JEE Main August- 2021

Question Paper With Text Solution

31 August. | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN AUGUST 2021 | 31 AUGUST SHIFT-1

SECTION – A

1. The number of real roots of the equation $e^{4x} + 2e^{3x} - e^x - 6 = 0$ is :

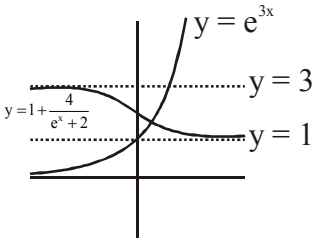
समीकरण $e^{4x} + 2e^{3x} - e^x - 6 = 0$ के वास्तविक मूलों की संख्या है :

(1) 0 (2) 2 (3) 4 (4) 1

Question ID : 86435121221
Option 1 ID : 86435170245
Option 2 ID : 86435170247
Option 3 ID : 86435170248
Option 4 ID : 86435170246

Ans. Official Answer NTA (4)

Sol.



$e^{3x}(e^x + 2) = e^x + 6$

$e^{3x} = 1 + \frac{4}{e^x + 2}$

Number of real roots = 1

2. The sum of 10 terms of the series $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots$ is :

श्रेणी $\frac{3}{1^2 \times 2^2} + \frac{5}{2^2 \times 3^2} + \frac{7}{3^2 \times 4^2} + \ldots$ के 10 पदों का योग है :

(1) $\frac{143}{144}$ (2) $\frac{120}{121}$ (3) 1 (4) $\frac{99}{100}$

Question ID : 86435121225
Option 1 ID : 86435170264
Option 2 ID : 86435170263
Option 3 ID : 86435170261
Option 4 ID : 86435170262

Ans. Official Answer NTA (2)

Sol. $S=\sum_{r=1}^{10}\frac{2r+1}{r^2(r+1)^2}=\sum_{r=1}^{10}\frac{(r+1)^2-r^2}{r^2(r+1)^2}$

$S=\sum_{r=1}^{10}\frac{1}{r^2}-\frac{1}{(r+1)^2}=\left(\left(\frac{1}{1^2}-\frac{1}{2^2}\right)+\left(\frac{1}{2^2}-\frac{1}{3^2}\right)+.....+\left(\frac{1}{10^2}-\frac{1}{11^2}\right)\right)$

$S=1-\frac{1}{121}=\frac{120}{121}$

3. If $a_r=\cos\frac{2r\pi}{9}+i\ \sin\frac{2r\pi}{9}$, $r=1,2,3,...,\ i=\sqrt{-1}$, then the determinant $\begin{vmatrix}a_1 & a_2 & a_3\\ a_4 & a_5 & a_6\\ a_7 & a_8 & a_9\end{vmatrix}$ is equal to :

यदि $a_r=\cos\frac{2r\pi}{9}+i\ \sin\frac{2r\pi}{9}$, $r=1,2,3,...,\ i=\sqrt{-1}$, तो सारणिक $\begin{vmatrix}a_1 & a_2 & a_3\\ a_4 & a_5 & a_6\\ a_7 & a_8 & a_9\end{vmatrix}$ बराबर है :

(1) $a_2a_6-a_4a_8$ (2) $a_1a_9-a_3a_7$ (3) a_9 (4) a_5

Question ID : 86435121222
Option 1 ID : 86435170251
Option 2 ID : 86435170250
Option 3 ID : 86435170249
Option 4 ID : 86435170252

Ans. Official Answer NTA (2)

Sol. $a_r=e^{i\frac{2r\pi}{9}}$, Let $\frac{2\pi}{9}=\theta\Rightarrow a_r=e^{ir\theta}$

$$\begin{vmatrix}a_1 & a_2 & a_3\\ a_4 & a_5 & a_6\\ a_7 & a_8 & a_9\end{vmatrix}=\begin{vmatrix}e^{i\theta} & e^{i2\theta} & e^{i3\theta}\\ e^{i4\theta} & e^{i5\theta} & e^{i6\theta}\\ e^{i7\theta} & e^{i8\theta} & e^{i9\theta}\end{vmatrix}$$

$C_2\to C_2-C_1$

$$=\begin{vmatrix}e^{i\theta} & e^{i\theta}(e^{i\theta}-1) & e^{i3\theta}\\ e^{i4\theta} & e^{i4\theta}(e^{i\theta}-1) & e^{i6\theta}\\ e^{i7\theta} & e^{i4\theta}(e^{i\theta}-1) & e^{i9\theta}\end{vmatrix}=0$$

Now $a_1a_9-a_3a_7=e^{i\theta}.e^{i9\theta}-e^{i3\theta}.e^{i7\theta}$

$=0$

4. If the following system of linear equations

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

has no solution, then :

यदि रैखिक समीकरण निकाय

$2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

का हल कोई नहीं हैं, तो :

(1) $a = -\frac{1}{3}, b \neq \frac{7}{3}$ (2) $a \neq \frac{1}{3}, b = \frac{7}{3}$ (3) $a = \frac{1}{3}, b \neq \frac{7}{3}$ (4) $a \neq -\frac{1}{3}, b = \frac{7}{3}$

