JEE Main January 2024

Question Paper With Text Solution

31 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN JANUARY 2024 | 31ST JANUARY SHIFT-1

SECTION – A

Question ID : 4058591106

1. Let S be the set of positive integral values of a for which $\frac{ax^2+2(a+1)x+9a+4}{x^2-8x+32}<0, \forall x \in R$. Then, the number of elements in S is :

(1) 3 (2) 0 (3) ∞ (4) 1

Ans. Official answer NTA (2)

Sol.

Question ID : 4058591117

2. If one of the diameters of the circle $x^2+y^2-10x+4y+13=0$ is a chord of another circle C, whose center is the point of intersection of the lines $2x+3y=12$ and $3x-2y=5$, then the radius of the circle is :

(1) 4 (2) $3\sqrt{2}$ (3) 6 (4) $\sqrt{20}$

Ans. Official answer NTA (3)

Sol.

Question ID : 4058591123

3. Two marbles are drawn in succession from a box containing 10 red, 30 white, 20 blue and 15 orange marbles, with replacement being made after each drawing. Then the probability, that first drawn marble is red and second drawn marble is white, is :

(1) $\frac{4}{25}$ (2) $\frac{2}{3}$ (3) $\frac{2}{25}$ (4) $\frac{4}{75}$

Ans. Official answer NTA (4)

Sol.

Question ID : 4058591105

4. If $f(x)=\frac{4x+3}{6x-4}$, $x \neq \frac{2}{3}$ and $(fof)(x)=g(x)$, where $g: R-\left\{\frac{2}{3}\right\} \to R-\left\{\frac{2}{3}\right\}$, then $(gogog)(4)$ is equal to :

(1) –4 (2) $-\frac{19}{20}$ (3) $\frac{19}{20}$ (4) 4

Ans. Official answer NTA (4)

Sol.

Question ID : 4058591109

5. The sum of the series $\frac{1}{1-3\cdot1^2+1^4}+\frac{2}{1-3\cdot2^2+2^4}+\frac{3}{1-3\cdot3^2+3^4}+\ldots\ldots$ upto 10-terms is :

(1) $\frac{45}{109}$ (2) $\frac{55}{109}$ (3) $-\frac{45}{109}$ (4) $-\frac{55}{109}$

Ans. Official answer NTA (4)

Sol.

Question ID : 4058591110

6. $\lim\limits_{x\to0}\frac{e^{2|\sin x|}-2|\sin x|-1}{x^2}$:

(1) does not exist (2) is equal to 1

(3) is equal to 2 (4) is equal to -1

Ans. Official answer NTA (3)

Sol.

Question ID : 4058591120

7. If the foci of a hyperbola are same as that of the ellipse $\frac{x^2}{9}+\frac{y^2}{25}=1$ and the eccentricity of the hyperbola is $\frac{15}{8}$ times the eccentricity of the ellipse, then the smaller focal distance of the point $\left(\sqrt{2},\frac{14}{3}\sqrt{\frac{2}{5}}\right)$ on the hyperbola, is equal to :

(1) $7\sqrt{\frac{2}{5}}+\frac{8}{3}$ (2) $7\sqrt{\frac{2}{5}}-\frac{8}{3}$ (3) $14\sqrt{\frac{2}{5}}-\frac{4}{3}$ (4) $14\sqrt{\frac{2}{5}}-\frac{16}{3}$

Ans. Official answer NTA (2)

Sol.

Question ID : 4058591111

8. Let a be the sum of all coefficients in the expansion of $(1 - 2x + 2x^2)^{2023}(3 - 4x^2 + 2x^3)^{2024}$ and

$b = \lim\limits_{x\to 0}\left(\dfrac{\int_0^x \frac{\log(1+t)}{t^{2024}+1}dt}{x^2}\right)$. If the equtions $cx^2 + dx + e = 0$ and $2bx^2 + ax + 4 = 0$ have a common root, where c, d, e ∈ R, then d : c : e equals :

(1) 1 : 1 : 4 (2) 4 : 1 : 4 (3) 2 : 1 : 4 (4) 1 : 2 : 4

Ans. Official answer NTA (1)

Sol.

Question ID : 4058591119

9. The distance of the point Q(0,2,–2) form the line passing through the point P(5,–4,3) and perpendicular to the lines $\vec{r} = \left(-3\hat{i} + 2\hat{k}\right) + \lambda\left(2\hat{i} + 3\hat{j} + 5\hat{k}\right)$, λ ∈ R and $\vec{r} = \left(\hat{i} - 2\hat{j} + \hat{k}\right) + \mu\left(-\hat{i} + 3\hat{j} + 2\hat{k}\right)$, μ ∈ R is :

(1) $\sqrt{20}$ (2) $\sqrt{54}$ (3) $\sqrt{74}$ (4) $\sqrt{86}$

Ans. Official answer NTA (3)

Sol.

