JEE Main April 2024

Question Paper With Text Solution

04 April | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN APRIL 2024 | 04 APRIL SHIFT-1

SECTION – A

Question ID : 87827055432

1. Let $\alpha \in (0, \infty)$ and $A = \begin{vmatrix} 1 & 2 & \alpha \\ 1 & 0 & 1 \\ 0 & 1 & 2 \end{vmatrix}$. If $\det(\text{adj}(2A - A^T) \cdot \text{adj}(A - 2A^T)) = 2^8$, then $(\det(A))^2$ is equal to :

(1) 49 (2) 16 (3) 1 (4) 36

Ans. Official answer NTA (2)

Sol.

Question ID : 87827055440

2. One of the points of intersection of the curves $y = 1 + 3x - 2x^2$ and $y = \frac{1}{x}$ is $\left(\frac{1}{2}, 2\right)$. Let the area of the region enclosed by the curves be $\frac{1}{24}\left(\ell\sqrt{5} + m\right) - n\log_e\left(1 + \sqrt{5}\right)$, where $\ell, m, n \in N$. Then $\ell + m + n$ is equal to :

(1) 32 (2) 29 (3) 31 (4) 30

Ans. Official answer NTA (4)

Sol.

Question ID : 87827055439

3. Let the sum of the maximum and the minimum values of the function $f(x) = \frac{2x^2 - 3x + 8}{2x^2 + 3x + 8}$ be $\frac{m}{n}$, where gcd(m, n) = 1. Then m + n is equal to :

(1) 182 (2) 217 (3) 195 (4) 201

Ans. Official answer NTA (4)

Sol.

Question ID : 87827055438

4. Let $f(x) = x^5 + 2e^{\frac{x}{4}}$ for all $x \in R$. Consider a function g(x) such that (gof) (x) = x for all $x \in R$. Then the value of $8g'(2)$ is :

(1) 8 (2) 4 (3) 2 (4) 16

Ans. Official answer NTA (4)

Sol.

Question ID : 87827055434

5. There are 5 points P_1, P_2, P_3, P_4, P_5 on the side AB, excluding A and B, of a triangle ABC. Similarly, there are 6 points $P_6, P_7, ..., P_{11}$ on the side BC and 7 points $P_{12}, P_{13}, ..., P_{18}$ on the side CA of the triangle. The number of triangles, that can be formed using the points $P_1, P_2, ..., P_{18}$ as vertices is :

(1) 751 (2) 776 (3) 771 (4) 796

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055446

6. Three urns A, B and C contain 7 red, 5 black; 5 red, 7 black and 6 red, 6 black balls, respectively. One of the urn is selected at random and a ball is drawn from it. If the ball drawn is black, then the probability that it is drawn from urn A is :

(1) $\frac{5}{18}$ (2) $\frac{7}{18}$ (3) $\frac{5}{16}$ (4) $\frac{4}{17}$

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055444

7. Let the point, on the line passing through the points P(1, –2,3) and Q(5, –4, 7), farther from the origin and at a distance of 9 units from the point P, be (α, β, γ). Then $\alpha^2 + \beta^2 + \gamma^2$ is equal to :

(1) 155 (2) 150 (3) 165 (4) 160

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055435

8. The sum of all rational terms in the expansion of $\left(2^{\frac{1}{5}}+5^{\frac{1}{3}}\right)^{15}$ is equal to :

(1) 6131 (2) 3133 (3) 931 (4) 633

Ans. Official answer NTA (2)

Sol.

Question ID : 87827055447

9. Let $\alpha, \beta \in R$. Let the mean and the variance of 6 observations $-3,4,7,-6, \alpha, \beta$ be 2 and 23 respectively. The mean deviation about the mean of these 6 observations is :

(1) $\frac{14}{3}$ (2) $\frac{13}{3}$ (3) $\frac{16}{3}$ (4) $\frac{11}{3}$

Ans. Official answer NTA (2)

Sol.

Question ID : 87827055445

10. Let a unit vector which makes and angle of 60° with $2\hat{i}+2\hat{j}-\hat{k}$ and an angle of 45° with $\hat{i}-\hat{k}$ be $\vec{C}$. Then $\vec{C}+\left(-\frac{1}{2}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{\sqrt{2}}{3}\hat{k}\right)$ is :

(1) $\left(\frac{1}{\sqrt{3}}+\frac{1}{2}\right)\hat{i}+\left(\frac{1}{\sqrt{3}}-\frac{1}{3\sqrt{2}}\right)\hat{j}+\left(\frac{1}{\sqrt{3}}+\frac{\sqrt{2}}{3}\right)\hat{k}$

(2) $\frac{\sqrt{2}}{3}\hat{i}+\frac{1}{3\sqrt{2}}\hat{j}-\frac{1}{2}\hat{k}$

(3) $\frac{\sqrt{2}}{3}\hat{i}-\frac{1}{2}\hat{k}$

(4) $-\frac{\sqrt{2}}{3}\hat{i}+\frac{\sqrt{2}}{3}\hat{j}+\left(\frac{1}{2}+\frac{2\sqrt{2}}{3}\right)\hat{k}$

Ans. Official answer NTA (3)

Sol.

