JEE Main August 2021

Question Paper With Text Solution

31 August. | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN AUGUST 2021 | 31 AUGUST SHIFT-2

SECTION – A

1. The mean and variance of 7 observations are 8 and 16 respectively. If two observations are 6 and 8, then the variance of the remaining 5 observations is :

7 प्रेक्षणों का माध्य तथा प्रसरण क्रमशः 8 तथा 16 हैं। यदि दो प्रेक्षण 6 तथा 8 हैं, तो शेष 5 प्रेक्षणों का प्रसरण है :

(1) $\frac{134}{5}$ (2) $\frac{112}{5}$ (3) $\frac{536}{25}$ (4) $\frac{92}{5}$

Question Type : MCQ
Question ID : 86435121326
Option 1 ID : 86435170578
Option 2 ID : 86435170577
Option 3 ID : 86435170576
Option 4 ID : 86435170575

Ans. Official Answer NTA (3)

Sol. $x_1, x_2, x_3, x_4, x_5, 8, 6$

$$\frac{x_1 + x_2 + x_3 + x_4 + x_5 + 14}{7} = 8$$

$x_1 + x_2 + x_3 + x_4 + x_5 = 42$ ... (1)

$$\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 + 8^2 + 6^2}{7} - 64 = 16$$

$$x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2 = 460$$

So variance of $x_1, x_2 ..... x_5$

$$= \frac{460}{5} - \left(\frac{42}{5}\right)^2 = \frac{536}{25}$$

2. The sum of the roots of the equation, $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$, is :

समीकरण $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 0$ के मूलों का योग है :

(1) $\log_2 13$ (2) $\log_2 11$ (3) $\log_2 12$ (4) $\log_2 14$

Question ID : 86435121312
Option 1 ID : 86435170521
Option 2 ID : 86435170519
Option 3 ID : 86435170520
Option 4 ID : 86435170522

Ans. Official Answer NTA (2)

Sol. $x + 1 - 2\log_2(3 + 2^x) + 2\log_4(10 - 2^{-x}) = 10$

$x + 1 = \log_2(3 + 2^x)^2 - \log_2(10 - 2^{-x})$

$$x+1=\log_2\left(\frac{(3+2^x)^2}{10-2^{-x}}\right)$$

$$2^{x+1}=\frac{9+2^{2x}+6\times 2^x}{10-2^{-x}}$$

$$2\times 2^x=\frac{9+2^{2x}+6\times 2^x}{10-\frac{1}{2^x}}$$

$$2^x=t$$

$$2t=\frac{9+t^2+6t}{10-\frac{1}{t}}$$

$$2t\left(10-\frac{1}{t}\right)=9+t^2+6t$$

$$t^2-14t+11=0 \quad \begin{cases} t_1 \\ t_2 \end{cases}$$

$t_1t_2=11$

$2^{x_1}\times 2^{x_2}=11$

$2^{x_1+x_2}=11$

$x_1+x_2=\log_2^{11}$

3. If $y\frac{dy}{dx}=x\left[\frac{y^2}{x^2}+\frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x>0$, $\phi>0$, and $y(1)=-1$, then $\phi\left(\frac{y^2}{4}\right)$ is equal to :

यदि $y\frac{dy}{dx}=x\left[\frac{y^2}{x^2}+\frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$, $x>0$, $\phi>0$, तथा $y(1)=-1$ हैं, तो $\phi\left(\frac{y^2}{4}\right)$ बराबर है :

(1) $2\phi(1)$ (2) $\phi(1)$ (3) $4\phi(2)$ (4) $4\phi(1)$

Question ID : 86435121320
Option 1 ID : 86435170552
Option 2 ID : 86435170551
Option 3 ID : 86435170554

Option 4 ID : 86435170553

Ans. Official Answer NTA (4)

Sol. $$y\frac{dy}{dx} = x\left[\frac{y^2}{x^2} + \frac{\phi\left(\frac{y^2}{x^2}\right)}{\phi'\left(\frac{y^2}{x^2}\right)}\right]$$

$$y = Vx$$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$vx\left(v + \frac{xdv}{dx}\right) = x\left(v^2 + \frac{\phi(v^2)}{\phi'(v^2)}\right)$$

$$vx\frac{dv}{dx} = \frac{\phi(v^2)}{\phi'(v^2)}$$

$$\int \frac{v\phi'(v^2)}{\phi(v^2)}dv = \int \frac{dx}{x}$$

$$\phi(v^2) = u$$

$$\phi'(v^2) \times 2vdv = du$$

$$v\phi'(v^2)dv = \frac{du}{2}$$

$$\frac{1}{2}\int \frac{du}{u} = \int \frac{dx}{x}$$

$$\frac{1}{2}l\,n\,u = \ln x + c$$

$$\frac{1}{2}l\,n(\phi(v^2)) = \ln x + c$$

$$\frac{1}{2}l\,n\left(\phi\left(\frac{y^2}{x^2}\right)\right) = \ln x + c$$

$$\frac{1}{2}l\,n\left(\phi\left(\frac{1}{1}\right)\right) = c$$

$$c = \frac{1}{2}l\,n(\phi(1))$$

$$\frac{1}{2}ln\left(\phi\left(\frac{y^2}{x^2}\right)\right) = ln\,x + \frac{1}{2}ln(\phi(1))$$

$$\frac{1}{2}ln\left(\phi\left(\frac{y^2}{4}\right)\right) = ln\,2 + \frac{1}{2}ln(\phi(1))$$

$$ln\left(\phi\left(\frac{y^2}{4}\right)\right) = ln\,4 + ln(\phi(1))$$

$$ln\left(\phi\left(\frac{y^2}{4}\right)\right) = ln(4\phi(1))$$

$$\phi\left(\frac{y^2}{4}\right) = 4\phi(1)$$

4. The distance of the point (–1, 2, –2) from the line of intersection of the planes $2x + 3y + 2z = 0$ and $x - 2y + z = 0$ is :

बिन्दु (–1, 2, –2) की समतलों $2x + 3y + 2z = 0$ और $x - 2y + z = 0$ की प्रतिच्छेदन रेखा से दूरी है :

(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{\sqrt{34}}{2}$ (3) $\frac{5}{2}$ (4) $\frac{\sqrt{42}}{2}$

Question ID : 86435121323
Option 1 ID : 86435170563
Option 2 ID : 86435170565
Option 3 ID : 86435170564
Option 4 ID : 86435170566

Ans. Official Answer NTA (2)

Sol. $P_1 : 2x + 3y + 2z = 0$

$\vec{n}_1 = 2\hat{i} + 3\hat{j} + 2\hat{k}$

$P_2 : x - 2y + z = 0$

$\vec{n}_2 = \hat{i} - 2\hat{j} + \hat{k}$

$\vec{b} = \vec{n}_1 \times \vec{n}_2$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 2 \\ 1 & -2 & 1 \end{vmatrix}$$

