JEE Main April 2024

Question Paper With Text Solution

04 April | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN APRIL 2024 | 04 APRIL SHIFT-2

SECTION – A

Question ID : 68019113798

1. The value of $\frac{1\times 2^2+2\times 3^2+....+100\times(101)^2}{1^2\times 2+2^2\times 3+....+100^2\times 101}$ is :

(1) $\frac{31}{30}$ (2) $\frac{306}{305}$ (3) $\frac{32}{31}$ (4) $\frac{305}{301}$

Ans. Official answer NTA (4)

Sol.

Question ID : 68019113797

2. If the coefficients of x^4, x^5 and x^6 in the expansion of $(1 + x)^n$ are in the arithmetic progression, then the maximum value of n is :

(1) 28 (2) 7 (3) 21 (4) 14

Ans. Official answer NTA (4)

Sol.

Question ID : 68019113813

3. Given that the inverse trigonometric function assumes principal values only. Let x, y be any two real numbers in $[-1,1]$ such that $\cos^{-1}x - \sin^{-1}y = \alpha$, $\frac{-\pi}{2} \le \alpha \le \pi$. Then the minimum value of $x^2 + y^2 + 2xy \sin\alpha$ is :

(1) -1 (2) $\frac{1}{2}$ (3) $\frac{-1}{2}$ (4) 0

Ans. Official answer NTA (4)

Sol.

Question ID : 68019113811

4. For $\lambda > 0$, let θ be the angle between the vectors $\vec{a} = \hat{i} + \lambda\hat{j} - 3\hat{k}$ and $\vec{b} = 3\hat{i} - \hat{j} + 2\hat{k}$. If the vectors $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are mutually perpendicular, then the value of $(14\cos\theta)^2$ is equal to:

(1) 20 (2) 25 (3) 40 (4) 50

Ans. Official answer NTA (2)

Sol.

Question ID : 68019113800

5. Let $f(x)=3\sqrt{x-2}+\sqrt{4-x}$ beareal valued function. If α and β are respectively the minimum and the maximum values of f, then $\alpha^2+2\beta^2$ is equal to :

(1) 24 (2) 38 (3) 44 (4) 42

Ans. Official answer NTA (4)

Sol.

Question ID : 68019113807

6. Consider a hyperbola H having centre at the origin and foci on the x-axis. Let C_1 be the circle touching the hyperbola H and having the centre at the origin. Let C_2 be the circle touching the hyperbola H at its vertex and having the centre at one of its foci. If areas (in sq units) of C_1 and C_2 are 36π and 4π, respectively, then the length (in units) of latus rectum of H is :

(1) $\frac{10}{3}$ (2) $\frac{14}{3}$ (3) $\frac{11}{3}$ (4) $\frac{28}{3}$

Ans. Official answer NTA (4)

Sol.

Question ID : 68019113803

7. If the value of the integral $\int_{-1}^{1}\frac{\cos\alpha x}{1+3^x}dx$ is $\frac{2}{\pi}$. Then, a value of α is :

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

Ans. Official answer NTA (3)

Sol.

Question ID : 68019113806

8. Let C be a circle with radius $\sqrt{10}$ units and centre at the origin. Let the line $x + y = 2$ intersects the circle C at the points P and Q. Let MN be a chord of C of length 2 unit and slope –1. Then, a distance (in units) between the chord PQ and the chord MN is :

(1) $\sqrt{2}+1$ (2) $\sqrt{2}-1$ (3) $3-\sqrt{2}$ (4) $2-\sqrt{3}$

Ans. Official answer NTA (3)

Sol.

Question ID : 68019113809

9. Let P be the point of intersection of the lines $\frac{x-2}{1}=\frac{y-4}{5}=\frac{z-2}{1}$ and $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-3}{2}$. Then, the shortest distance of P from the line $4x = 2y = z$ is :

(1) $\frac{6\sqrt{14}}{7}$ (2) $\frac{3\sqrt{14}}{7}$ (3) $\frac{5\sqrt{14}}{7}$ (4) $\frac{\sqrt{14}}{7}$

Ans. Official answer NTA (2)

Sol.

Question ID : 68019113808

10. Let PQ be a chord of the parabola $y^2 = 12x$ and the midpoint of PQ be at (4, 1). Then, which of the following points lies on the line passing through the points P and Q?