Question ID : 86435121223
Option 1 ID : 86435170255
Option 2 ID : 86435170254
Option 3 ID : 86435170253
Option 4 ID : 86435170256

Ans. Official Answer NTA (3)

Sol. $2x + y + z = 5$

$x - y + z = 3$

$x + y + az = b$

Eliminate y

$3x + 2z = 8$

$2x + (a+1)z = (b+3)$

for no solution $\frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$

$$\Rightarrow \frac{3}{2} = \frac{2}{a+1} \neq \frac{8}{b+3}$$

$$\Rightarrow a = \frac{1}{3},\ b \neq \frac{7}{3}$$

5. $\lim\limits_{x\to0}\dfrac{\sin^2(\pi\cos^4 x)}{x^4}$ is equal to :

$\lim\limits_{x\to0}\dfrac{\sin^2(\pi\cos^4 x)}{x^4}$ बराबर है :

(1) $2\pi^2$ (2) $4\pi^2$ (3) π^2 (4) 4π

Question ID : 86435121226
Option 1 ID : 86435170265
Option 2 ID : 86435170267
Option 3 ID : 86435170266
Option 4 ID : 86435170268

Ans. Official Answer NTA (2)

Sol. $\lim\limits_{x\to0}\dfrac{\sin^2(\pi-\pi\cos^4 x)}{x^4}\left(\dfrac{0}{0}\right)$

$\Rightarrow \lim\limits_{x\to0}\dfrac{(\pi-\pi\cos^4 x)^2}{x^4}$

$= \lim\limits_{x\to0}\dfrac{\pi^2\sin^4 x(1+\cos^2 x)^2}{x^4}$

$= 4\pi^2$

6. Let f be a non-negative function in [0, 1] and twice differentiable in (0, 1). If $\int_0^x \sqrt{1-(f'(t))^2}\ dt = \int_0^x f(t)\,dt, 0\le x\le 1$ and $f(0)=0$, then $\lim\limits_{x\to0}\dfrac{1}{x^2}\int_0^x f(t)\ dt$:

(1) equals 0 (2) does not exist (3) equals 1 (4) equals $\dfrac{1}{2}$

माना f, [0, 1] में ऋणेतर तथा (0, 1) में दो बार अवकलनीय है। यदि $\int_0^x \sqrt{1-(f'(t))^2}\ dt = \int_0^x f(t)\,dt, 0\le x\le 1$ तथा $f(0)=0$ हैं, तो $\lim\limits_{x\to0}\dfrac{1}{x^2}\int_0^x f(t)\ dt$:

(1) 0 के बराबर (2) का अस्तित्व नहीं है (3) 1 के बराबर है (4) $\dfrac{1}{2}$ के बराबर है

Question ID : 86435121231
Option 1 ID : 86435170285
Option 2 ID : 86435170288
Option 3 ID : 86435170287
Option 4 ID : 86435170286

Ans. Official Answer NTA (4)

Sol. $\int_0^x \sqrt{1-(f'(t))^2}\,dt = \int_0^x f(t)dt$

use leibintz theorem

$\sqrt{1-(f'(x))^2} = f(x)$

$f'(x) = \sqrt{1-(f(x))^2}$

$\int \frac{f'(x)}{\sqrt{1-(f(x))^2}}dx = \int dx \Rightarrow \sin^{-1}(f(x)) = x + c$

$f(0) = 0 \Rightarrow C = 0$

$f(x) = \sin x$

$\lim_{x\to 0} \frac{\int_0^x f(t)dt}{x^2}\left(\frac{0}{0}\right)$

Use L'Hospital Rule

$\lim_{x\to 0} \frac{f(x)}{2x} = \lim_{x\to 0} \frac{\sin x}{2x} = \frac{1}{2}$

7. cosec18º is a root of the equation :

cosec18º निम्न में से किस समीकरण का एक मूल है ?

(1) $x^2 + 2x - 4 = 0$ (2) $x^2 - 2x + 4 = 0$ (3) $4x^2 + 2x - 1 = 0$ (4) $x^2 - 2x - 4 = 0$

Question Type : MCQ
Question ID : 86435121236
Option 1 ID : 86435170307
Option 2 ID : 86435170306
Option 3 ID : 86435170308
Option 4 ID : 86435170305
Status : Answered

Ans. Official Answer NTA (4)

Sol. $x = \cos ec18^\circ = \frac{4}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1}$

$x = \sqrt{5}+1$

$(x-1)^2 = 5$

$x^2 - 2x - 4 = 0$

8. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|2\vec{a} + 3\vec{b}| = |3\vec{a} + \vec{b}|$ and the angle between $\vec{a}$ and $\vec{b}$ is 60º.