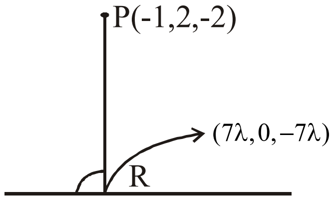
$$= \hat{i}(7) - \hat{j}(0) + \hat{k}(-7)$$

$\vec{b} = 7\hat{i} - 7\hat{k}$

$\vec{r} = \vec{a} + \lambda\vec{b}$

$= (0,0,0) + \lambda(7\hat{i} - 7\hat{k})$

$= 7\lambda(\hat{i} - \hat{k})$



$\overrightarrow{PR} = (7\lambda+1)\hat{i} - 2\hat{j} + (2-7\lambda)\hat{k}$

$\overrightarrow{PR}.\vec{b} = 0$

$7(7\lambda+1) - 7(2-7\lambda) = 0$

$\lambda = \frac{1}{14}$

$R = \left(-\frac{1}{2}, 0, -\frac{1}{2}\right)$

$PR = \sqrt{\left(\frac{3}{2}\right)^2 + 4 + \left(\frac{3}{2}\right)^2}$

$= \sqrt{\frac{9}{2} + 4} = \sqrt{\frac{17}{2}}$

5. The number of solutions of the equation $32^{\tan^2 x} + 32^{\sec^2 x} = 81, 0 \le x \le \frac{\pi}{4}$ is :

समीकरण $32^{\tan^2 x} + 32^{\sec^2 x} = 81, 0 \le x \le \frac{\pi}{4}$ के हलों की संख्या है :

(1) 3 (2) 2 (3) 0 (4) 1

Question ID : 86435121327
Option 1 ID : 86435170582
Option 2 ID : 86435170581
Option 3 ID : 86435170579
Option 4 ID : 86435170580

Ans. Official Answer NTA (4)

Sol. $(32)^{\tan^2 x} + (32)^{\sec^x x} = 81$

$(32)^{\tan^2 x} + 32 \times 32^{\tan^2 x} = 81$

$33 \times (32)^{\tan^2 x} = 81$

$(32)^{\tan^2 x} = \frac{81}{33} = \frac{27}{11}$

$(32)^{\tan^2 x} = \frac{27}{11}$

$x \in \left[0, \frac{\pi}{4}\right]$

$\tan^2 x \in [0,1]$

$32^{\tan^2 x} \in [1,32]$

$\Rightarrow$ one solution

6. The locus of mid-points of the line segments joining (–3, –5) and the points on the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$ is :

बिन्दु (–3, –5) को दीर्घवृत्त $\frac{x^2}{4} + \frac{y^2}{9} = 1$ के बिन्दुओं से मिलाने वाले रेखाखण्डों के मध्य–बिन्दुओं का बिन्दुपथ है :

(1) $36x^2 + 16y^2 + 90x + 56y + 145 = 0$

(2) $9x^2 + 4y^2 + 18x + 8y + 145 = 0$

(3) $36x^2 + 16y^2 + 72x + 32y + 145 = 0$

(4) $36x^2 + 16y^2 + 108x + 80y + 145 = 0$

Question ID : 86435121322
Option 1 ID : 86435170559
Option 2 ID : 86435170560
Option 3 ID : 86435170561
Option 4 ID : 86435170562

Ans. Official Answer NTA (4)

Sol. $\frac{x^2}{4} + \frac{y^2}{9} = 1$

$P(2\cos\theta, 3\sin\theta)$ $\quad$ Q (–3,–5)

$\text{mid point} = \left(\frac{2\cos\theta - 3}{2}, \frac{3\sin\theta - 5}{2}\right) = (h, k)$

$2h = 2\cos\theta - 3$

$\frac{2h+3}{2} = \cos\theta \quad ....(1)$

$2k = 3\sin\theta - 5$

$\frac{2k+5}{3} = \sin\theta$ ....(2)

$(1)^2 + (2)^2$

$\left(\frac{2h+3}{2}\right)^2 + \left(\frac{2k+5}{3}\right)^2 = 1$

$\frac{4h^2+9+12h}{4} + \frac{4k^2+25+20k}{9} = 1$

$36h^2 + 81 + 108h + 16k^2 + 100 + 80k = 36$

$36h^2 + 16k^2 + 108h + 80k + 145 = 0$

$36x^2 + 16y^2 + 108x + 80y + 145 = 0$

7. An angle of intersection of the curves, $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and $x^2 + y^2 = ab$, $a > b$, is :

वक्रों $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ तथा $x^2 + y^2 = ab$, $a > b$ का एक प्रतिच्छेदन कोण है :

(1) $\tan^{-1}(2\sqrt{ab})$ (2) $\tan^{-1}\left(\frac{a+b}{\sqrt{ab}}\right)$ (3) $\tan^{-1}\left(\frac{a-b}{2\sqrt{ab}}\right)$ (4) $\tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$

Question ID : 86435121315
Option 1 ID : 86435170534
Option 2 ID : 86435170532
Option 3 ID : 86435170533
Option 4 ID : 86435170531

Ans. Official Answer NTA (4)

Sol. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ $x^2 + y^2 = ab$

$\frac{2x}{a^2} + \frac{2yy'}{b^2} = 0$

$\frac{yy'}{b^2} = -\frac{x}{a^2}$

$y' = -\frac{b^2}{a^2} \times \frac{x}{y}$

$y'_{(x_1, y_1)} = -\frac{b^2}{a^2}\left(\frac{x_1}{y_1}\right)$

$2x + 2yy' = 0$

$$y' = -\frac{x}{y} = -\frac{x_1}{y_1}$$

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{\frac{x_1}{y_1} - \frac{b^2}{a^2}\left(\frac{x_1}{y_2}\right)}{1 + \frac{b^2}{a^2}\left(\frac{x_1^2}{y_1^2}\right)}\right|$$

$$\tan\theta = \left|\frac{x_1 y_1 (a^2 - b^2)}{a^2 y_1^2 + b^2 x_1^2}\right|$$

$b^2x^2 + a^2y^2 = a^2b^2$

$b^2x_1^2 + a^2y_1^2 = a^2b^2$

$$\tan\theta = \left|\frac{x_1 y_1 (a^2 - b^2)}{a^2 b^2}\right|$$

$y^2 = ab - x^2$

$$\frac{x^2}{a^2} + \frac{ab - x^2}{b^2} = 1$$

$$\frac{x^2}{a^2} + \frac{a}{b} - \frac{x^2}{b^2} = 1$$

$$x^2\left(\frac{1}{a^2} - \frac{1}{b^2}\right) = 1 - \frac{a}{b}$$

$$x^2\left(\frac{b^2 - a^2}{a^2 b^2}\right) = \frac{b - a}{b}$$

$$x^2 = \frac{a^2 b}{b + a}$$

$$y^2 = ab - \frac{a^2 b}{a + b}$$

$$y^2 = \frac{a^2 b + ab^2 - a^2 b}{a + b} = \frac{ab^2}{a + b}$$

$$x^2 y^2 = \frac{a^3 b^3}{(a + b)^2}$$

$$xy = \frac{(a^3b^3)^{1/2}}{a+b}$$

$$\tan\theta = \left|\frac{a^{3/2}b^{3/2}}{a+b} \times \frac{(a-b)(a+b)}{a^2b^2}\right|$$

$$\tan\theta = \left|\frac{a-b}{\sqrt{ab}}\right|$$

$$\tan\theta = \frac{a-b}{\sqrt{ab}}$$

$$\theta = \tan^{-1}\left(\frac{a-b}{\sqrt{ab}}\right)$$

8. Let $S = \{1, 2, 3, 4, 5, 6\}$. Then the probability that a randomly chosen onto function g from S to S satisfies $g(3) = 2g(1)$ is :

