

**JEE Main January 2024**  
**Question Paper With Text Solution**  
**31 January | Shift-2**

**MATHEMATICS**



JEE Main & Advanced | XI-XII Foundation | VI-X Pre-Foundation

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**JEE MAIN JANUARY 2024 | 31<sup>TH</sup> JANUARY SHIFT-2****SECTION – A**

Question ID : 4058591211

1. The shortest distance, between lines  $L_1$  and  $L_2$ , where  $L_1 : \frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+4}{2}$  and  $L_2$  is the line, passing A

$(-4, 4, 3)$ , B  $(-1, 6, 3)$  and perpendicular to the line  $\frac{x-3}{-2} = \frac{y}{3} = \frac{z-1}{1}$ , is :

- (1)  $\frac{141}{\sqrt{221}}$       (2)  $\frac{42}{\sqrt{117}}$       (3)  $\frac{24}{\sqrt{117}}$       (4)  $\frac{121}{\sqrt{221}}$

**Ans.** Official answer NTA(1)

**Sol.**

Question ID : 4058591207

2. The temperature  $T(t)$  of a body at time  $t = 0$  is  $160^\circ\text{F}$  and it decreases continuously as per the differential

equation  $\frac{dT}{dt} = -k(T - 80)$ , where  $K$  is a positive constant. If  $T(15) = 120^\circ\text{F}$ , then  $T(45)$  is equal to :

- (1)  $80^\circ\text{F}$       (2)  $85^\circ\text{F}$       (3)  $95^\circ\text{F}$       (4)  $90^\circ\text{F}$

**Ans.** Official answer NTA(4)

**Sol.**

Question ID : 4058591197

3. Let  $z_1$  and  $z_2$  be two complex numbers such that  $z_1 + z_2 = 5$  and  $z_1^3 + z_2^3 = 20 + 15i$ . Then  $|z_1^4 + z_2^4|$  equals :

- (1)  $30\sqrt{3}$       (2)  $25\sqrt{3}$       (3)  $15\sqrt{15}$       (4) 75

**Ans.** Official answer NTA(4)

**Sol.**

Question ID : 4058591201

4. Let  $2^{\text{nd}}$ ,  $8^{\text{th}}$  and  $44^{\text{th}}$  terms of a non-constant A.P. be respectively the  $1^{\text{st}}$ ,  $2^{\text{nd}}$  and  $3^{\text{rd}}$  of a G.P. If the first terms of a G.P. If the first term of the A.P. is 1, then the sum of its first 20 terms is equal to :

- (1) 990      (2) 980      (3) 960      (4) 970

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**Ans.** Official answer NTA(4)

**Sol.**

Question ID : 4058591202

5. Consider the function  $f : (0, \infty) \rightarrow \mathbb{R}$  defined by  $f(x) = e^{-|\log_e x|}$ . if  $m$  and  $n$  be respectively the number of points at which  $f$  is not continuous and  $f$  is not differentiable, then  $m + n$  is :

- (1) 2                      (2) 3                      (3) 0                      (4) 1

**Ans.** Official answer NTA(4)

**Sol.**

Question ID : 4058591205

6. The area of the region enclosed by the parabola  $y = 4x - x^2$  and  $3y = (x - 4)^2$  is equal to :

- (1)  $32/9$                       (2) 4                      (3) 6                      (4)  $14/3$

**Ans.** Official answer NTA(3)

**Sol.**

Question ID : 4058591206

7. Let  $f, g : (0, \infty) \rightarrow \mathbb{R}$  be two functions defined by  $f(x) = \int_{-x}^x (|t| - t^2)e^{-t^2} dt$  and  $g(x) = \int_0^{x^2} t^{1/2} e^{-t} dt$ . Then

the value of  $9\left(f\left(\sqrt{\log_e 9}\right) + \left(\sqrt{\log_e 9}\right)\right)$  is equal to :

- (1) 9                      (2) 6                      (3) 10                      (4) 8

**Ans.** Official answer NTA(4)

**Sol.**

Question ID : 4058591204

8. Let a variable line passing through the centre of the circle  $x^2 + y^2 - 16x - 4y = 0$ , meet the positive co-ordinate axes at the points A and B. Then the minimum value of  $OA + OB$ , where O is the origin, is equal to :

- (1) 18                      (2) 12                      (3) 24                      (4) 20

**Ans.** Official answer NTA(1)

**Sol.**

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Question ID : 4058591195

9. If the function  $f : (-\infty, -1) \rightarrow (a, b)$  defined by  $f(x) = e^{x^3-3x+1}$  is one-to-one and onto, then the distance of the point  $P(2b+4, a+2)$  from the line  $x + e^{-3}y = 4$  is :