Question ID : 87827055441

11. If the solution $y = y(x)$ of the differential equation $(x^4 + 2x^3 + 3x^2 + 2x + 2)dy - (2x^2 + 2x + 3)dx = 0$ satisfies $y(-1) = -\frac{\pi}{4}$, then $y(0)$ is equal to :

(1) 0 (2) $-\frac{\pi}{12}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{4}$

Ans. Official answer NTA (4)

Sol.

Question ID : 87827055443

12. The vertices of a triangle are A(–1,3), B(–2,2) and C(3, –1). A new triangle is formed by shifting the sides of the triangle by one unit inwards. Then the equation of the side of the new triangle nearest to origin is :

(1) $x + y - \left(2 - \sqrt{2}\right) = 0$ (2) $x - y - \left(2 + \sqrt{2}\right) = 0$

(3) $-x + y - \left(2 - \sqrt{2}\right) = 0$ (4) $x + y + \left(2 - \sqrt{2}\right) = 0$

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055431

13. If the system of equations

$$x + \left(\sqrt{2}\sin\alpha\right)y + \left(\sqrt{2}\cos\alpha\right)z = 0$$

$$x + \left(\cos\alpha\right)y + \left(\sin\alpha\right)z = 0$$

$$x + \left(\sin\alpha\right)y - \left(\cos\alpha\right)z = 0$$

has a non-trivial solution, then $\alpha \in \left(0, \frac{\pi}{2}\right)$ is equal to :

(1) $\frac{3\pi}{4}$ (2) $\frac{7\pi}{24}$ (3) $\frac{11\pi}{24}$ (4) $\frac{5\pi}{24}$

Ans. Official answer NTA (4)

Sol.

Question ID : 87827055433

14. Let $f : R \to R$ be a function given by

$$f(x) = \begin{cases} \frac{1-\cos 2x}{x^2}, & x < 0 \\ \alpha, & x = 0 \\ \frac{\beta\sqrt{1-\cos x}}{x}, & x > 0 \end{cases}$$

where $\alpha, \beta \in R$. If f is continuous at $x = 0$, then $\alpha^2 + \beta^2$ is equal to :

(1) 6 (2) 48 (3) 3 (4) 12

Ans. Official answer NTA (4)

Sol.

Question ID : 87827055436

15. Let the first three terms 2, p and q with $q \neq 2$, of a G.P. be respectively the 7^{th}, 8^{th} and 13^{th} terms of an A.P. If the 5^{th} term of the G.P. is the n^{th} term of the A.P., then n is equal to :

(1) 163 (2) 169 (3) 151 (4) 177

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055428

16. If the domain of the function $\sin^{-1}\left(\frac{3x-22}{2x-19}\right) + \log_e\left(\frac{3x^2-8x+5}{x^2-3x-10}\right)$ is $(\alpha, \beta]$, then $3\alpha + 10\beta$ is equal to :

(1) 97 (2) 98 (3) 100 (4) 95

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055442

17. A square is inscribed in the circle $x^2 + y^2 - 10x - 6y + 30 = 0$. One side of this square is parallel to $y = x + 3$. If (x_i, y_i) are the vertices of the square, then $\sum\left(x_i^2 + y_i^2\right)$ is equal to :

(1) 156 (2) 148 (3) 152 (4) 160

Ans. Official answer NTA (3)

Sol.

Question ID : 87827055437

18. Let $f(x) = \begin{cases} -2, & -2 \le x \le 0 \\ x - 2, & 0 < x \le 2 \end{cases}$ and $h(x) = f(|x|) + |f(x)|$. Then $\int_{-2}^{2} h(x)\,dx$ is equal to :

(1) 2 (2) 6 (3) 4 (4) 1

Ans. Official answer NTA (1)

Sol.

Question ID : 87827055429

19. Let α and β be the sum and the product of all the non-zero solutions of the equation $(\bar{z})^2 + |z| = 0$, $z \in C$. Then $4(\alpha^2 + \beta^2)$ is equal to :

(1) 2 (2) 8 (3) 4 (4) 6

Ans. Official answer NTA (3)

Sol.