माना $S = \{1, 2, 3, 4, 5, 6\}$ है तो S से S में एक यादृच्छिक चुने गये आच्छादक फलन g के $g(3) = 2g(1)$ को संतुष्ट करने की प्रायिकता है :

(1) $\frac{1}{15}$ (2) $\frac{1}{5}$ (3) $\frac{1}{30}$ (4) $\frac{1}{10}$

Question ID : 86435121325
Option 1 ID : 86435170572
Option 2 ID : 86435170574
Option 3 ID : 86435170571
Option 4 ID : 86435170573

Ans. Official Answer NTA (4)

Sol. $g(3) = 2g(1)$ $S = \{1,2,3,4,5,6\}$

can be defined in 3 ways

number of onto functions in this

condition $= 3 \times 4!$

Total umber of onto function $= 6!$

Required probability $= \frac{3 \times \lfloor 4}{\lfloor 6} = \frac{3}{6 \times 5} = \frac{1}{10}$

9. Let A be the set of all points (α, β) such that the area of triangle by the points (5, 6), (3, 2) and (α, β) is 12 square units. Then the least possible length of a line segment joining the origin to a point in A, is :

माना A उन सभी बिन्दुओं (α, β) जिनके लिए बिन्दुओं (5, 6), (3, 2) तथा (α, β) द्वारा बनाए गए त्रिभुज का क्षेत्रफल 12 वर्ग इकाई है तो मूलबिन्दु को A में एक बिन्दु से मिलाने वाले रेखाखण्ड की निम्नतम सम्भव लम्बाई है :

(1) $\frac{8}{\sqrt{5}}$ (2) $\frac{4}{\sqrt{5}}$ (3) $\frac{12}{\sqrt{5}}$ (4) $\frac{16}{\sqrt{5}}$

Question ID : 86435121321
Option 1 ID : 86435170555
Option 2 ID : 86435170556
Option 3 ID : 86435170558
Option 4 ID : 86435170557

Ans. Official Answer NTA (1)

Sol. $$\left|\frac{1}{2}\begin{vmatrix}\alpha & \beta & 1\\ 5 & 6 & 1\\ 3 & 2 & 1\end{vmatrix}\right| = 12$$

$|\alpha(4)-\beta(2)+1(-8)| = 24$

$|4\alpha-2\beta-8| = 24$

$|2\alpha-\beta-4| = 12$

$2\alpha-\beta-4 = \pm 12$

$2\alpha-\beta-4 = 12$ $\qquad$ $2\alpha-\beta-4 = -12$

$2\alpha-\beta = 16$ $\qquad$ $2\alpha-\beta = -8$

$2x-y = 16$ $\qquad$ $2x-y = -8$

$d_1 = \left|\frac{-16}{\sqrt{5}}\right|$ $\qquad$ $d_2 = \left|\frac{8}{\sqrt{5}}\right|$

$d_1 = \frac{16}{\sqrt{5}}$ $\qquad$ $d_2 = \frac{8}{\sqrt{5}}$

10. If z is a complex number such that $\frac{z-i}{z-1}$ is purely, imaginary, then the minimum value of $|z-(3+3i)|$ is :

माना z एक सम्मिश्र संख्या है, जिसके लिए $\frac{z-i}{z-1}$ पूर्ण रूप से काल्पनिक है, तो $|z-(3+3i)|$ का निम्नतम मान है :

(1) $3\sqrt{2}$ (2) $6\sqrt{2}$ (3) $2\sqrt{2}$ (4) $2\sqrt{2}-1$

Question ID : 86435121311
Option 1 ID : 86435170517

Option 2 ID : 86435170518
Option 3 ID : 86435170516
Option 4 ID : 86435170515

Ans. Official Answer NTA (3)

Sol. $z = x + iy$

$$\frac{x+iy-i}{x+iy-1} = \frac{x+i(y-1)}{x-1+iy} \times \frac{(x-1)-iy}{(x-1)-iy}$$

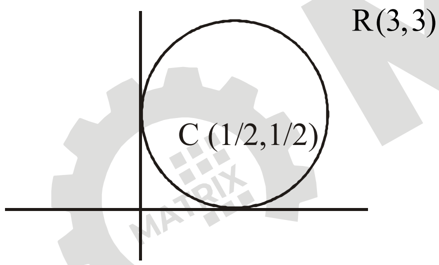
$$= \frac{x(x-1)-ixy+i(y-1)(x-1)+y(y-1)}{(x-1)^2+y^2}$$

$$= \frac{x^2-x+y^2-y+i((y-1)(x-1)-xy)}{(x-1)^2+y^2}$$

$$\frac{x^2+y^2-x-y}{(x-1)^2+y^2} = 0$$

$$x^2+y^2-x-y=0$$

$$C = \left(\frac{1}{2}, \frac{1}{2}\right) \qquad r = \frac{1}{\sqrt{2}}$$



$$|z-(3+3i)|\min = CR - r$$

$$= \sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2} - \frac{1}{\sqrt{2}}$$

$$= \frac{5}{\sqrt{2}} - \frac{1}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

11. Let $f : N \to N$ be a function such that $f(m+n) = f(m) + f(n)$ for every $m, n \in N$. If $f(6) = 18$, then $f(2) \cdot f(3)$ is equal to :

माना $f : N \to N$ एक फलन है, जिसके लिए $f(m+n) = f(m) + f(n)$ $m, n \in N$ है। यदि $f(6) = 18$ है, तो $f(2) \cdot f(3)$ बराबर है :

(1) 36 (2) 54 (3) 6 (4) 18

Question ID : 86435121310
Option 1 ID : 86435170513
Option 2 ID : 86435170514
Option 3 ID : 86435170511
Option 4 ID : 86435170512

Ans. Official Answer NTA (2)