If $\frac{1}{8}\vec{a}$ is a unit vector, then $|\vec{b}|$ is equal to :

माना $\vec{a}$ तथा $\vec{b}$ दो सदिशों हैं जिनके लिए $|2\vec{a} + 3\vec{b}| = |3\vec{a} + \vec{b}|$ हैं और $\vec{a}$ तथा $\vec{b}$ के बीच का कोण 60º है। यदि $\frac{1}{8}\vec{a}$ एक इकाई सदिश हैं, तो $|\vec{b}|$ बराबर है :

(1) 6 (2) 4 (3) 8 (4) 5

Question Type : MCQ
Question ID : 86435121237
Option 1 ID : 86435170311
Option 2 ID : 86435170309
Option 3 ID : 86435170312
Option 4 ID : 86435170310

Ans. Official Answer NTA (4)

Sol. $\vec{a}\hat{}\vec{b} = 60^o$

$|\vec{a}| = 8$

$|2\vec{a} + 3\vec{b}|^2 = |3\vec{a} + \vec{b}|^2$

$4a^2 + 12ab\cos 60^o + 9b^2 = 9a^2 + 6ab\cos 60^o + b^2$

$\Rightarrow b^2 + 3b - 40 = 0$

$\Rightarrow b = 5 (b > 0)$

9. The length of the latus rectum of a parabola, whose vertex and focus are on the positive x-axis at a distance R and S (> R) respectively from the origin, is :

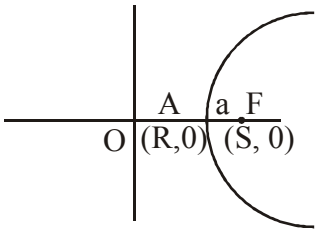
एक परवलय, जिसके शीर्ष तथा नाभि धनात्मक x-अक्ष पर मूल बिन्दु से क्रमशः R तथा S (> R) की दूरी पर हैं, कि नाभिलम्ब जीवा की लम्बाई है :

(1) 2 (S – R) (2) 4 (S – R) (3) 2 (S + R) (4) 4 (S + R)

Question Type : MCQ
Question ID : 86435121233
Option 1 ID : 86435170294
Option 2 ID : 86435170296
Option 3 ID : 86435170293
Option 4 ID : 86435170295

Ans. Official Answer NTA (2)

Sol.



$a = S - R$

$\ell(LR) = 4a$

$= 4(S - R)$

10. Which of the following is not correct for relation R on the set of real numbers ?

निम्न में से कौनसा वास्तविक संख्याओं के समुच्चय पर संबंध R के लिए सही नहीं है ?

(1) $(x, y) \in R \Leftrightarrow 0 < |x| - |y| \le 1$ is neither transitive nor symmetric

$(x, y) \in R \Leftrightarrow 0 < |x| - |y| \le 1$ स्वतुल्य है किन्तु सममित नहीं है

(2) $(x, y) \in R \Leftrightarrow |x - y| \le 1$ is reflexive and symmetric

$(x, y) \in R \Leftrightarrow |x - y| \le 1$ स्वतुल्य तथा सममित है

(3) $(x, y) \in R \Leftrightarrow 0 < |x - y| \le 1$ is symmetric and tansitive

$(x, y) \in R \Leftrightarrow 0 < |x - y| \le 1$ सममित तथा संक्रामक है

(4) $(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ is reflexive but not symmetric

$(x, y) \in R \Leftrightarrow |x| - |y| \le 1$ न तो संक्रामक है न ही सममित है

Question Type : MCQ
Question ID : 86435121220
Option 1 ID : 86435170243
Option 2 ID : 86435170241
Option 3 ID : 86435170244
Option 4 ID : 86435170242

Ans. Official Answer NTA (3)

Sol. $(x, y) \in R \Leftrightarrow 0 < |x - y| \le 1$ is symmetric but not transitive.

11. If p and q are the lengths of perpendiculars from the origin on the lines, $x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$ and $x \sin \alpha + y \cos \alpha = k \sin 2\alpha$ respectively, then k^2 is equal to :

यदि रेखाओं $x \operatorname{cosec} \alpha - y \sec \alpha = k \cot 2\alpha$ तथा $x \sin \alpha + y \cos \alpha = k \sin 2\alpha$ पर मूल बिन्दु से डाले गए लम्बों की लम्बाईयाँ क्रमशः p तथा q है, तो k^2 बराबर है :

(1) $p^2 + 4q^2$ (2) $4p^2 + q^2$ (3) $p^2 + 2q^2$ (4) $2p^2 + q^2$

Question Type : MCQ
Question ID : 86435121232
Option 1 ID : 86435170290
Option 2 ID : 86435170292
Option 3 ID : 86435170289
Option 4 ID : 86435170291

Ans. Official Answer NTA (2)

Sol. $L_1 : x\cos ec\,\alpha - y\sec\alpha = k\cot 2\alpha$

$L_2 : x\sin\alpha - y\cos\alpha = k\sin 2\alpha$

$$P = \left|\frac{0-0-k\cot 2\alpha}{\sqrt{\cos ec^2\alpha + \sec^2\alpha}}\right|$$

$$P^2 = \frac{k^2\cot^2 2\alpha}{\cos ec^2\alpha + \sec^2\alpha} = k^2\cot^2 2\alpha.\sin^2\alpha\cos^2\alpha$$

$$P^2 = \frac{k^2}{4}\cos^2 2\alpha$$

$$q = \left|\frac{0-0-k\sin^2\alpha}{\sqrt{\sin^2\alpha + \cos^2\alpha}}\right|$$

$$q^2 = k^2\sin^2 2\alpha$$

$$4p^2 + q^2 = k^2\cos^2 2\alpha + k^2\sin^2 2\alpha$$

$$k^2 = 4p^2 + q^2$$

12. The line $12x\cos\theta + 5y\sin\theta = 60$ is tangent to which of the following curves ?

रेखा $12x\cos\theta + 5y\sin\theta = 60$ निम्न में से किस वक्र की स्पर्श रेखा है ?

