $\int_0^{\frac{\pi}{4}}\frac{136\sin x}{3\sin x+5\cos x}dx$ is equal to :

(1) $3\pi-30\log_e 2+20\log_e 5$ (2) $3\pi-50\log_e 2+20\log_e 5$

(3) $3\pi-10\log_e\left(2\sqrt{2}\right)+10\log_e 5$ (4) $3\pi-25\log_e 2+10\log_e 5$

Ans. Official answer NTA (2)

Sol.

Question ID : 87827055615

4. Let $f(x)=x^5+2x^3+3x+1, x\in\mathbf{R}$, and g(x) be a function such that $g(f(x))=x$ for all $x\in\mathbf{R}$. the $\frac{g(7)}{g'(7)}$ is equal to :

(1) 42 (2) 7 (3) 14 (4) 1

Ans. Official answer NTA (3)

Sol.

Question ID : 87827055611

5. If the system of equations

$11x + y + \lambda z = -5$

$2x + 3y + 5z = 3$

$8x - 19y - 39z = \mu$

has infinitely many solutions, then $\lambda^4 - \mu$ is equal to :

(1) 47 (2) 49 (3) 51 (4) 45

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055625

6. If A (1, –1, 2), B(5, 7, –6), C(3, 4 –10) and D(–1, –4, –2) are the vertices of quadrilateral ABCD, then its are is :

(1) $24\sqrt{29}$ (2) $12\sqrt{29}$ (3) $48\sqrt{7}$ (4) $24\sqrt{7}$

Ans. Official answer NTA (2)

Sol.

Question ID : 87827055627

7. Suppose $\theta \in \left[0, \frac{\pi}{4}\right]$ is a solutions of $4\cos\theta - 3\sin\theta = 1$. Then $\cos\theta$ is equal to :

(1) $\frac{4}{\left(3\sqrt{6}+2\right)}$ (2) $\frac{4}{\left(3\sqrt{6}-2\right)}$ (3) $\frac{6-\sqrt{6}}{\left(3\sqrt{6}-2\right)}$ (4) $\frac{6+\sqrt{6}}{\left(3\sqrt{6}+2\right)}$

Ans. Official answer NTA (2)

Sol.

Question ID : 87827055608

8. Let $A = \{1, 3, 7, 9, 11\}$ and $B = \{2, 4, 5, 7, 8, 10, 12\}$. Then the total number of one-one maps $f: A \to B$, such that $f(1) + f(3) = 14$, is :

(1) 180 (2) 120 (3) 480 (4) 240

Ans. Official answer NTA (4)

Sol.

Question ID : 87827055620

9. Let a circle C of radius 1 and closer to the origin be such that the lines passing through the point (3, 2) and parallel to the coordinate axes touch it. Then the shortest distance of the circle C from the point (5, 5) is :

(1) 4 (2) 5 (3) $4\sqrt{2}$ (4) $2\sqrt{2}$

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055616

10. Let a rectangle ABCD of sides 2 and 4 be inscribed in another PQRS such that the vertices of the rectangle ABCD lie on the sides of the rectangle PQRS. Let a and b be the sides of the rectangle PQRS when its area is maximum. Then $(a + b)^2$ is equal to :

(1) 80 (2) 72 (3) 60 (4) 64

Ans. Official answer NTA (2)

Sol.

Question ID : 87827055612

11. For the function $f(x) = \sin x + 3x - \frac{2}{\pi}\left(x^2 + x\right)$, where $x \in \left[0, \frac{\pi}{2}\right]$ consider the following two statements :

(I) f is increasing in $\left(0, \frac{\pi}{2}\right)$

(II) f' is decreasing in $\left(0, \frac{\pi}{2}\right)$

Between the above two statements :

(1) only (I) is true (2) only (II) is true (II) both (I) and (II) are true (4) neither (I) nor (II) is true

Ans. Official answer NTA (3)

Sol.

Question ID : 87827055617

12. The value of $\int_{-\pi}^{\pi} \frac{2y(1+\sin y)}{1+\cos^2 y}dy$ is :

(1) π^2 (2) $\frac{\pi}{2}$ (3) $\frac{\pi^2}{2}$ (4) $2\pi^2$

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055621

13. Let two straight lines drawn from the origin O intersect the line $3x + 4y = 12$ at the points P and Q such that ΔOPQ is an isosceles triangle and $\angle POQ = 90^\circ$. If $l = OP^2 + PQ^2 + QO^2$, then the greatest integer less than or equal to l is :

(1) 44 (2) 42 (3) 46 (4) 48

Ans. Official answer NTA (3)

Sol.

Question ID : 87827055624

14. Let d be the distance of the point of intersection of the lines $\frac{x+6}{3} = \frac{y}{2} = \frac{z+1}{1}$ and $\frac{x-7}{4} = \frac{y-9}{3} = \frac{z-4}{2}$ from the point (7, 8, 9). Then $d^2 + 6$ is equal to:

(1) 69 (2) 72 (3) 75 (4) 78

Ans. Official answer NTA (3)

Sol.

Question ID : 87827055613

15. If $\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+.....+\frac{1}{\sqrt{99}+\sqrt{100}}=m$ and $\frac{1}{1.2}+\frac{1}{2.3}+.....+\frac{1}{99.100}=n$, then then point (m, n) lies on the line :

(1) 11 (x– 2) –100 (y – 1) = 0 (2) 11 (x– 1) –100 y = 0

(3) 11 –100y = 0 (4) 11 (x– 1) –100 (y – 2) = 0

Ans. Official answer NTA (3)

Sol.