Sol. $f(m + n) = f(m) + f(n)$

$m = n = 1$

$f(2) = 2f(1)$

$f(3) = f(2) + f(1)$

$f(3) = 3f(1)$

$f(2)\, f(3) = 6\,(f(1))^2$

$m = 3 \quad n = 3$

$f(6) = 2f(3)$

$18 = 2f(3)$

$f(3) = 9$

$3f(1) = 9 \Rightarrow f(1) = 3$

$f(2)\, f(3) = 6 \times 9 = 54$

12. If $\alpha = \lim\limits_{x\to\pi/4} \dfrac{\tan^3 x - \tan x}{\cos\left(x + \dfrac{\pi}{4}\right)}$ and $\beta = \lim\limits_{x\to 0}(\cos x)^{\cot x}$ are the roots of the equation, $ax^2 + bx - 4 = 0$, then the ordered pair (a, b) is :

यदि समीकरण $ax^2 + bx - 4 = 0$ के मूल $\alpha = \lim\limits_{x\to\pi/4} \dfrac{\tan^3 x - \tan x}{\cos\left(x + \dfrac{\pi}{4}\right)}$ तथा $\beta = \lim\limits_{x\to 0}(\cos x)^{\cot x}$ हैं, तो क्रमित युग्म (a, b) है :

(1) (–1, –3) (2) (1, –3) (3) (1, 3) (4) (–1, 3)

Question ID : 86435121316
Option 1 ID : 86435170538
Option 2 ID : 86435170536
Option 3 ID : 86435170537
Option 4 ID : 86435170535

Ans. Official Answer NTA (3)

Sol. $\alpha = \lim_{x \to \frac{\pi}{4}} \frac{\tan^3 x - \tan x}{\cos\left(x + \frac{\pi}{4}\right)}$

$= \lim_{x \to \frac{\pi}{4}} \frac{3\tan^2 x \times \sec^2 x - \sec^2 x}{-\sin\left(x + \frac{\pi}{4}\right)}$

$= \frac{3 \times 1 \times 2 - 2}{-1} = -4 = \alpha$

$\beta = \lim_{x \to 0} (\cos x)^{\cot x}$

$= e^{\lim_{x \to 0} \cot x (\cos x - 1)}$

$= e^{\lim_{x \to 0} -\left(\frac{1 - \cos x}{\tan x}\right)}$

$= e^{\lim_{x \to 0} -x\left(\frac{1 - \cos x}{x^2}\right)} = 1 = \beta$

$-\frac{b}{a} = -4 + 1$

$-\frac{b}{a} = -3 \Rightarrow \quad b = 3a$

$-\frac{4}{a} = -4 \times 1$

a = 1

b = 3

(a, b) = (1, 3)

13. If [x] is the greatest integer $\le$ x, then $\pi^2 \int_0^2 \left(\sin \frac{\pi x}{2}\right)\left(x - [x]\right)^{[x]} dx$ is equal to :

यदि [x] महतम पूर्णांक $\le$ x है, तो $\pi^2 \int_0^2 \left(\sin \frac{\pi x}{2}\right)\left(x - [x]\right)^{[x]} dx$ बराबर है :

(1) 4 (π + 1) (2) 2 (π + 1) (3) 4 (π – 1) (4) 2 (π – 1)

Question ID : 86435121318
Option 1 ID : 86435170545
Option 2 ID : 86435170543
Option 3 ID : 86435170546
Option 4 ID : 86435170544

Ans. Official Answer NTA (3)

Sol. $\pi^2\int_0^2\left(\sin\frac{\pi x}{2}\right)\left(x-[x]\right)^{[x]}dx$

$$=\pi^2\left[\int_0^1\sin\frac{\pi x}{2}dx+\int_1^2(x-1)\sin\frac{\pi x}{2}dx\right]$$

$$=\pi^2\left[-\frac{2}{\pi}\cos\frac{\pi x}{2}\Big|_0^1\left((x-1)\left(-\frac{2}{\pi}\cos\frac{\pi x}{2}\right)+\int\frac{2}{\pi}\cos\frac{\pi x}{2}dx\right)\right]$$

$$=\pi^2\left[-\frac{2}{\pi}(0-1)-(x-1)\frac{2}{\pi}\cos\frac{\pi}{x}\Big|_1^2+\frac{4}{\pi^2}\sin\frac{\pi x}{2}\Big|_1^2\right]$$

$$=\pi^2\left[\frac{2}{\pi}-\left(1\times0+1\times\frac{2}{\pi}\times(-1)\right)+\frac{4}{\pi^2}(0-1)\right]$$

$$=\pi^2\left[\frac{2}{\pi}+\frac{2}{\pi}-\frac{4}{\pi^2}\right]$$

$$=\pi^2\left[\frac{4}{\pi}-\frac{4}{\pi^2}\right]=4\pi-4=4(\pi-1)$$

14. The domain of the function $f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$ is :

फलन $f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$ का प्रांत है :

(1) $\left[0,\frac{1}{4}\right]$ (2) $\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$ (3) $\left[0,\frac{1}{2}\right]$ (4) $[-2,0]\cup\left[\frac{1}{4},\frac{1}{2}\right]$

Question ID : 86435121328
Option 1 ID : 86435170583
Option 2 ID : 86435170585
Option 3 ID : 86435170584
Option 4 ID : 86435170586

Ans. Official Answer NTA (2)

Sol. $f(x)=\sin^{-1}\left(\frac{3x^2+x-1}{(x-1)^2}\right)+\cos^{-1}\left(\frac{x-1}{x+1}\right)$

$$-1\le\frac{3x^2+x-1}{x^2-2x+1}\le1 \qquad -1\le\frac{x-1}{x+1}\le1$$

$$\frac{3x^2+x-1}{x^2-2x+1}\ge -1$$

$$\frac{4x^2-x}{(x-1)^2}\ge 0 \Rightarrow \frac{x(4x-1)}{(x-1)^2}\ge 0$$

$$x\in(-\infty,0]\cup\left[\frac{1}{4},1\right)\cup(1_1+\infty)$$

$$\frac{3x^2+x-1}{x^2-2x+1}-1\le 0$$

$$\frac{(x+2)(2x-1)}{(x-1)^2}\le 0$$

+   −   +   +

−2 $\frac{1}{2}$ 1

$$x\in\left[-2,\frac{1}{2}\right]$$

$$x\in[-2,0]\cup\left[\frac{1}{4},\frac{1}{2}\right]$$

$$\frac{x-1}{x+1}\ge -1$$

$$\frac{x}{x+1}\ge 0$$

+   −   +

−1   0

$$x\in(-\infty,-1)\cup[0,+\infty)$$

$$\frac{x-1}{x+1}\le 1$$

$$\frac{1}{x+1}\ge 0$$

$$x>-1$$

$$x\in[0,+\infty)$$

$$x\in\left[\frac{1}{4},\frac{1}{2}\right]\cup\{0\}$$

15. If $\frac{dy}{dx}=\frac{2^x y+2^y\cdot 2^x}{2^x+2^{x+y}\log_e 2}$, $y(0)=0$, then for $y=1$, the value of x lies in the interval :