Question ID : 4058591118

10. Let α, β, γ, δ ∈ Z and let A(α, β), B(1,0), C(γ, δ) and D(1,2) be the vertices of a parallelogram ABCD. If AB = $\sqrt{10}$ and the points A and C lie on the line 3y = 2x + 1, then 2(α + β + γ + δ) is equal to :

(1) 12 (2) 10 (3) 5 (4) 8

Ans.

Ans. Official answer NTA (4)

Question ID : 4058591107

11. If $f(x)=\begin{vmatrix} x^3 & 2x^2+1 & 1+3x \\ 3x^2+2 & 2x & x^3+6 \\ x^3-x & 4 & x^2-2 \end{vmatrix}$ for all $x \in R$, then $2f(0)+f'(0)$ is equal to :

(1) 48 (2) 18 (3) 24 (4) 42

Ans. Official answer NTA (4)

Sol.

Question ID : 4058591112

12. For $0 < c < b < a$, let $(a+b-2c)x^2+(b+c-2a)x+(c+a-2b)=0$ and $\alpha \neq 1$ be one of its root. Then, among the two statements :

(I) If $\alpha \in (-1, 0)$, then b cannot be the geometric mean of a and c.

(II) If $\alpha \in (0, 1)$, then b may be the geometric mean of a and c.

(1) Neither (I) nor (II) is true. (2) Only (I) is true

(3) Only (II) is true. (4) Both (I) and (II) are true.

Ans. Official answer NTA (4)

Sol.

Question ID : 4058591115

13. Let $y = y(x)$ be the solution of the differential equation $\frac{dy}{dx}=\frac{(\tan x)+y}{\sin x(\sec x-\sin x\tan x)}$, $x \in \left(0,\frac{\pi}{2}\right)$ satisfying the condition $y\left(\frac{\pi}{4}\right)=2$. Then $y\left(\frac{\pi}{3}\right)$, is :

(1) $\sqrt{3}\left(2+\log_e\sqrt{3}\right)$ (2) $\frac{\sqrt{3}}{2}\left(2+\log_e 3\right)$ (3) $\sqrt{3}\left(2+\log_e 3\right)$ (4) $\sqrt{3}\left(1+2\log_e 3\right)$

Ans. Official answer NTA (1)

Sol.

Question ID : 4058591124

14. For $\alpha, \beta, \gamma \neq 0$, if $\sin^{-1}\alpha + \sin^{-1}\beta + \sin^{-1}\gamma = \pi$ and $(\alpha + \beta + \gamma)(\alpha - \gamma + \beta) = 3\alpha\beta$, then γ equals :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{\sqrt{3}}{2}$ (3) $\sqrt{3}$ (4) $\frac{\sqrt{3}-1}{2\sqrt{2}}$

Ans. Official answer NTA (2)

Sol.

Question ID : 4058591108

15. If the system of linear equations

$x - 2y + z = -4$

$2x + \alpha y + 3z = 5$

$3x - y + \beta z = 3$

has infinitely many solutions, then $12\alpha + 13\beta$ is equal to :

(1) 54 (2) 64 (3) 60 (4) 58

Ans. Official answer NTA (4)

Sol.

Question ID : 4058591116

16. The solution curve of the differential equation $y\frac{dx}{dy} = x\left(\log_e x - \log_e y + 1\right)$, $x > 0$, $y > 0$ passing through the point (e, 1) is :

(1) $\left|\log_e \frac{y}{x}\right| = y^2$ (2) $\left|\log_e \frac{y}{x}\right| = x$ (3) $2\left|\log_e \frac{x}{y}\right| = y + 1$ (4) $\left|\log_e \frac{x}{y}\right| = y$

Ans. Official answer NTA (4)

Sol.

Question ID : 4058591121

17. Three rotten apples are accidently mixed with fifteen good apples. Assuming the random variable x to be the number of rotten apples in a draw of two apples, the variance of x is :

(1) $\frac{40}{153}$ (2) $\frac{37}{153}$ (3) $\frac{47}{153}$ (4) $\frac{57}{153}$

Ans. Official answer NTA (1)

Sol.

Question ID : 4058591113

18. Let g(x) be a linear function and $f(x)=\begin{cases} g(x) & , \quad x \le 0 \\ \left(\frac{1+x}{2+x}\right)^{\frac{1}{x}} & , \quad x>0 \end{cases}$, is continuous at $x=0$. If $f'(1)=f(-1)$, then the value of g(3) is :

(1) $\log_e\left(\frac{4}{9e^{\frac{1}{3}}}\right)$ (2) $\log_e\left(\frac{4}{9}\right)-1$ (3) $\frac{1}{3}\log_e\left(\frac{4}{9}\right)+1$ (4) $\frac{1}{3}\log_e\left(\frac{4}{9e^{\frac{1}{3}}}\right)$

Ans. Official answer NTA (1)

Sol.