(1) (3, –3) (2) $\left(\frac{3}{2},-16\right)$ (3) $\left(\frac{1}{2},-20\right)$ (4) (2, –9)

Ans.

Ans. Official answer NTA (3)

Question ID : 68019113795

11. The area (in sq. units) of the region $S=\left\{z\in C:|z-1|\le 2;(z+\bar{z})+i(z-\bar{z})\le 2,\text{Im}(z)\ge 0\right\}$ is :

(1) $\frac{7\pi}{4}$ (2) $\frac{7\pi}{3}$ (3) $\frac{17\pi}{8}$ (4) $\frac{3\pi}{2}$

Ans. Official answer NTA (4)

Sol.

Question ID : 68019113801

12. If the function $f(x)=\begin{cases}\dfrac{72^x-9^x-8^x+1}{\sqrt{2}-\sqrt{1+\cos x}}, & x\neq 0\\ a\log_e 2\log_e 3, & x=0\end{cases}$ is continuous at $x=0$, then the value of a^2 is equal to :

(1) 1152 (2) 1250 (3) 746 (4) 968

Ans. Official answer NTA (1)

Sol.

Question ID : 68019113794

13. Let a relation R on $N \times N$ be defined as :

$(x_1, y_1)\, R(x_2, y_2)$ if and only if $x_1 \le x_2$ $y_1 \le y_2$.

Consider the two statements :

(I) R is reflexive but not symmetric.

(II) R is transitive.

Then which of the following is true ?

(1) Only (I) is correct. (2) Only (II) is correct.

(3) Neither (I) nor (II) is correct. (4) Both (I) and (II) are correct.

Ans. Official answer NTA (1)

Sol.

Question ID : 68019113799

14. Let three real numbers a, b, c be in arithmetic progression and a + 1, b, c + 3 be in geometric progression. If $a > 10$ and the arithmetic mean of a, b and c is 8, then the cube of the geometric mean of a, b and c is :

(1) 316 (2) 312 (3) 128 (4) 120

Ans. Official answer NTA (4)

Sol.

Question ID : 68019113802

15. Let $f(x)=\int_0^x\left(t+\sin\left(1-e^t\right)\right)dt$, $x\in R$. Then, $\lim\limits_{x\to0}\frac{f(x)}{x^3}$ is equal to :

(1) $\frac{1}{6}$ (2) $-\frac{1}{6}$ (3) $-\frac{2}{3}$ (4) $\frac{2}{3}$

Ans. Official answer NTA (2)

Sol.

Question ID : 68019113796

16. Let $A=\begin{bmatrix}1 & 2\\ 0 & 1\end{bmatrix}$ and $B = I + adj(A) + (adj\ A)^2 + ... + (adj\ A)^{10}$. Then, the sum of all the elements of the matrix B is :

(1) –110 (2) 22 (3) –124 (4) –88

Ans. Official answer NTA (4)

Sol.

Question ID : 68019113804

17. The are (in sq. units) of the region described by $\{(x, y) : y^2 \le 2x, \text{ and } y \ge 4x - 1\}$ is :

(1) $\frac{8}{9}$ (2) $\frac{9}{32}$ (3) $\frac{11}{12}$ (4) $\frac{11}{32}$

Ans. Official answer NTA (2)

Sol.

Question ID : 68019113805

18. Let $y = y(x)$ be the solution of the differential equation $(x^2 + 4)^2dy + (2x^3y + 8xy - 2)dx = 0$. If $y(0) = 0$, then $y(2)$ is equal to :

(1) $\frac{\pi}{32}$ (2) $\frac{\pi}{16}$ (3) 2π (4) $\frac{\pi}{8}$

Ans. Official answer NTA (1)

Sol.

Question ID : 68019113810

19. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$, $\vec{b}=2\hat{i}+4\hat{j}-5\hat{k}$ and $\vec{c}=x\hat{i}+2\hat{j}+3\hat{k}$, $x \in R$. If $\vec{d}$ is the unit vector in the direction of $\vec{b}+\vec{c}$ such that $\vec{a}\cdot\vec{d}=1$, then $(\vec{a}\times\vec{b})\cdot\vec{c}$ is equal to :

(1) 11 (2) 3 (3) 6 (4) 9

Ans. Official answer NTA (1)

Sol.