- (1)  $\sqrt{1+e^6}$       (2)  $2\sqrt{1+e^6}$       (3)  $4\sqrt{1+e^6}$       (4)  $3\sqrt{1+e^6}$

**Ans.** Official answer NTA(2)

**Sol.**

Question ID : 4058591196

10. The number of solutions, of the equation  $e^{\sin x} - 2e^{-\sin x} = 2$ , is :

- (1) 0      (2) more than 2      (3) 2      (4) 1

**Ans.**

**Ans.** Official answer NTA(1)

Question ID : 4058591212

11. Let the mean and the variance of 6 observations  $a, b, 68, 44, 48, 60$  be 55 and 194, respectively. If  $a > b$ , then  $a + 3b$  is :

- (1) 180      (2) 210      (3) 200      (4) 190

**Ans.** Official answer NTA(1)

**Sol.**

Question ID : 4058591203

12. Let  $f : \mathbb{R} \rightarrow (0, \infty)$  be strictly increasing function such that  $\lim_{x \rightarrow \infty} \frac{f(7x)}{f(x)} = 1$ . Then, the value of

$\lim_{x \rightarrow \infty} \left[ \frac{f(5x)}{f(x)} = 1 \right]$  is equal to :

- (1)  $7/5$       (2) 0      (3) 1      (4) 4

**Ans.** Official answer NTA(2)

**Sol.**

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Question ID : 4058591198

13. Let A be a  $3 \times 3$  real matrix such that  $A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ ,  $A \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} = 4 \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$ ,  $A \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = 2 \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ . Then, the system

$$(A - 3I) \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \text{ has :}$$

- (1) no solution (2) exactly two solutions  
 (3) infinitely many solutions (4) unique solutions

**Ans.** Official answer NTA(4)**Sol.**

Question ID : 4058591213

14. A coin is biased so that a head is twice as likely to occur as a tail. If the coin is tossed 3 times, then the probability of getting two tails and one head is :

- (1)  $2/27$  (2)  $1/27$  (3)  $2/9$  (4)  $1/9$

**Ans.** Official answer NTA(3)**Sol.**

Question ID : 4058591208

15. Let P be a parabola with vertex (2, 3) and directrix  $2x + y = 6$ . Let an ellipse  $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b$ , of eccentricity  $\frac{1}{\sqrt{2}}$  pass through the focus of the parabola P. Then the square of the length of the latus rectum of

E is :

- (1)  $656/25$  (2)  $374/8$  (3)  $385/8$  (4)  $512/5$

**Ans.** Official answer NTA(1)**Sol.**



Question ID : 4058591199

16. The number of ways in which 21 identical apples can be distributed among three children such that each child gets at least 2 apples is :

- (1) 142                      (2) 406                      (3) 136                      (4) 130

**Ans.** Official answer NTA(3)

**Sol.**

Question ID : 4058591200

17. If for some  $m, n$ ;  ${}^6C_m + 2({}^6C_{m+1}) + {}^6C_{m+2} > {}^8C_3$  and  ${}^{n-1}P_3 : {}^nP_4 = 1 : 8$ , then  ${}^nP_{m+1} + {}^{n+1}C_m$  is equal to :

- (1) 372                      (2) 384                      (3) 380                      (4) 376

**Ans.** Official answer NTA(1)

**Sol.**

Question ID : 4058591210

18. Let  $(\alpha, \beta, \gamma)$  be the mirror image of the point  $(2, 3, 5)$  in the line  $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ . Then  $2\alpha + 3\beta + 4\gamma$

is equal to :

- (1) 34                      (2) 32                      (3) 31                      (4) 33

**Ans.** Official answer NTA(4)

**Sol.**

Question ID : 4058591214

19. if  $a = \sin^{-1}(\sin(5))$  and  $b = \cos^{-1}(\cos(5))$ , then  $a^2 + b^2$  is equal to :

- (1)  $4\pi^2 + 25$                       (2)  $4\pi^2 - 20\pi^2 + 50$                       (3)  $8\pi^2 - 40\pi^2 + 50$                       (4) 25

**Ans.** Official answer NTA(3)

**Sol.**



Question ID : 4058591209

20. Let  $A(a, b)$ ,  $B(3, 4)$  and  $C(-6, -8)$  respectively denote the centroid, circumcentre and orthocentre of a triangle. Then the distance of the point  $P(2a + 3, 7b + 5)$  from the line  $2x + 3y - 4 = 0$  measured parallel to the line  $x - 2y - 1 = 0$  is :

- (1)  $\frac{17\sqrt{5}}{6}$       (2)  $\frac{17\sqrt{5}}{7}$       (3)  $\frac{\sqrt{5}}{17}$       (4)  $\frac{15\sqrt{5}}{7}$

**Ans.** Official answer NTA(2)**Sol.****SECTION - B**

Question ID : 4058591216

21. Let  $A$  be a  $3 \times 3$  matrix  $\det(A) = 2$ . If  $n = \det(\underbrace{\text{adj}(\text{adj}(\dots(\text{adj}A)))}_{2024\text{-times}})$ , then the remainder when  $n$  is divided by 9 is equal to \_\_\_\_\_.

