JEE Main April 2025

Question Paper With Text Solution

04 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2025 | 04TH APRIL SHIFT-2

SECTION – A

Question ID : 603421766

1. Let f be a differentiable function on R such that $f(2) = 1$, $f'(2) = 4$. Let $\lim\limits_{x\to 0}(f(2+x))^{3/x} = e^{\alpha}$.

Then the number of times the curve $y = 4x^3 - 4x^2 - 4(\alpha - 7)x - \alpha$ meets x-axis is :

(1) 2 (2) 1 (3) 3 (4) 0

Ans. Official answer NTA (1)

Sol.

Question ID : 603421753

2. Let the product of $\omega_1 = (8+i)\sin\theta + (7+4i)\cos\theta$ and $\omega_2 = (1+8i)\sin\theta + (4+7i)\cos\theta$ be $\alpha + i\beta, i = \sqrt{-1}$.

Let p and q be the maximum and the minimum values of $\alpha + \beta$ respectively. Then $p + q$ is equal to :

(1) 160 (2) 150 (3) 140 (4) 130

Ans. Official answer NTA (4)

Sol.

Question ID : 603421751

3. Let the domains of the function $f(x) = \log_4 \log_3 \log_7 \left(8 - \log_2\left(x^2 + 4x + 5\right)\right)$ and $g(x) = \sin^{-1}\left(\frac{7x+10}{x-2}\right)$

be (α, β) and $[\gamma, \delta]$, respectively. Then $\alpha^2 + \beta^2 + \gamma^2 + \delta^2$ is equal to :

(1) 13 (2) 15 (3) 14 (4) 16

Ans. Official answer NTA (2)

Sol.

Question ID : 603421767

4. Let $a > 0$. If the function $f(x) = 6x^3 - 45ax^2 + 108a^2x + 1$ attains its local maximum and minimum values at the points x_1 and x_2 respectively such that $x_1 x_2 = 54$, then $a + x_1 + x_2$ is equal to :

(1) 15 (2) 13 (3) 24 (4) 18

Ans. Official answer NTA (4)

Sol.

Question ID : 603421768

5. Let $f(x)+2f\left(\frac{1}{x}\right)=x^2+5$ and $2g(x)-3g\left(\frac{1}{2}\right)=x, x>0$. If $\alpha=\int_1^2 f(x)dx$, and $\beta=\int_1^2 g(x)dx$, then the value of $9\alpha+\beta$ is :

(1) 0 (2) 10 (3) 11 (4) 1

Ans. Official answer NTA (3)

Sol.

Question ID : 603421770

6. If a curve y = y(x) passes through the point $\left(1,\frac{\pi}{2}\right)$ and satisfies the differential equation $\left(7x^4\cot y-e^x \operatorname{cosec} y\right)\frac{dx}{dy}=x^5, x\geq 1$, then at x = 2, the value of cosy is :

(1) $\frac{2e^2-e}{64}$ (2) $\frac{2e^2+e}{128}$ (3) $\frac{2e^2-e}{128}$ (4) $\frac{2e^2+e}{64}$

Ans. Official answer NTA (3)

Sol.

Question ID : 603421757

7. If $1^2\cdot\left({}^{15}C_1\right)+2^2\cdot\left({}^{15}C_2\right)+3^2\cdot\left({}^{15}C_3\right)+\ldots+15^2\cdot\left({}^{15}C_{15}\right)=2^m\cdot3^n\cdot5^k$, where m, n, k $\in$ N, then m + n + k is equal to :

(1) 20 (2) 19 (3) 18 (4) 21

Ans. Official answer NTA (2)

Sol.

Question ID : 603421752

8. Let A = {–3, –2, –1, 0, 1, 2, 3} and R be a relation on A defined by xRy if and only if $2x-y\in\{0,1\}$. Let *l* be the number of elements in R. Let m and n be the minimum number of elements required to be added in R to

make it reflexive and symmetric relations, respectively. Then $l + m + n$ is equal to :

(1) 15 (2) 16 (3) 17 (4) 18

Ans. Official answer NTA (3)

Sol.

Question ID : 603421765

9. Let A be the point of intersection of the lines $L_1 : \frac{x-7}{1} = \frac{y-5}{0} = \frac{z-3}{-1}$ and $L_2 : \frac{x-1}{3} = \frac{y+3}{4} = \frac{z+7}{5}$. Let B and C be the points on the lines L_1 and L_2 respectively such that $AB = AC = \sqrt{15}$. Then the square of the area of the triangle ABC is :

(1) 57 (2) 63 (3) 60 (4) 54

Ans. Official answer NTA (4)

Sol.

Question ID : 603421756

10. Consider two sets A and B, each containing three numbers in A.P. Let the sum and the product of the elements of A be 36 and p respectively and the sum and the product of the elements of B be 36 and q respectively. Let d and D be the common differences of AP' s in A and B respectively such that $D = d + 3$, $d > 0$. If $\frac{p+q}{p-q} = \frac{19}{5}$. then $p - q$ is equal to :

(1) 600 (2) 540 (3) 450 (4) 630

Ans. Official answer NTA (2)

Sol.

Question ID : 603421759

11. The axis of a parabola is the line $y = x$ and its vertex and focus are in the first quadrant at distances $\sqrt{2}$ and $2\sqrt{2}$ units from the origin, respectively. If the point (1, k) lies on the parabola, then a possible value of k is :

(1) 4 (2) 9 (3) 8 (4) 3

Ans. Official answer NTA (2)

Sol.

Question ID : 603421769

12. A line passing through the point A(–2, 0), touches the parabola P : $y^2 = x - 2$ at the point B in the first quadrant. The area, of the region bounded by the line AB, parabola P and the x-axis, is :

(1) 2 (2) 3 (3) $\frac{8}{3}$ (4) $\frac{7}{3}$

Ans. Official answer NTA (3)

Sol.