(1) $x^2 + y^2 = 169$ (2) $25x^2 + 12y^2 = 3600$

(3) $144x^2 + 25y^2 = 3600$ (4) $x^2 + y^2 = 60$

Question Type : MCQ
Question ID : 86435121234
Option 1 ID : 86435170298
Option 2 ID : 86435170300
Option 3 ID : 86435170299
Option 4 ID : 86435170297

Ans. Official Answer NTA (3)

Sol. $$\frac{x\cos\theta}{5} + \frac{y\sin\theta}{12} = 1$$

is tangent to the ellipse

$$\frac{x^2}{25}+\frac{y^2}{144}=1$$

$$\Rightarrow 144x^2+252=3600$$

13. A vertical pole fixed to the horizontal ground is divided in the ratio 3 : 7 by a mark on it with lower part shorter than the upper part. If the two parts subtend equal angles at a point on the ground 18 m away from the base of the pole, then the height of the pole (in meters) is :

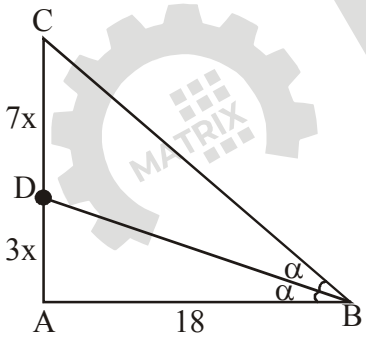
एक ऊर्ध्वाधर पोल एक क्षैतिज धरातल पर स्थित है। इसको 3 : 7 के अनुपात में दो भागों में बांटा गया है जिनमें नीचे का भाग ऊपर के भाग से छोटा है। यदि वह दो भाग, पोल के आधार से धरातल पर 18 मीटर दूर एक बिन्दु पर समान कोण बनाते हैं, तो पोल की ऊँचाई (मीटर में) है :

(1) $12\sqrt{10}$ (2) $8\sqrt{10}$ (3) $12\sqrt{15}$ (4) $6\sqrt{10}$

Question Type : MCQ
Question ID : 86435121238
Option 1 ID : 86435170313
Option 2 ID : 86435170314
Option 3 ID : 86435170316
Option 4 ID : 86435170315

Ans. Official Answer NTA (1)

Sol.



BD is angle bisector

$$\frac{BC}{BA}=\frac{CD}{}=\frac{7}{3}$$

$$\Rightarrow BC=42$$

$$AC=\sqrt{BC^2-AB^2}$$

$$AC=\sqrt{42^2-18^2}=12\sqrt{10}m$$

14. If $\frac{dy}{dx}=\frac{2^{x+y}-2^x}{2^y}$, $y(0)=1$, then $y(1)$ is equal to :

यदि $\frac{dy}{dx}=\frac{2^{x+y}-2^x}{2^y}$, $y(0)=1$ हैं, तो $y(1)$ बराबर है :

(1) $\log_2(2+e)$ (2) $\log_2(1+e)$ (3) $\log_2(2e)$ (4) $\log_2(1+e^2)$

Question Type : MCQ
Question ID : 86435121230
Option 1 ID : 86435170281
Option 2 ID : 86435170284
Option 3 ID : 86435170282
Option 4 ID : 86435170283

Ans. Official Answer NTA (2)

Sol. $\int\frac{2^y dy}{2^y-1}=\int 2^x dx$

$\ln(2^y-1)=2^x+C$

$y(0)=1\Rightarrow C=-1$

$\ln(2^y-1)=2^x-1$

$2^y=1+e^{2^x-1}$

$y=\log_2(1+e^{2^x-1})$

$y(1)=\log_2(1+e)$

15. The function $f(x)=|x^2-2x-3|.\ e^{|9x^2-12x+4|}$ is not differentiable at exactly :

(1) one point (2) three points (3) two points (4) four points

फलन $f(x)=|x^2-2x-3|.\ e^{|9x^2-12x+4|}$:

(1) ठीक एक बिन्दु पर अवकलनीय नहीं है (2) ठीक तीन बिन्दुओं पर अवकलनीय नहीं है

(3) ठीक दो बिन्दुओं पर अवकलनीय नहीं है (4) ठीक चार बिन्दुओं पर अवकलनीय नहीं है

Question Type : MCQ
Question ID : 86435121228
Option 1 ID : 86435170273
Option 2 ID : 86435170275
Option 3 ID : 86435170274
Option 4 ID : 86435170276

Ans. Official Answer NTA (3)