Question ID : 68019113812

20. If the mean of the following probability distribution of a random variable X :

X	0	2	4	6	8
P(X)	a	2a	a + b	2b	3b

is $\frac{46}{9}$, then the variance of the distribution is :

(1) $\frac{173}{27}$ (2) $\frac{566}{81}$ (3) $\frac{581}{81}$ (4) $\frac{151}{27}$

Ans. Official answer NTA (2)

Sol.

SECTION – B

Question ID : 68019113822

21. Consider a line L passing through the points P(1,2,1) and Q(2,1,–1). If the mirror image of the point A(2,2,2) in the line L is (α, β, γ), then $\alpha + \beta + 6\gamma$ is equal to _____.

Ans. Official answer NTA (6)

Sol.

Question ID : 68019113816

22. Let A be a 2×2 symmetric matrix such that $A\begin{bmatrix}1\\1\end{bmatrix}=\begin{bmatrix}3\\7\end{bmatrix}$ and the determinant of A be 1. If $A^{-1} = \alpha A + \beta I$, where I is an identity matrix of order 2×2, then $\alpha + \beta$ equals _____.

Ans. Official answer NTA (5)

Sol.

Question ID : 68019113819

23. If $\int \cos ec^5 x dx = \alpha \cot x \cos ecx\left(\cos ec^2 x + \frac{3}{2}\right) + \beta \log_e \left|\tan \frac{x}{2}\right| + C$ where $\alpha, \beta \in R$ and C is the constant integration, then the value of $8(\alpha + \beta)$ equals _____.

Ans. Official answer NTA (1)

Sol.

Question ID : 68019113814

24. Consider the function $f : R \to R$ defined by $f(x) = \frac{2x}{\sqrt{1+9x^2}}$. If the composition of $f, \underbrace{(fofofo.....f)}_{10 \text{ times}}(x) = \frac{2^{10}x}{\sqrt{1+9\alpha x^2}}$, then the vlaue of $\sqrt{3\alpha+1}$ is equal to _____.

Ans. Official answer NTA (1024)

Sol.

Question ID : 68019113817

25. There are 4 men and 5 women in Group A, and 5 men and 4 women in Group B. If 4 persons are selected from each grooup, then the number of ways of selecting 4 men and 4 women is _____.

Ans. Official answer NTA (5626)

Sol.

Question ID : 68019113823

26. In a tournament, a team plays 10 matches with probabilities of winning and losing each match as $\frac{1}{3}$ and $\frac{2}{3}$ respectively. Let x be the number of matches that the team wins, and y be the number of matches that team loses. If the probability $P(|x-y| \le 2)$ is p, then $3^9 p$ equals _____.

Ans. Official answer NTA (8288)

Sol.

Question ID : 68019113820

27. Let $y = y(x)$ be the solution of the differential equation $(x + y + 2)^2\,dx = dy$; $y(0) = -2$. Let the maximum and minimum values of the function $y = y(x)$ in $\left[0, \frac{\pi}{3}\right]$ be α and β, respectively. If $(3\alpha + \pi)^2 + \beta^2 = \gamma + \delta\sqrt{3}$, $\gamma, \delta \in Z$, then $\gamma + \delta$ equals _____.

Ans. Official answer NTA (31)

Sol.

Question ID : 68019113815

28. Let $S = \{\sin^2 2\theta : (\sin^4\theta + \cos^4\theta)x^2 + (\sin 2\theta)x + (\sin^6\theta + \cos^6\theta) = 0$ has real roots$\}$. If α and β be the smallest and largest elements of the set S, respectively, then $3((\alpha - 2)^2 + (\beta - 1)^2)$ equals _____.

Ans. Official answer NTA (4)

Sol.

Question ID : 68019113818

29. Let $f : R \to R$ be a thrice diffrentiable function such that $f(0) = 0$, $f(1) = 1$, $f(2) = -1$, $f(3) = 2$ and $f(4) = -2$. Then, the minimum number of zeroes of $(3f'f'' + ff''')(x)$ is _____.

Ans. Official answer NTA (5)

Sol.

Question ID : 68019113821

30. Consider a triangle ABC having the vertices A(1,2), B(α, β) and C(γ, δ) and angles $\angle ABC = \frac{\pi}{6}$ and $\angle BAC = \frac{2\pi}{3}$. If the points B and C lie on the line $y = x + 4$, then $\alpha^2 + \gamma^2$ is equal to _____.

Ans. Official answer NTA (14)

Sol.