यदि $\frac{dy}{dx}=\frac{2^x y+2^y\cdot 2^x}{2^x+2^{x+y}\log_e 2}$, $y(0)=0$ है, तो $y=1$ के लिए x का मान निम्न में से किस अंतराल में है :

(1) $\left(\frac{1}{2},1\right]$ (2) (1, 2)   (3) $\left(0,\frac{1}{2}\right]$ (4) (2, 3)

Question ID : 86435121319
Option 1 ID : 86435170548
Option 2 ID : 86435170549
Option 3 ID : 86435170547
Option 4 ID : 86435170550

Ans. Official Answer NTA (2)

Sol. $\frac{dy}{dx} = \frac{2^x y + 2^y.2^x}{2^x + 2^{x+y} l n_2}$

$\frac{dy}{dx} = \frac{2^x(y+2^y)}{2^x(1+2^y l n_2)}$

$\int \frac{1+2^y l n 2}{y+2^y} dy = \int dx$

$y + 2^y = t$

$(1+2^y l n 2)dy = dt$

$\int \frac{dt}{t} = \int dx$

$lnt = x + c$

$ln(y + 2^y) = x + c$

$ln(0 + 1) = 0 + c$

$c = 0$

$ln(y + 2^y) = x$

$ln(1 + 2) = x$

$x = ln3$

$x \in (1, 2)$

16. Negation of the statement $(p \vee r) \Rightarrow (q \vee r)$ is :

कथन $(p \vee r) \Rightarrow (q \vee r)$ का निषेधन है :

(1) $p \wedge q \wedge r$ (2) $\sim p \wedge q \wedge r$ (3) $\sim p \wedge q \wedge \sim r$ (4) $p \wedge \sim q \wedge \sim r$

Question ID : 86435121329
Option 1 ID : 86435170587
Option 2 ID : 86435170588
Option 3 ID : 86435170590
Option 4 ID : 86435170589

Ans. Official Answer NTA (4)

Sol. $(p \vee r) \Rightarrow (q \vee r)$

$\sim (A \Rightarrow B) = A \wedge \sim B$

$\therefore \sim(p \vee r) \Rightarrow q \vee r$

$= (p \vee r) \wedge (\sim q \wedge \sim r)$

$= (p \vee r) \wedge (\sim r) \wedge (\sim q)$

$= p \wedge (\sim r) \wedge (\sim q)$

17. If $\alpha + \beta + \gamma = 2\pi$, then the system of equations

$x + (\cos\gamma)y + (\cos\beta)z = 0$

$(\cos\gamma)x + y + (\cos\alpha)z = 0$

$(\cos\beta)x + (\cos\alpha)y + z = 0$

has :

यदि $\alpha + \beta + \gamma = 2\pi$ है, तो समीकरण निकाय

$x + (\cos\gamma)y + (\cos\beta)z = 0$

$(\cos\gamma)x + y + (\cos\alpha)z = 0$

$(\cos\beta)x + (\cos\alpha)y + z = 0$

(1) a unique solution

के अनंत हल है

(2) exactly two solutions

के ठीक दो हल है

(3) no solution

का कोई हल नहीं है

(4) infinitely many solutions

का अद्वितीय हल है

Question ID : 86435121313
Option 1 ID : 86435170525
Option 2 ID : 86435170526
Option 3 ID : 86435170524
Option 4 ID : 86435170523

Ans. Official Answer NTA (4)

Sol. $\alpha + \beta + \gamma = 2\pi$

$$\begin{vmatrix} 1 & \cos\gamma & \cos\beta \\ \cos\gamma & 1 & \cos\alpha \\ \cos\beta & \cos\alpha & 1 \end{vmatrix}$$

$1(1-\cos^2\alpha) - \cos\gamma(\cos\gamma - \cos\beta\cos\alpha)$

$+\cos\beta(\cos\gamma(\cos\alpha-\cos\beta)$

$1-\cos^2\alpha-\cos^2\gamma+\cos\alpha\cos\beta\cos\gamma+\cos\alpha\cos\beta\cos\gamma-\cos^2\beta$

$1+2\cos\alpha\cos\beta\cos\gamma-\cos^2\gamma-\cos^2\beta-\cos^2\gamma$

$1-\cos^2\alpha-\cos^2\beta-\cos^2\gamma+2\cos(\beta+\gamma)\cos\beta\cos\gamma$

$\sin^2\alpha-\cos^2\beta-\cos^2\gamma+\cos(\beta+\gamma)(\cos(\beta+\gamma)+\cos(\beta-\gamma)$

$\sin^2\alpha-\cos^2\beta-\cos^2\gamma+\cos^2(\beta+\gamma)+\cos(\beta+\gamma)\cos(\beta-\gamma)$

$\sin^2\alpha-\cos^2\beta-\cos^2\gamma+\cos^2(\beta+\gamma)+\cos^2\beta-\sin^2\gamma$

$\sin^2\alpha-(1-\sin^2\gamma)+\cos^2(\beta+\gamma)-\sin^2\gamma$

$\sin^2\alpha-1+\sin^2\gamma+\cos^2(\alpha)-\sin^2\gamma$

$-\cos^2\alpha+\cos^2\alpha=0$

18. Let f be any continuous function on [0, 2] and twice differentiable on (0, 2). If $f(0)=0$, $f(1)=1$ and $f(2)=2$, then :

माना कोई फलन f अंतराल [0, 2] में संतत है तथा (0, 2) में दो बार अवकलनीय है। यदि $f(0)=0, f(1)=1$ तथा $f(2)=2$ हैं तो :

(1) $f'(x)=0$ for some $x\in[0,2]$

किसी $x\in[0,2]$ के लिए $f'(x)=0$ है

(2) $f''(x)=0$ for some $x\in(0,2)$

सभी $x\in(0,2)$ के लिए $f''(x)=0$ है

(3) $f''(x)=0$ for some $x\in(0,2)$

$x\in(0,2)$ के लिए $f''(x)=0$ है

(4) $f''(x)>0$ for some $x\in(0,2)$

सभी $x\in(0,2)$ के लिए $f''(x)>0$ है

Question ID : 86435121317
Option 1 ID : 86435170539
Option 2 ID : 86435170541
Option 3 ID : 86435170542
Option 4 ID : 86435170540

Ans. Official Answer NTA (2)

Sol. $f(0)=0 \quad f(1)=1 \quad f(2)=2$

$g(x)=f(x)-x$

$\left.\begin{array}{l} g(0)=0 \\ g(1)=0 \end{array}\right\} \Rightarrow$ g(x) has three roots

$g(2) = 0$

$g'(x) = f'(x) - 1 \Rightarrow g'(x)$ has atleast two roots

$g''(x) = f''(x) \Rightarrow$ has atleast one roots

19. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors mutually perpendicular to each other and have same magnitude. If a vector $\vec{r}$ satisfies $\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0}$ then $\vec{r}$ is equal to :