Question ID : 87827055430

20. If 2 and 6 are the roots of the equation $ax^2 + bx + 1 = 0$, then the quadratic equation, whose roots are $\frac{1}{2a+b}$ and $\frac{1}{6a+b}$, is :

(1) $2x^2 + 11x + 12 = 0$ (2) $x^2 + 8x + 12 = 0$ (3) $4x^2 + 14x + 12 = 0$ (4) $x^2 + 10x + 16 = 0$

Ans. Official answer NTA (2)

Sol.

SECTION – B

Question ID : 87827055456

21. If the shortest distance between the lines $\frac{x+2}{2}=\frac{y+3}{3}=\frac{z-5}{4}$ and $\frac{x-3}{1}=\frac{y-2}{-3}=\frac{z+4}{2}$ is $\frac{38}{3\sqrt{5}}k$, and $\int_0^k\left[x^2\right]dx=\alpha-\sqrt{\alpha}$, where [x] denotes the greatest integer function, then $6\alpha^3$ is equal to _____.

Ans. Official answer NTA (48)

Sol.

Question ID : 87827055449

22. Let A be a 3×3 matrix of non-negative real elements such that $A\begin{bmatrix}1\\1\\1\end{bmatrix}=3\begin{bmatrix}1\\1\\1\end{bmatrix}$. Then the maximum value of det(A) is _____.

Ans. Official answer NTA (27)

Sol.

Question ID : 87827055451

23. If $\lim\limits_{x\to1}\frac{(5x+1)^{\frac{1}{3}}-(x+5)^{\frac{1}{3}}}{(2x+3)^{\frac{1}{2}}-(x+4)^{\frac{1}{2}}}=\frac{m\sqrt{5}}{n(2n)^{\frac{2}{3}}}$, where gcd(m, n) = 1, then 8m + 12n is equal to _____.

Ans. Official answer NTA (100)

Sol.

Question ID : 87827055452

24. If $\int_0^{\frac{\pi}{4}}\frac{\sin^2x}{1+\sin x\cos x}dx=\frac{1}{a}\log_e\left(\frac{a}{3}\right)+\frac{\pi}{b\sqrt{3}}$, where $a, b\in N$, then a + b is equal to _____.

Ans. Official answer NTA (8)

Sol.

Question ID : 87827055454

25. Let A be a square matrix of order 2 such that $|A| = 2$ and the sum of its diagonal elements is –3. If the points (x, y) satisfying $A^2 + xA + yI = 0$ lie on a hyperbola, whose transverse axis is parallel to the x-axis, eccentricity is e and the length of the latus rectum is ℓ, then $e^4 + \ell^4$ is equal to _____.

Ans. Official answer NTA (25)

Answer by Matrix is (Bonus)

Sol.

Question ID : 87827055457

26. Let ABC be a triangle of area $15\sqrt{2}$ and the vectors $\overrightarrow{AB} = \hat{i} + 2\hat{j} - 7\hat{k}$, $\overrightarrow{BC} = a\hat{i} + b\hat{j} + c\hat{k}$ and $\overrightarrow{AC} = 6\hat{i} + d\hat{j} - 2\hat{k}$, $d > 0$. Then the square of the length of the largest side of the triangle ABC is _____.

Ans. Official answer NTA (54)

Sol.

Question ID : 87827055455

27. Let the length of the focal chord PQ of the parabola $y^2 = 12x$ be 15 units. If the distance of PQ from the origin is p, then $10p^2$ is equal to _____.

Ans. Official answer NTA (72)

Sol.

Question ID : 87827055450

28. Let $a = 1 + \frac{{}^2C_2}{3!} + \frac{{}^3C_2}{4!} + \frac{{}^4C_2}{5!} + ...,$

$$b = 1 + \frac{{}^1C_0 + {}^1C_1}{1!} + \frac{{}^2C_0 + {}^2C_1 + {}^2C_2}{2!} + \frac{{}^3C_0 + {}^3C_1 + {}^3C_2 + {}^3C_3}{3!} + ...$$

Then $\frac{2b}{a^2}$ is equal to _____.

Ans. Official answer NTA (8)

Sol.

Question ID : 87827055448

29. In a survey of 220 students of a higher secondary school, it was found that at least 125 and at most 130 students studied Mathematics; at least 85 and at most 95 studied Physics; at least 75 and at most 90 studied Chemistry; 30 studied both Physics and Chemistry; 50 studied both Chemistry and Mathematics; 40 studied both Mathematics and Physics and 10 studied none of these subjects. Let m and n respectively be the least and the most number of students whose studied all the three subjects. Then m + n is equal to _____.

Ans. Official answer NTA (45)

Sol.

Question ID : 87827055453

30. Let the solution $y = y(x)$ of the differential equation $\frac{dy}{dx} - y = 1 + 4\sin x$ satisfy $y(\pi) = 1$. Then $y\left(\frac{\pi}{2}\right) + 10$ is equal to _____.

Ans. Official answer NTA (7)

Sol.