Question ID : 603421758

13. Let the mean and the standard deviation of the observation 2, 3, 3, 4, 5, 7, a, b be 4 and $\sqrt{2}$ respectively. Then the mean deviation about the mode of these observations is :

(1) $\frac{3}{4}$ (2) $\frac{1}{2}$ (3) 1 (4) 2

Ans. Official answer NTA (3)

Sol.

Question ID : 603421762

14. Let the sum of the focal distances of the point P(4, 3) on the hyperbola $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $8\sqrt{\frac{5}{3}}$. If for H, the length of the latus rectum is l and the product of the focal distances of the point P is m, then $9l^2 + 6m$ is equal to :

(1) 186 (2) 187 (3) 184 (4) 185

Ans. Official answer NTA (4)

Sol.

Question ID : 603421754

15. Let the matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ satisfy $A^n = A^{n-2} + A^2 - I$ for $n \geq 3$. Then the sum of all the elements of A^{50} is :

(1) 39 (2) 52 (3) 53 (4) 44

Ans. Official answer NTA (3)

Sol.

Question ID : 603421760

16. Let for two distinct values of p the lines $y = x + p$ touch the ellipse $E : \frac{x^2}{4^2} + \frac{y^2}{3^2} = 1$ at the points A and B . Let the line $y = x$ intersect E at the points C and D . Then the area of the quadrilateral ABCD is equal to :

(1) 36 (2) 48 (3) 24 (4) 20

Ans. Official answer NTA (3)

Sol.

Question ID : 603421764

17. Let the values of p, for which the shortest distance between the lines $\frac{x+1}{3} = \frac{y}{4} = \frac{z}{5}$ and $\vec{r} = (p\hat{i} + 2\hat{j} + \hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$ is $\frac{1}{\sqrt{6}}$, be a, b, $(a < b)$. Then the length of the latus rectum of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is :

(1) $\frac{2}{3}$ (2) 18 (3) 9 (4) $\frac{3}{2}$

Ans. Official answer NTA (1)

Sol.

Question ID : 603421761

18. The centre of a circle C is at the centre of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$. Let C pass through the foci F_1 and F_2 of E such that the circle C and the ellipse E intersect at four points. Let P be one of these four points. If the area of the triangle PF_1F_2 is 30 and the length of the major axis of E is 17, then the distance between the foci of E is :

(1) 13 (2) $\frac{13}{2}$ (3) 12 (4) 26

Ans. Official answer NTA (1)

Sol.

Question ID : 603421763

19. The sum of the infinite series $\cot^{-1}\left(\frac{7}{4}\right)+\cot^{-1}\left(\frac{19}{4}\right)+\cot^{-1}\left(\frac{39}{4}\right)+\cot^{-1}\left(\frac{67}{4}\right)+\ldots$ is :

(1) $\frac{\pi}{2}-\cot^{-1}\left(\frac{1}{2}\right)$ (2) $\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{2}\right)$ (3) $\frac{\pi}{2}+\tan^{-1}\left(\frac{1}{2}\right)$ (4) $\frac{\pi}{2}+\cot^{-1}\left(\frac{1}{2}\right)$

Ans. Official answer NTA (2)

Sol.

Question ID : 603421755

20. If the sum of the first 20 terms of the series $\frac{4\cdot1}{4+3\cdot1^2+1^4}+\frac{4\cdot2}{4+3\cdot2^2+2^4}+\frac{4\cdot3}{4+3\cdot3^2+3^4}+\frac{4\cdot4}{4+3\cdot4^2+4^4}+\ldots$ is $\frac{m}{n}$, where m and n are coprime, then m + n is equal to :

(1) 423 (2) 422 (3) 420 (4) 421

Ans. Official answer NTA (4)

Sol.

SECTION – B

Question ID : 603421772

21. Let m and n, (m < n), be two 2-digit numbers. Then the total numbers of pairs (m, n), such that gcd(m,n) = 6, is _____.

Ans. Official answer NTA (64)

Sol.

Question ID : 603421774

22. Let the three sides of a triangle ABC be given by the vectors $2\hat{i}-\hat{j}+\hat{k}, \hat{i}-3\hat{j}-5\hat{k}$ and $3\hat{i}-4\hat{j}-4\hat{k}$. Let G be the centroid of the triangle ABC. Then $6\left(|\overrightarrow{AG}|^2+|\overrightarrow{BG}|^2+|\overrightarrow{CG}|^2\right)$ is equal to_____.

Ans. Official answer NTA (164)

Sol.

Question ID : 603421771

23. If α is a root of the equation $x^2 + x + 1 = 0$ and $\sum_{k=1}^{n}\left(\alpha^k + \frac{1}{\alpha^k}\right)^2 = 20$, then n is equal to_____.

Ans. Official answer NTA (11)

Sol.

Question ID : 603421773

24. A card from a pack of 52 cards is lost. From the remaining 51 cards, n cards are drawn and are found to be spades. If the probability of the lost card to be a spade is $\frac{11}{50}$, then n is equal to_____.

Ans. Official answer NTA (2)

Sol.

Question ID : 603421775

25. If $\int \frac{\left(\sqrt{1+x^2}+x\right)^{10}}{\left(\sqrt{1+x^2}-x\right)^{9}}dx = \frac{1}{m}\left(\left(\sqrt{1+x^2}+x\right)^{n}\left(n\sqrt{1+x^2}-x\right)\right)+C$ where C is the constant of integration and m, $n \in N$, then $m + n$ is equal to _____.

Ans. Official answer NTA (379)

Sol.