माना तीन सदिश $\vec{a}, \vec{b}, \vec{c}$ परस्पर लम्बवत हैं तथा इनके परिमाण बराबर है। यदि एक सदिश $\vec{r}$,

$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = \vec{0}$ को सन्तुष्ट करता है, तो $\vec{r}$ बराबर है :

(1) $\frac{1}{2}(\vec{a} + \vec{b} + 2\vec{c})$ (2) $\frac{1}{3}(2\vec{a} + \vec{b} - \vec{c})$ (3) $\frac{1}{3}(\vec{a} + \vec{b} + \vec{c})$ (4) $\frac{1}{2}(\vec{a} + \vec{b} + \vec{c})$

Question ID : 86435121324
Option 1 ID : 86435170567
Option 2 ID : 86435170570
Option 3 ID : 86435170569
Option 4 ID : 86435170568

Ans. Official Answer NTA (4)

Sol. $\vec{r} = x\vec{a} + y\vec{b} + z\vec{c}$

$|\vec{a}| = |\vec{b}| = |\vec{c}| = k$

$\vec{a} \times \{(\vec{r} - \vec{b}) \times \vec{a}\} + \vec{b} \times \{(\vec{r} - \vec{c}) \times \vec{b}\} + \vec{c} \times \{(\vec{r} - \vec{a}) \times \vec{c}\} = 0$

$\vec{a} \times \left((\vec{r} - \vec{b}) \times \vec{a}\right)$

$= |\vec{a}|^2 (\vec{r} - \vec{b}) - \left(\vec{a}.(\vec{r} - \vec{b})\right)\vec{a}$

$= k^2(\vec{r} - \vec{b}) - \left(\vec{a}.\vec{r}\right)\vec{a}$

$= k^2(\vec{r} - \vec{b}) - \left(\vec{a}.(x\vec{a} + y\vec{b} + z\vec{c})\right)\vec{a}$

$= k^2(\vec{r} - \vec{b}) - \left(k^2 x\right)\vec{a}$

$= k^2(\vec{r} - \vec{b}) - k^2 x\vec{a} + k^2(\vec{r} - \vec{c}) - k^2 y\vec{b} + k^2(\vec{r} - \vec{a}) - k^2 z\vec{c} = 0$

$\vec{r} - \vec{b} - x\vec{a} + \vec{r} - \vec{c} - y\vec{b} + \vec{r} - \vec{a} - z\vec{c} = 0$

$3\vec{r} - \vec{a} - \vec{b} - \vec{c} - x\vec{a} - y\vec{b} - z\vec{c} = 0$

$3\vec{r} - (\vec{a} + \vec{b} + \vec{c}) - (x\vec{a} + y\vec{b} + z\vec{c}) = 0$

$2\vec{r} = \vec{a} + \vec{b} + \vec{c}$

$\vec{r} = \dfrac{\vec{a} + \vec{b} + \vec{c}}{2}$

20. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\dfrac{a_1 + a_2 + \ldots + a_{10}}{a_1 + a_2 + \ldots + a_p} = \dfrac{100}{p^2}$, $p \neq 10$, then $\dfrac{a_{11}}{a_{10}}$ is equal to :

माना $a_1, a_2, a_3, \ldots$ एक A.P. है। यदि $\dfrac{a_1 + a_2 + \ldots + a_{10}}{a_1 + a_2 + \ldots + a_p} = \dfrac{100}{p^2}$, $p \neq 10$ है, तो $\dfrac{a_{11}}{a_{10}}$ बराबर है :

(1) $\dfrac{121}{100}$ (2) $\dfrac{21}{19}$ (3) $\dfrac{100}{121}$ (4) $\dfrac{19}{21}$

Question ID : 86435121314
Option 1 ID : 86435170527
Option 2 ID : 86435170528
Option 3 ID : 86435170530
Option 4 ID : 86435170529

Ans. Official Answer NTA (2)

Sol. $$\frac{\frac{10}{2}(2a + 9d)}{\frac{P}{2}(2a + (P-1)d)} = \frac{100}{P^2}$$

$$\frac{2a + 9d}{2a + (P-1)d} = \frac{10}{P}$$

$$2aP + 9dP = 20a + 10(P-1)d$$

$$2aP = 20a = 10(P-1)d - 9Pd$$

$$a(2P - 20) = d(10P - 10 - 9P)$$

$$2a(P-10) = d(P-10)$$

$$2a = d$$

$$\frac{a_{11}}{a_{10}} = \frac{a + 10d}{a + 9d} = \frac{a + 10(2a)}{a + 9(2a)} = \frac{21a}{19a} = \frac{21}{19}$$

SECTION - B

1. The number of 4-digit numbers which are neither multiple of 7 nor multiple of 3 is______.

4-अंको की संख्याएँ, जो न तो 7 की गुणज हैं न ही 3 की गुणज हैं, की संख्या है______।

Question ID : 86435121331

Ans. Official Answer NTA (5143)

Sol. 4 digit numbers divisible by 3

1002, 1005,............9999

9999 = 1002 + (n –) 3

$$\frac{9999-1002}{3}+1=n$$

$$\frac{8997}{3}+1=n$$

2999 + 1 = n

n = 3000

4-digit numbers divisible by 7

1001, 1008,..............9996

9996 = 1001 + (n – 1)7

$$\frac{9996-1001}{7}+1=n$$

$$\frac{8995}{7}+1=n$$

1285 + 1 = n

n = 1286

4-digit numbers divisible by both 3 and 7

1008, 1029........9996

9996 = 1008 + (n – 1)21

$$\frac{9996-1008}{21}+1=n$$

$$\frac{8988}{21}+1=n$$

428 + 1 = n

n = 429

No. divisible by either 3 or 7

= 3000 + 1286 – 429

= 3857

No. not divisible by 3 and 7

= 9000 – 3857

= 5143

2. If the coefficient of a^7b^8 in the expansion of $(a + 2b + 4ab)^{10}$ is $K \cdot 2^{16}$, then K is equal to______.

यदि $(a + 2b + 4ab)^{10}$ के प्रसार में a^7b^8 का गुणांक $K \cdot 2^{16}$ है, तो K बराबर है ______ ।

Question ID : 86435121332

Ans. Official Answer NTA (315)

Sol. $\frac{\lfloor 10}{\lfloor P_1 \lfloor P_2 \lfloor P_3}(a)^{P_1}(2b)^{P_2}(4ab)^{P_3}$

$P_1 + P_3 = 7$ $\qquad$ $P_2 + P_3 = 8$

$P_1 + P_2 + P_3 = 10$

$7 + P_2 = 10$

$P_2 = 3$ $\qquad$ $P_3 = 5$

$P_1 = 2$

$\frac{\lfloor 10}{\lfloor 2 \lfloor 3 \lfloor 5} \times 2^{13}$

$\frac{10 \times 9 \times 8 \times 7 \times 6}{6 \times 2} \times 2^{13}$

$5 \times 9 \times 7 \times 2^{16} = K \times 2^{16}$

$K = 5 \times 9 \times 7$

$K = 45 \times 7 = 315$

3. The number of elements in the set, $\left\{A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \{-1, 0, 1\} \text{ and } (I - A)^3 = I - A^3\right\}$ where I is 2×2 identity matrix, is_____.