Sol. $f(x)=|x^2-2x-3|\,e^{|(3x-2)^2|}$

$f(x) = |(x-3)(x+1)|\, e^{(3x-2)^2}$

Not differentiable at $x = -183$

16. Let $*, \square \in \{\wedge, \vee\}$ be such that the Bolean expression $(p * \sim q) \Rightarrow (p \square q)$ is a tautology. Then :

माना $*, \square \in \{\wedge, \vee\}$ इस प्रकार है कि बूलीय व्यंजक $(p * \sim q) \Rightarrow (p \square q)$ एक पूनरूक्ति है, तो :

(1) $* = \wedge, \square = \vee$ (2) $* = \wedge, \square = \wedge$ (3) $* = \vee, \square = \wedge$ (4) $* = \vee, \square = \vee$

Question Type : MCQ
Question ID : 86435121239
Option 1 ID : 86435170318
Option 2 ID : 86435170317
Option 3 ID : 86435170319
Option 4 ID : 86435170320

Ans. Official Answer NTA (1)

Sol. $(p \wedge \sim q) \rightarrow (p \vee q)$

$\Rightarrow \sim (p \wedge \sim q) \vee (p \vee q)$

$\Rightarrow (\sim p \vee q) \vee (p \vee q)$

$\Rightarrow (\sim p \vee p) \vee (q \vee q)$

$\Rightarrow t \vee q = t$

17. Three numbers are in an increasing geometric progression with common ratio r. If the middle number is doubled, then the new numbers are in an arithmetic progression with common difference d. If the fourth term of GP is $3r^2$, then $r^2 - d$ is equal to :

तीन संख्याऐं एक वर्धमान गुणोत्तर श्रेढ़ी, जिसका सार्व अनुपात r है, में हैं। यदि बीच की संख्या को दुगुना कर दिया जाये, तो नयी संख्याऐं एक समान्तर श्रेढ़ी, जिसका सार्वअंतर d हैं, में हैं। यदि गुणोत्तर श्रेढ़ी का चौथ पद $3r^2$ हैं, तो $r^2 - d$ बराबर है :

(1) $7 - \sqrt{3}$ (2) $7 - 7\sqrt{3}$ (3) $7 + 3\sqrt{3}$ (4) $7 + \sqrt{3}$

Question Type : MCQ
Question ID : 86435121224
Option 1 ID : 86435170259
Option 2 ID : 86435170257
Option 3 ID : 86435170260
Option 4 ID : 86435170258

Ans. Official Answer NTA (4)

Sol. $\frac{3}{r}, 3, 3r, 3r^2, ...$

$\frac{3}{r}, 6, 3r$ are in A.P.

$$12 = \frac{3}{r} + 3r$$

$$\Rightarrow r^2 - 4r + 1 = 0 \quad (r > 0)$$

$$r = 2 + \sqrt{3}$$

$$d = 3r - 6 = 3\sqrt{3}$$

$$r^2 - d = (2+\sqrt{3})^2 - (3\sqrt{3})$$

$$= 7 + \sqrt{3}$$

18. Let the equation of the plane, that passes through the point (1, 4, –3) and contains the line of intersection of the planes $3x - 2y + 4z - 7 = 0$ and $x + 5y - 2z + 9 = 0$, be $\alpha x + \beta y + \gamma z + 3 = 0$, then $\alpha + \beta + \gamma$ is equal to :
माना समतल, जो बिन्दु (1, 4, –3) से होकर जाता है तथा जिसमें समतलों $3x - 2y + 4z - 7 = 0$ तथा $x + 5y - 2z + 9 = 0$ की प्रतिच्छेदन रेखा स्थित है, का समीकरण $\alpha x + \beta y + \gamma z + 3 = 0$ है, तो $\alpha + \beta + \gamma$ बराबर है :

(1) 15 (2) 23 (3) – 23 (4) – 15

Question Type : MCQ
Question ID : 86435121235
Option 1 ID : 86435170302
Option 2 ID : 86435170301
Option 3 ID : 86435170304
Option 4 ID : 86435170303

Ans. Official Answer NTA (3)

Sol. $P_1 : 3x - 2y + 4z - 7 = 0$

$P_2 : x + 5y - 2z + 9 = 0$

$P : P_1 + \lambda P_2 = 0$

$(3x - 2y + 4z - 7) + \lambda(x + 5y - 2z + 9) = 0$

$\theta = \frac{\pi}{4}$

$$P : (3x - 2y + 4z - 7) + \frac{2}{3}(x + 5y - 2z + 9) = 0$$

$$P : -11x - 4y - 8z + 3 = 0$$

$$\alpha + \beta + \gamma = -11 - 4 - 8 = -23$$

19. If the function $f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , \quad x<0 \\ k & , \quad x=0 \\ \frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1} & , \quad x>0\end{cases}$ is continuous at $x=0$, then $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ is equal to :

यदि फलन $f(x)=\begin{cases}\frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right) & , \quad x<0 \\ k & , \quad x=0 \\ \frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1} & , \quad x>0\end{cases}$

$x=0$ पर संतत हैं, तो $\frac{1}{a}+\frac{1}{b}+\frac{4}{k}$ बराबर है :