**Ans.** Official answer NTA(7)**Sol.**

Question ID : 4058591222

22. Let  $A(-2, -1)$ ,  $B(1, 0)$ ,  $C(\alpha, \beta)$  and  $(\gamma, \delta)$  be the vertices of a parallelogram  $ABCD$ . If the point  $C$  lies on  $2x - y = 5$  and the point  $D$  lies on  $3x - 2y = 6$ , then the value of  $|\alpha + \beta + \gamma + \delta|$  is equal to \_\_\_\_\_.

**Ans.** Official answer NTA(32)**Sol.**

Question ID : 4058591215

23. Let  $A = \{1, 2, 3, \dots, 100\}$ . Let  $R$  be a relation on  $A$  defined  $(x, y) \in R$  if and only if  $2x = 3y$ . Let  $R_1$  be a symmetric relation on  $A$  such that  $R \subset R_1$  and the number of elements in  $R_1$  is  $n$ . Then, the minimum value of  $n$  is \_\_\_\_\_.

**Ans.** Official answer NTA(66)**Sol.****MATRIX JEE ACADEMY****Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911****Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in**



Question ID : 4058591218

24. Let  $a, b, c$  be the lengths of three sides of a triangle satisfying the condition  $(a^2 + b^2)x^2 - 2b(a + c)x + (b^2 + c^2) = 0$ . If the set of all possible values of  $x$  is the interval  $(\alpha, \beta)$ , then  $12(\alpha^2 + \beta^2)$  is equal to \_\_\_\_\_.

**Ans.** Official answer NTA(36)**Sol.**

Question ID : 4058591219

25. If  $\lim_{x \rightarrow 0} \frac{ax^2 e^x - b \log_e(1+x) + cxe^{-x}}{x^2 \sin x} = 1$ , then  $16(a^2 + b^2 + c^2)$  is equal to \_\_\_\_\_.

**Ans.** Official answer NTA(81)**Sol.**

Question ID : 4058591223

26. A line passes through  $A(4, -6, -2)$  and  $B(16, -2, 4)$ . Then point  $P(a, b, c)$ , where  $a, b, c$  are non-negative integers, on the line  $AB$  lies at a distance of 21 units, from the point  $A$ . The distance between the points  $P(a, b, c)$  and  $Q(4, -12, 3)$  is equal to \_\_\_\_\_.

**Ans.** Official answer NTA(22)**Sol.**

Question ID : 4058591224

27. Let  $\vec{a} = 3\hat{i} + 2\hat{j} + \hat{k}$ ,  $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$  and  $\vec{c}$  be a vector such that  $(\vec{a} + \vec{b}) \times \vec{c} = 2(\vec{a} \times \vec{b}) + 24\hat{j} - 6\hat{k}$  and  $(\vec{a} - \vec{b} + \hat{i}) \cdot \vec{c} = -3$ . Then is equal to \_\_\_\_\_.

**Ans.** Official answer NTA(38)**Sol.**





Question ID :4058591220

28.  $\left| \frac{120}{\pi^3} \int_0^{\pi} \frac{x^2 \sin x \cos x}{\sin^4 x + \cos^4 x} dx \right|$  is equal to \_\_\_\_\_.

**Ans.** Official answer NTA(15)**Sol.**

Question ID : 4058591221

29. Let  $y = y(x)$  be the solution of the differential equation

$$\sec^2 x dx + (e^{2y} \tan^2 x + \tan x) dy = 0, 0 < x < \frac{\pi}{2}, y\left(\frac{\pi}{4}\right) = 0. \text{ If } y\left(\frac{\pi}{6}\right) = \alpha, \text{ then } e^{8\alpha} \text{ is equal to } \underline{\hspace{2cm}}.$$

**Ans.** Official answer NTA(9)**Sol.**

Question ID : 4058591217

30. Let the coefficient of  $x^r$  in the expansion of

$$(x+3)^{n-1} + (x+3)^{n-2}(x+2) + (x+3)^{n-3}(x+2)^2 + \dots + (x+2)^{n-1} \text{ be } \alpha_r. \text{ If } \sum_{r=0}^n \alpha_r = \beta^n - \gamma^n, \beta, \gamma \in \mathbb{N},$$

then the value of  $\beta^2 + \gamma^2$  equals \_\_\_\_\_.**Ans.** Official answer NTA(25)**Sol.**