Question ID : 4058591114

19. The area of the region $\left\{(x,y): y^2 \le 4x, x<4, \frac{xy(x-1)(x-2)}{(x-3)(x-4)}>0, x \ne 3\right\}$ is :

(1) $\frac{32}{3}$ (2) $\frac{16}{3}$ (3) $\frac{64}{3}$ (4) $\frac{8}{3}$

Ans. Official answer NTA (1)

Sol.

Question ID : 4058591122

20. Let $\vec{a}=3\hat{i}+\hat{j}-2\hat{k}$, $\vec{b}=4\hat{i}+\hat{j}+7\hat{k}$ and $\vec{c}=\hat{i}-3\hat{j}+4\hat{k}$ be three vectors. If a vector $\vec{p}$ satisfies $\vec{p}\times\vec{b}=\vec{c}\times\vec{b}$ and $\vec{p}\cdot\vec{a}=0$, then $\vec{p}\cdot\left(\hat{i}-\hat{j}-\hat{k}\right)$ is equal to :

(1) 32 (2) 36 (3) 28 (4) 24

Ans. Official answer NTA (1)

Sol.

SECTION – B

Question ID : 4058591128

21. In the expansion of $(1+x)\left(1-x^2\right)\left(1+\frac{3}{x}+\frac{3}{x^2}+\frac{1}{x^3}\right)^5$, $x\neq 0$, the sum of the coefficients of x^3 and x^{-13} is equal to _____ .

Ans. Official answer NTA (118)

Sol.

Question ID : 4058591130

22. Let $f:R\to R$ be a function defined by $f(x)=\frac{4^x}{4^x+2}$ and $M=\int_{f(a)}^{f(1-a)} x\sin^4\left(x(1-x)\right)dx$,

$N=\int_{f(a)}^{f(1-a)} \sin^4\left(x(1-x)\right)dx$; $a\neq\frac{1}{2}$. If $\alpha M=\beta N$, $\alpha,\beta\in N$, then the least value of $\alpha^2+\beta^2$ is equal to _____.

Ans. Official answer NTA (5)

Sol.

Question ID : 4058591131

23. If the integral $525\int_0^{\frac{\pi}{2}} \sin 2x\cos^{\frac{11}{2}}x\left(1+\cos^{\frac{5}{2}}x\right)^{\frac{1}{2}}dx$ is equal to $\left(n\sqrt{2}-64\right)$, then n is equal to _____.

Ans. Official answer NTA (176)

Sol.

Question ID : 4058591125

24. Let A = {1, 2, 3, 4} and R = {(1, 2), (2, 3), (1, 4)} be a relation on A. Let S be the equivalence relation on A such that R $\subset$ S and the number of elements in S is n. Then, the minimum value of n is _____.

Ans. Official answer NTA (16)

Sol.

Question ID : 4058591132

25. Let the foci and length of latus rectum of an ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b$ be $(\pm 5, 0)$ and $\sqrt{50}$, respectively.

Then, the square of the eccentricity of the hyperbola $\frac{x^2}{b^2}-\frac{y^2}{a^2b^2}=1$ equals _____.

Ans. Official answer NTA (51)

Sol.

Question ID : 4058591126

26. If α denotes the number of solutions of $|1-i|^x = 2^x$ and $\beta=\left(\frac{|z|}{\arg(z)}\right)$, where $z=\frac{\pi}{4}(1+i)^4\left[\frac{1-\sqrt{\pi}i}{\sqrt{\pi}+i}+\frac{\sqrt{\pi}-i}{1+\sqrt{\pi}i}\right]$, $i=\sqrt{-1}$, then the distance of the point (α, β) from the line $4x-3y=7$ is _____.

Ans. Official answer NTA (3)

Sol.

Question ID : 4058591133

27. Let Q and R be the feet of perpendiculars from the point (a,a,a) on the lines $x = y$, $z = 1$ and $x = -y$, $z = -1$ respectively. If $\angle QPR$ is a right angle, then $12a^2$ is equal to _____.

Ans. Official answer NTA (12)

Sol.

Question ID : 4058591127

28. The total number of words (with or without meaning) that can be formed out of the letters of the word 'DISTRIBUTION' taken four at a time, is equal to _____.

Ans. Official answer NTA (3734)

Sol.

Question ID : 4058591129

29. Let S = $(-1, \infty)$ and $f : S \to R$ be defined as $f(x) = \int_{-1}^{x} (e^t - 1)^{11} (2t-1)^5 (t-2)^7 (t-3)^{12} (2t-10)^{61} dt$.

Let p = Sum of squares of the values of x, where f(x) attains local maxima on S, and q = Sum of the values of x, where f(x) attains local minima on S. Then, the value of $p^2 + 2q$ is _____.

Ans. Official answer NTA (27)

Sol.

Question ID : 4058591134

30. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}| = 1$, $|\vec{b}| = 4$, and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ and the angle between $\vec{b}$ and $\vec{c}$ is α, then $192 \sin^2\alpha$ is equal to _____.

Ans. Official answer NTA (48)

Sol.