(1) 5 (2) -5 (3) -4 (4) 4

Question Type : MCQ
Question ID : 86435121227
Option 1 ID : 86435170270
Option 2 ID : 86435170269
Option 3 ID : 86435170271
Option 4 ID : 86435170272

Ans. Official Answer NTA (2)

Sol. $f(0)=k$

$$f(0^+)=\lim_{x\to 0^+}\frac{\cos^2 x-\sin^2 x-1}{\sqrt{x^2+1}-1}\times\frac{\sqrt{x^2+1}+1}{\sqrt{x^2+1}+1}$$

$$f(0^+)=\lim_{x\to 0^+}\frac{-2\sin^2 x}{x^2}(\sqrt{x^2+1}+1)$$

$$f(0^+)=-4$$

$$f(0^-)=\lim_{x\to 0^-}\frac{1}{x}\log_e\left(\frac{1+\frac{x}{a}}{1-\frac{x}{b}}\right)$$

$(9\sqrt{2},2)$

$$f(0^+)=\frac{1}{a}+\frac{1}{b}$$

$$f(0)=f(0^+)=f(0^-)$$

$$k=-4=\frac{1}{a}+\frac{1}{b}\Rightarrow\frac{1}{a}+\frac{1}{b}+\frac{4}{k}=-5$$

20. The integral $\int\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ is equal to :

(where C is a constant of integration)

समाकलन $\int\frac{1}{\sqrt[4]{(x-1)^3(x+2)^5}}dx$ का बराबर है :

(जहाँ C एक समाकलन अचर है)

(1) $\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{\frac{1}{4}}+C$

(2) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{1}{4}}+C$

(3) $\frac{3}{4}\left(\frac{x-1}{x+2}\right)^{\frac{5}{4}}+C$

(4) $\frac{3}{4}\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}+C$

Question Type : MCQ
Question ID : 86435121229
Option 1 ID : 86435170277
Option 2 ID : 86435170278
Option 3 ID : 86435170279
Option 4 ID : 86435170280

Ans. Official Answer NTA (1)

Sol. $$I=\int\frac{dx}{4\sqrt{(x-1)^8.\frac{(x+2)^5}{(x-1)^5}}}$$

$$I=\int\frac{dx}{(x-1)^2\left(\frac{x+2}{x-1}\right)^{\frac{5}{4}}}$$

Let, $\frac{x+2}{x-1}=t$

$\Rightarrow \frac{-3dx}{(x-1)^2}=dt$

$$I=\frac{-1}{3}\int\frac{dt}{t^{5/4}}=\frac{-1}{3}.\frac{t^{-1/4}}{\frac{-1}{4}}+C$$

$$I=\frac{4}{3}\left(\frac{x+2}{x-1}\right)^{-1/4}+C$$

$$I=\frac{4}{3}\left(\frac{x-1}{x+2}\right)^{1/4}+C$$

SECTION - B

1. The square of the distance of the point of intersection of the line $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ and the plane $2x-y+z=6$ from the point $(-1,-1,2)$ is _____.

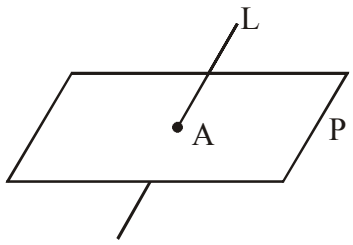
बिन्दु $(-1,-1,2)$ से रेखा $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}$ तथा समतल $2x-y+z=6$ के प्रतिच्छेदन बिन्दु की दूरी का वर्ग बराबर है _____ ।

Question Type : SA

Question ID : 86435121246

Ans. Official Answer NTA (61)

Sol.



$P: 2x-y+z=6$

$$L: \frac{x-1}{2}=\frac{y-2}{3}=\frac{z+1}{6}=\lambda$$

$A(1+2\lambda, 2+3\lambda, -1+6\lambda)$

$\Rightarrow 2+4\lambda-2-3\lambda-1+6\lambda=6$

$\Rightarrow \lambda=1$

$A(3,5,5)$

$B(-1,-1,2)$

$AB^2=61$

2. If the variable line $3x + 4y = \alpha$ lies between the two circles $(x-1)^2 + (y-1)^2 = 1$ and $(x-9)^2 + (y-1)^2 = 4$, without intercepting a chord on either circle, then the sum of all the integral values of α is _____.

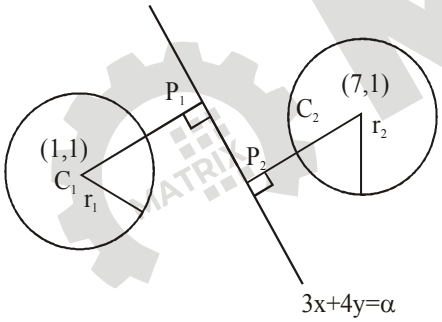
यदि चर रेखा $3x + 4y = \alpha$, तो वृत्तों $(x-1)^2 + (y-1)^2 = 1$ तथा $(x-9)^2 + (y-1)^2 = 4$ के बीच इस प्रकार स्थित है कि यह किसी भी वृत्त से जीवा नहीं बनाती, तो α के सभी पूर्णांक मानों का योग है _____।