समुच्चय $\left\{A = \begin{pmatrix} a & b \\ 0 & d \end{pmatrix} : a, b, d \in \{-1, 0, 1\} \text{ तथा } (I - A)^3 = I - A^3\right\}$, I, 2×2 का तत्समक आव्यूह है में अवयवों की संख्या है_____ ।

Question ID : 86435121330

Ans. Official Answer NTA (8)

Sol. $(I - A)^3 = I - A^3$

$(I - A)(I - A)(I - A) = I - A^3$

$(I - A - A + A^2)(I - A) = I - A^3$

$(I - 2A + A^2)(I - A) = I - A^3$

$I - 2A + A^2 - A + 2A^2 - A^3 = I - A^3$

$I - 3A + 3A^2 - A^3 = I - A^3$

$3A^2 - 3A = 0$

$A^2 - A = 0$

$A^2 - A$

$$\begin{bmatrix} a & b \\ 0 & d \end{bmatrix}\begin{bmatrix} a & b \\ 0 & d \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

$$\begin{bmatrix} a^2 & ab+bd \\ 0 & d^2 \end{bmatrix} = \begin{bmatrix} a & b \\ 0 & d \end{bmatrix}$$

$a^2 = a$	$ab + bd = b$
	$b(a + d) = b$
$d^2 = d$	$b(a + d - 1) = 0$
$a(a - 1) = 0$	$b = 0$ or $a + d = 1$
$a = 0, 1$	$d = 0, 1$

Total 8 Matrices

4. If $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \dots$, then 160 S is equal to______.

यदि $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \dots$ है, तो 160 S बराबर है ______।

Question ID : 86435121333

Ans. Official Answer NTA (305)

Sol. $S = \frac{7}{5} + \frac{9}{5^2} + \frac{13}{5^3} + \frac{19}{5^4} + \dots\dots$

$\frac{S}{5} = \frac{7}{5^2} + \frac{9}{5^3} + \frac{13}{5^4} + \dots\dots$

$\frac{4S}{5} = \frac{7}{5} + \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^9} + \dots\dots$

$S_0 = \frac{2}{5^2} + \frac{4}{5^3} + \frac{6}{5^4} + \ldots$

$\frac{S_0}{5} = \qquad \frac{2}{5^3} \quad + \quad \frac{4}{5^4} + \ldots$

$\frac{4S_0}{5} = \frac{2}{5^2} + \frac{2}{5^3} + \frac{2}{5^4} + \ldots$

$2S_0 = \frac{1}{5} + \frac{1}{5^2} + \frac{1}{5^3} + \ldots$

$2S_0 = \frac{\frac{1}{5}}{1 - \frac{1}{5}} = \frac{\frac{1}{5}}{\frac{4}{5}} = \frac{1}{4}$

$S_0 = \frac{1}{8}$

$\frac{4S}{5} - \frac{7}{5} = \frac{1}{8}$

$\frac{4S}{5} = \frac{1}{8} + \frac{7}{5} = \frac{61}{40}$

$S = \frac{5}{4} \times \frac{61}{40}$

$160S = 160 \times \frac{5}{4} \times \frac{61}{40} = 61 \times 5 = 305$

5. A tangent line L is drawn at the point (2, –4) on the parabola $y^2 = 8x$. If the line L is also tangent to the circle $x^2 + y^2 = a$, then 'a' is equal to______.

परवलय $y^2 = 8x$ के बिन्दु (2, –4) पर एक स्पर्श रेखा L खींची गई है। यदि रेखा L वृत्त $x^2 + y^2 = a$ की भी स्पर्श रेखा है, तो 'a' बराबर है______।

Question ID : 86435121338

Ans. Official Answer NTA (2)

Sol. $y \times (-4) = 8\left(\frac{x+2}{2}\right)$

$-y = x + 2$

$x + y + 2 = 0$

$\left|\frac{\sqrt{D}}{a}\right| = \frac{C}{a}$

$|\sqrt{2}| = \sqrt{a}$

$a = 2$

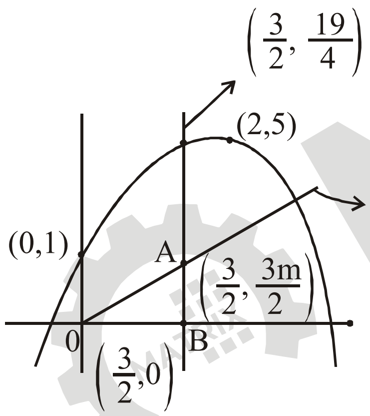
6. If the line $y = mx$ bisects the area enclosed by the lines $x = 0, y = 0, x = \frac{3}{2}$ and the curve $y = 1 + 4x - x^2$, then 12 m is equal to_____.

यदि रेखाओं $x = 0, y = 0, x = \frac{3}{2}$ तथा वक्र $y = 1 + 4x - x^2$ से घिरे क्षेत्र के क्षेत्रफल को रेखा $y = mx$ समद्विभाजित करती है, तो 12 m बराबर है _____।

Question ID : 86435121336

Ans. Official Answer NTA (26)

Sol.



$v = \left(-\frac{b}{2a}, \frac{-D}{4a}\right)$

$v = (2,5)$

$\text{Total Area} = \int_0^{3/2} (1 + 4x - x^2)dx$

$= x + 2x^2 - \frac{x^3}{3}\Big|_0^{3/2}$

$= \frac{3}{2} + 2\left(\frac{9}{4}\right) - \frac{1}{3}\left(\frac{27}{8}\right)$

$= \frac{3}{2} + \frac{9}{2} - \frac{9}{8}$

$= \frac{12 + 36 - 9}{8} = \frac{39}{8}$

Area of $\Delta OAB = \frac{39}{16}$

$\frac{39}{16} = \frac{1}{2} \times \frac{3}{2} \times \frac{3m}{2}$

$\frac{13}{2} = 3m$

$12m = \frac{13}{2} \times 4 = 26$

7. If $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx = \alpha \log_e |1 + \tan x| + \beta \log_e |1 - \tan x + \tan^2 x| + \gamma \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$, when C is constant of integration, then the value of $18(\alpha + \beta + \gamma^2)$ is______.