Question ID : 87827055614

16. If the function $f(x)=\frac{\sin 3x+\alpha \sin x-\beta \cos 3x}{x^3}$, $x \in \mathbf{R}$, is continuous at x = 0, then f(0) is equal to :

(1) 2 (2) 4 (3) –2 (4) – 4

Ans. Official answer NTA (4)

Sol.

Question ID : 87827055622

17. Let the line 2x + 3y – k = 0, k > 0, intersect the x-axis and y-axis at the points A and B, respectively. If the equation of the circle having the line segment AB as a diameter is $x^2+y^2-3x-2y=0$ and the length of the latus rectum of the ellipse $x^2+9y^2=k^2$ is $\frac{m}{n}$, where m and n are coprime, then 2m + n is equal to :

(1) 12 (2) 13 (3) 11 (4) 10

Ans. Official answer NTA (3)

Sol.

Question ID : 87827055626

18. The coefficients a, b, c in quadratic equation $ax^2+bx+c=0$ are chosen from the set {1, 2, 3, 4, 5, 6, 7, 8}. The probability of this equations having repeated roots is :

(1) 1/128 (2) 1/64 (3) 3/256 (4) 3/128

Ans. Official answer NTA (2)

Sol.

Question ID : 87827055619

19. If $y = y(x)$ is the solution of the differential equation $\frac{dy}{dx} + 2y = \sin(2x)$, $y(0) = \frac{3}{4}$, then $y\left(\frac{\pi}{8}\right)$ is equal to :

(1) $e^{-\pi/4}$ (2) $e^{-\pi/8}$ (3) $e^{\pi/8}$ (4) $e^{\pi/4}$

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055609

20. Consider the following two statements :

Statement I : For any two non-zero complex numbers z_1, z_2.

$$(|z_1| + |z_2|)\left|\frac{z_1}{|z_1|} + \frac{z_2}{|z_2|}\right| \le 2(|z_1| + |z_2|) \text{ and}$$

Statement II : If x, y, z are three distinct complex numbers and a, b, c are three positive real numbers such that $\frac{a}{|y-z|} = \frac{b}{|z-x|} = \frac{c}{|x-y|}$, then $\frac{a^2}{y-z} + \frac{b^2}{z-x} + \frac{c^2}{x-y} = 1$.

Between the above two statemetns :

(1) Statement I is correct but Statement II is incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both Statement I and Statement II are incorrect

(4) Both Statement I and Statement II are correct

Ans. Official answer NTA (1)

Sol.

SECTION – B

Question ID : 87827055629

21. The number of distinct real roots of the equation $|x|\,|x+2|-5\,|x+1|-1=0$ is _____.

Ans. Official answer NTA (3)

Sol.

Question ID : 87827055630

22. The number of ways of getting a sum 16 on throwing a dice four times is_____.

Ans. Official answer NTA (125)

Sol.

Question ID : 87827055632

23. The $a_1, a_2, a_3, \ldots\ldots$ be in the arithemetic progression of positive terms.

Let $A_k = a_1^2 - a_2^2 + a_3^2 - a_4^2 + \ldots\ldots + a_{2k-1}^2 - a_{2k}^2$.

If $A_3 = -153, A_5 = -435$ and $a_1^2 + a_2^2 + a_3^2 = 66$, then $a_{17} - A_7$ is equal to_____.

Ans. Official answer NTA (910)

Sol.

Question ID : 87827055628

24. If $S = \{a \in R : |2a-1| = 3[a] + 2[a]\}$, where [t] deonts the greatest integer less than or equal to t and {t} represents the fractional part t, then $72\sum_{a \in S} a$ is equal to _____.

Ans. Official answer NTA (18)

Sol.

Question ID : 87827055634

25. The area of the region enclosed by the parabolas $y = x^2 - 5x$ and $y = 7x - x^2$ is_____.

Ans. Official answer NTA (198)

Answer by Matrix is (72)

Sol.

Question ID : 87827055637

26. From a lot of 10 items, which include 3 defective items, a sample of 5 items is drawn at random. Let the random variable X denote the number of defective items in the sample. If the variance of X is σ^2, then $96\sigma^2$ is equal to _____.

Ans. Official answer NTA (56)

Sol.

Question ID : 87827055636

27. Let $\vec{a} = \hat{i} - 3\hat{j} + 7\hat{k}, \vec{b} = 2\hat{i} - \hat{i} + \hat{k}$ and $\vec{c}$ be a vector such that $\left(\vec{a} + 2\vec{b}\right) \times \vec{c} = 3\left(\vec{c} \times \vec{a}\right)$. If $\vec{a} \cdot \vec{c} = 130$, then $\vec{b} \cdot \vec{c}$ is equal to_____.

Ans. Official answer NTA (30)

Sol.

Question ID : 87827055633

28. Let f be a differentiable function in the interval $(0, \infty)$ such that $f(1) = 1$ and $\lim\limits_{t \to x} \dfrac{t^2 f(x) - x^2 f(t)}{t - x} = 1$ for each $x > 0$. Then $2\,f(2) + 3\,f(3)$ is equal to _____.

Ans. Official answer NTA (24)

Sol.

Question ID : 87827055635

29. Suppose AB is a focal chord of the parabola $y^2 = 12x$ of length l and slope $m < \sqrt{3}$. If the distance of the chord AB from the origin is d, then ld^2 is equal to_____.

Ans. Official answer NTA (108)

Sol.

Question ID : 87827055631

30. If the constant term in the expansion of $\left(1+2x-3x^3\right)\left(\frac{3}{2}x^2-\frac{1}{3x}\right)^9$ is p, then 108p is equal to _____.

Ans. Official answer NTA (54)

Sol.