Question Type : SA

Question ID : 86435121245

Ans. Official Answer NTA (165)

Sol.



$$\left|z-9\sqrt{2}-2i\right|^2$$

$|7-\alpha| \geq 5$

$7-\alpha \leq -5$ or $7-\alpha \geq 5$

$\alpha \geq 12$ or $\alpha \leq 2$

$\alpha \in (-\infty, 2] \cup [2, \infty)$ .....(1)

$$p_2 > r_2 \Rightarrow \left|\frac{27+4-\alpha}{5}\right| \geq 2$$

$|31-\alpha| \geq 10$

$31-\alpha \geq 10$ or $31-\alpha \leq -10$

$\alpha \leq 21$ or $\alpha \geq 41$

$\alpha \in (-\infty, 21] \cup [41, \infty)$ .....(2)

C_1, C_2 lie on the opposite side of line

$(7-\alpha)(31-\alpha) < 0$

(i) $\cap$ (ii) $\cap$ (iii)

$\alpha \in [12, 21]$

Sum = 12 + 13 + ... + 20 + 21

Sum = 165

3. A point z moves in the complex plane such that $\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$, then the minimum value of $\left|z-9\sqrt{2}-2i\right|^2$ is equal to _____.

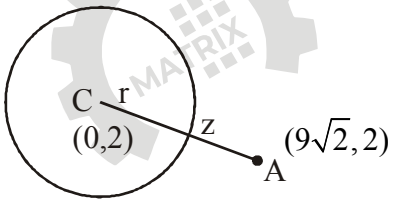
एक बिन्दु z सम्मिश्र समतल में इस प्रकार चलायमान है कि $\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}$ है, तो $\left|z-9\sqrt{2}-2i\right|^2$ का न्यूनतम मान बराबर है _____।

Question Type : SA

Question ID : 86435121241

Ans. Official Answer NTA (98)

Sol.



Let z = x + iy

$$\frac{z-2}{z+2}=\frac{(x-2)+iy}{(x+2)+iy}\times\frac{(x+2)-iy}{(x+2)-iy}$$

$$\frac{z-2}{z+2}=\frac{(x^2+y^2-4)+iy4}{x^2+y^2+4x+4}$$

$$\arg\left(\frac{z-2}{z+2}\right)=\frac{\pi}{4}\Rightarrow x^2+y^2-4=4y$$

$$\Rightarrow x^2+y^2-4y=4$$

$\Rightarrow x^2+(y-2)^2=8 \Rightarrow z$ lies on the circle

$|z-(9\sqrt{2}+2i)|^2_{min}=(AC-r)^2$

$=98$

4. Let [t] denote the greatest integer $\le t$. Then the value of $8.\int_{-\frac{1}{2}}^{1}\left([2x]+|x|\right)dx$ is _____.

माना [t] महत्तम पूर्णांक $\le t$ है। तो $8.\int_{-\frac{1}{2}}^{1}\left([2x]+|x|\right)dx$ का मान है _____।

Question Type : SA

Question ID : 86435121243

Ans. Official Answer NTA (5)

Sol. $I=8\int_{\frac{-1}{2}}^{1}[2x]dx+8\int_{\frac{-1}{2}}^{1}|x|dx$

$I=8\left(\int_{\frac{-1}{2}}^{0}-1.dx+\int_{0}^{\frac{1}{2}}0dx+\int_{\frac{1}{2}}^{1}1dx\right)+8\left(\int_{\frac{-1}{2}}^{0}-xdx+\int_{0}^{1}xdx\right)$

$I=8\left(\frac{-1}{2}+0+\frac{1}{2}\right)+8\left(\frac{-x^2}{2}\Big|_{\frac{-1}{2}}^{0}+\frac{x^2}{2}\Big|_{0}^{1}\right)$

$I=8\left(\frac{1}{8}+\frac{1}{2}\right)=5$

5. The mean of 10 numbers $7\times8, 10\times10, 13\times12, 16\times14,$ .....is _____.

10 संख्याओं $7\times8, 10\times10, 13\times12, 16\times14,$ .....का माध्य है _____।

Question Type : SA

Question ID : 86435121249

Ans. Official Answer NTA (398)

Sol. $T_r=(3r+4).(2r+6)$

$T_r=6r^2+26r+24$

$$\bar{x} = \frac{\sum_{r=1}^{10} T_r}{10} = \frac{6.\frac{10\times11\times21}{6} + 26.\frac{10\times11}{2} + 24\times10}{10}$$

$$\bar{x} = 398$$

6. An electric instrument consists of two units. Each unit must function independently for the instrument to operate. The probability that the first unit functions is 0.9 and that of the second unit is 0.8. The instrument is switched on and it fails to operate. If the probability that only the first unit failed and second unit is functioning is p, then 98 p is equal to _____.