यदि $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx = \alpha \log_e |1 + \tan x| + \beta \log_e |1 - \tan x + \tan^2 x| + \gamma \tan^{-1}\left(\frac{2\tan x - 1}{\sqrt{3}}\right) + C$, जहाँ C एक समाकलन अचर है, तो $18(\alpha + \beta + \gamma^2)$ का मान बराबर है______ ।

Question ID : 86435121335

Ans. Official Answer NTA (3)

Sol. $\int \frac{\sin x}{\sin^3 x + \cos^3 x} dx$

$\int \frac{\tan x \sec^2 x}{\tan^3 x + 1} dx$

$\tan x = t$

$\sec^2 x dx = dt$

$\int \frac{t dt}{t^3 + 1}$

$\int \frac{t dt}{(t+1)(t^2 - t + 1)}$

$\frac{t}{(t+1)(t^2 - t + 1)} = \frac{A}{t+1} + \frac{Bt + c}{t^2 - t + 1}$

$t = A(t^2 - t + 1) + (t + 1)(Bt + c)$

$t = At^2 - At + A + Bt^2 + Ct + Bt + c$

$t = (A + B)t^2 + (B + C - A)t + A + C$

$A + B = 0$ $\quad$ $B + C - A = 1$ $\quad$ $A + C = 0$

$B + C - A = 1$ $\quad$ $C = -A$

$-A - A - A = 1$ $\quad$ $B = -A$

$A=-\frac{1}{3}$ $\quad C=\frac{1}{3}$ $\quad B=\frac{1}{3}$

$$\int\frac{tdt}{(t+1)(t^2-t+1)}=-\frac{1}{3}\int\frac{dt}{t+1}+\frac{1}{3}\int\frac{t+1}{t^2-t+1}dt=-\frac{1}{3}\ln|t+1|+\frac{1}{3}\int\frac{t+1}{t^2-t+1}dt$$

$$\int\frac{t+1}{t^2-t+1}dt=\frac{1}{2}\int\frac{2t}{t^2-t+1}dt+\int\frac{dt}{t^2-t+1}$$

$$=\frac{1}{2}\int\frac{2t-1}{t^2-t+1}+\frac{3}{2}\int\frac{dt}{t^2-t+1}$$

$$=\frac{1}{2}\ln|t^2-t+1|+\frac{3}{2}\int\frac{dt}{\left(t-\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2}$$

$$=\frac{1}{2}\ln|t^2-t+1|+\frac{3}{2}\times\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{t-\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}}\right)$$

$$=-\frac{1}{3}\ln|t+1|+\frac{1}{6}\ln|t^2-t+1|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2t-1}{\sqrt{3}}\right)$$

$$=-\frac{1}{3}\ln|1+\tan x|+\frac{1}{6}\ln|\tan^2x-\tan x+1|+\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2\tan x-1}{\sqrt{3}}\right)$$

$\alpha=-\frac{1}{3}$ $\quad \beta=\frac{1}{6}$ $\quad \gamma=\frac{1}{\sqrt{3}}$

$$18(\alpha+\beta+\gamma^2)=18\left(-\frac{1}{3}+\frac{1}{6}+\frac{1}{3}\right)$$

$$=18\times\frac{1}{6}=3$$

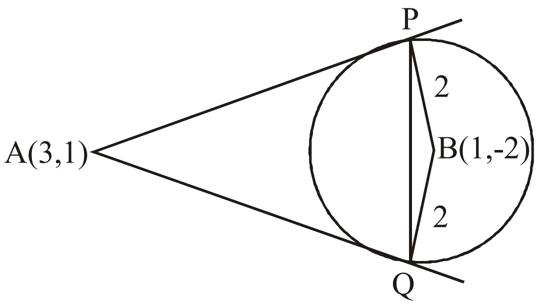
8. Let B be the centre of the circle $x^2+y^2-2x+4y+1=0$. Let the tangents at two points P and Q on the circle intersect at the point A (3, 1). Then $8\cdot\left(\frac{\text{area }\Delta APQ}{\text{area }\Delta BPQ}\right)$ is equal to_____.

माना वृत्त $x^2+y^2-2x+4y+1=0$ का केन्द्र B है। माना वृत्त के दो बिन्दुओं P तथा Q पर स्पर्श रेखाओं का प्रतिच्छेदन बिन्दु A (3, 1) है तो $8\cdot\left(\frac{\Delta APQ \text{ का क्षेत्रफल}}{\Delta BPQ \text{ का क्षेत्रफल}}\right)$ बराबर है ___।

Question ID : 86435121337

Ans. Official Answer NTA (18)

Sol.



$r=\sqrt{1+4-1}=2$

$AP=\sqrt{S_1}=3$

$$8\cdot\left(\frac{\text{area }\Delta APQ}{\text{area }\Delta BPQ}\right)=8\left(\frac{\frac{1}{2}\times 3\times 3\times \sin\theta}{\frac{1}{2}\times 2\times 2\times \sin\theta}\right)=18$$

9. Let f(x) be a cubie polynomical with $f(1)=-10$, $f(-1)=6$, and has a local minima at $x=1$, and $f'(x)$ has a local minima at $x=-1$. Then $f(3)$ is equal to_______.

माना $f(x)$ एक त्रिघातीय बहुपद है जिसके लिए $f(1)=-10$, $f(-1)=6$ हैं, तथा f का एक स्थानीय निम्नतम बिन्दु $x=1$ है और $f'(x)$ का एक स्थानीय निम्नतम बिन्दु $x=-1$ है तो $f(3)$ बराबर है _______ ।

Question ID : 86435121334

Ans. Official Answer NTA (22)

Sol. $f(1)=-10 \qquad f(-1)=6$

$f''(x)=a(x+1)$

$$f'(x)=\frac{ax^2}{2}+ax+c$$

$$f'(1)=0 \quad\Rightarrow\quad c=\frac{-3a}{2}$$

$$f'(x)=\frac{ax^2}{2}+ax-\frac{3a}{2}$$

$f(x) = \frac{ax^3}{6} + \frac{ax^2}{2} - \frac{3ax}{2} + c$

$f(1) = -10$

$f(-1) = 6$

$= 1$ $c = -5$ $a = 6$

$f(x) = x^3 + 3x^2 - 9x - 5$

$f(3) = 22$

10. Suppose the line $\frac{x-2}{\alpha} = \frac{y-2}{-5} = \frac{z+2}{2}$ lies on the plane $x + 3y - 2z + \beta = 0$. The $(\alpha + \beta)$ is equal to_____.

माना रेखा $\frac{x-2}{\alpha} = \frac{y-2}{-5} = \frac{z+2}{2}$ समतल $x + 3y - 2z + \beta = 0$ में स्थित है तो $(\alpha + \beta)$ बराबर है _____ ।

Question ID : 86435121339

Ans. Official Answer NTA (7)

Sol. $\frac{x-2}{\alpha} = \frac{y-2}{-5} = \frac{z+2}{2}$

$\vec{b} = \alpha\hat{i} - 5\hat{j} + 2\hat{k}$

$\vec{n} = \hat{i} + 3\hat{j} - 2\hat{k}$

$\vec{b}.\vec{n} = 0$

$\alpha - 15 - 4 = 0$

$\alpha = 19$

$2 + 6 + 4 + \beta = 0$

$\beta = -12$

$\alpha + \beta = 19 - 12 = 7$