एक बिजली के उपकरण में दो भाग हैं। उपकरण के काम करने के लिए प्रत्येक भाग का स्वतंत्र रूप से काम करना आवश्यक है। पहले भाग के काम करने की प्रायिकता 0.9 है तथा दूसरे भाग के काम करने प्रायिकता 0.8 है। उपकरण को चालू किया जाता है परन्तु यह काम नहीं करता। यदि केवल पहले भाग के काम न करने तथा दूसरे भाग के काम करने की प्रायिकता p है, तो 98 p बराबर है _____।

Question Type : SA

Question ID : 86435121247

Ans. Official Answer NTA (28)

Sol. $P(A) = 0.9$

$P(B) = 0.8$

$$P\left(\frac{\bar{A}\cap B}{\text{Instrument not working}}\right) = \frac{P(\bar{A}\cap B)}{P(\bar{A}\cap\bar{B}) + P(\bar{A}\cap B) + P(A\cap\bar{B})}$$

$$P = \frac{0.1\times0.8}{0.1\times0.2 + 0.1\times0.8 + 0.9\times0.2}$$

$$P = \frac{2}{7}$$

$$98P = 98\times\frac{2}{7} = 28$$

7. If $x\,\phi(x) = \int_5^x \left(3t^2 - 2\phi'(t)\right) dt$, $x > -2$, and $\phi(0) = 4$, then $\phi(2)$ is _____.

यदि $x\,\phi(x) = \int_5^x \left(3t^2 - 2\phi'(t)\right) dt$, $x > -2$, तथा $\phi(0) = 4$ है, तो $\phi(2)$ बराबर है _____।

Question Type : SA

Question ID : 86435121244

Ans. Official Answer NTA (4)

Sol. $x.\phi(x) = \int_5^x (3t^2 - 2\phi'(t))dt$

$\Rightarrow x.\phi'(x) + \phi(x) = 3x^2 - 2\phi'(x)$

$\Rightarrow \phi'(x) + \frac{1}{x+2}\phi(x) = \frac{3x^2}{x+2}$

Let $\phi(x) = y$

$\Rightarrow \frac{dy}{dx} + \frac{y}{x+2} = \frac{3x^2}{x+2}$

I.F. $= e^{\int \frac{1}{x+2}dx} = x+2$

$y.(x+2) = \int \frac{3x^2}{(x+2)}.(x+2)dx$

$y(x+2) = x^3 + C$

$y(0) = 4 \Rightarrow C = 8$

$\phi(x) = \frac{x^3+8}{x+2}$

$\phi(2) = \frac{16}{4} = 4$

8. If 'R' is the least value of 'a' such that function $f(x) = x^2 + ax + 1$ is increasing on $[1, 2]$ and 'S' is the greatest value of 'a' such that the function $f(x) = x^2 + ax + 1$ is decreasing on $[1, 2]$, then the value of $|R - S|$ is _____.

यदि 'a' का न्यूनतम मान, जिसके लिए फलन $f(x) = x^2 + ax + 1$, अंतराल $[1, 2]$ पर वर्धमान हैं, 'R' है तथा 'a' का अधिकतम मान, जिसके लिए फलन $f(x) = x^2 + ax + 1$, अंतराल $[1, 2]$ पर ह्रासमान है, तो $|R - S|$ का मान _____।

Question Type : SA

Question ID : 86435121242

Ans. Official Answer NTA (2)

Sol. $f(x) = x^2 + ax + 1$

$f'(x) = 2x + a$

For increasing $f'(x) \geq 0$

$2x + a \geq \theta$

$a \geq -2x$

$a \geq -2[1, 2]$

$a \geq [-4, -2]$

$R = -2$

For decreasing $f'(x) \leq 0$

$a \leq -2x$

$a \leq [-4, -2]$

$a \leq -4$

$S = -4$

$|R - S| = 2$

9. If $\left(\frac{3^6}{4^4}\right)$k is term, independent of x, in the binomial expansion of $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$, then k is equal to ____.

यदि $\left(\frac{x}{4} - \frac{12}{x^2}\right)^{12}$ के द्विपद प्रसार में x से स्वतंत्र पद $\left(\frac{3^6}{4^4}\right)$k है, तो k बराबर है____।

Question Type : SA

Question ID : 86435121240

Ans. Official Answer NTA (55)

Sol. $T_{r+1} = {}^{12}C_r \left(\frac{x}{4}\right)^{12-r} \left(\frac{-12}{x^2}\right)^r$

$T_{r+1} = {}^{12}C_r \frac{(-12)^r}{4^{12-r}} . x^{12-3r}$

$12 - 3r = 0 \Rightarrow r = 4$

$T_5 = {}^{12}C_4 \frac{12^4}{4^8} = \frac{3^6}{4^4} . k \quad \Rightarrow k = 55$

10. The number of six letter words (with or without meaning), formed using all the letters of the word 'VOWELS', so that all the consonants never come together, is _____.

'VOWELS' शब्द के सभी अक्षरों से, 6 अक्षरों शब्द (अर्थपूर्ण या अर्थहीन) बनाए जाने की संख्या, जिनमें सभी व्यंजन एक साथ न हों, है _____।

Question Type : SA

Question ID : 86435121248

Ans. Official Answer NTA (576)

Sol. Number of ways = Total – all consonants together

$= 6! - 3!.4!$

$= 576$