JEE Main January 2023

Question Paper With Text Solution

31 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN JANUARY 2023 | 31TH JANUARY SHIFT-2

SECTION – A

Question ID : 7155051788

1. Let $(a,b)\subset(0,2\pi)$ be the largest interval for which $\sin^{-1}(\sin\theta)-\cos^{-1}(\sin\theta)>0$, $\theta\in(0,2\pi)$, holds. If $\alpha x^2+\beta x+\sin^{-1}\left(x^2-6x+10\right)+\cos^{-1}\left(x^2-6x+10\right)=0$ and $\alpha-\beta=b-a$, then α is equal to :

माना $(a,b)\subset(0,2\pi)$ सबसे बड़ा अंतराल है, जिसके लिए $\sin^{-1}(\sin\theta)-\cos^{-1}(\sin\theta)>0$, $\theta\in(0,2\pi)$ है। यदि $\alpha x^2+\beta x+\sin^{-1}\left(x^2-6x+10\right)+\cos^{-1}\left(x^2-6x+10\right)=0$ तथा $\alpha-\beta=b-a$ है, तो α बराबर है :

(1) $\frac{\pi}{48}$ (2) $\frac{\pi}{16}$ (3) $\frac{\pi}{8}$ (4) $\frac{\pi}{12}$

Ans. Official Answer NTA (4)

Sol. $\sin^{-1}\sin\theta-\left(\frac{\pi}{2}-\sin^{-1}\sin\theta\right)>0$

$\Rightarrow \sin^{-1}\sin\theta>\frac{\pi}{4}$

$\Rightarrow \sin\theta>\frac{1}{\sqrt{2}}$

So, $\theta\in\left(\frac{\pi}{4},\frac{3\pi}{4}\right)$

$\theta\in\left(\frac{\pi}{4},\frac{3\pi}{4}\right)=(a,b)$

$\Rightarrow b-a=\frac{\pi}{2}=\alpha-\beta$

$\Rightarrow \beta=\alpha-\frac{\pi}{2}$

$\Rightarrow \alpha x^2+\beta x+\sin^{-1}\left[(x-3)^2+1\right]+\cos^{-1}\left[(x-3)^2+1\right]=0$

$x=3, 9\alpha+3\beta+\frac{\pi}{2}+0=0$

$\Rightarrow 9\alpha+3\left(\alpha-\frac{\pi}{2}\right)+\frac{\pi}{2}=0$

$\Rightarrow 12\alpha-\pi=0$

$\alpha = \frac{\pi}{12}$

Question ID : 7155051781

2. Let y = y(x) be the solution of the differential equation $(3y^2 - 5x^2)y\,dx + 2x(x^2 - y^2)\,dy = 0$ such that y(1) = 1. Then $|(y(2))^3 - 12y(2)|$ is equal to :

माना अवकल समीकरण $(3y^2 - 5x^2)y\,dx + 2x(x^2 - y^2)\,dy = 0$, y(1) = 1 का हल y = y(x) है तो $|(y(2))^3 - 12y(2)|$ बराबर है :

(1) $32\sqrt{2}$ (2) $16\sqrt{2}$ (3) 32 (4) 64

Ans. Official Answer NTA (1)

Sol. $(3y2 - 5x^2)\,ydx + 2x\,(x^2 - y^2)\,dy = 0$

$$\Rightarrow \frac{dy}{dx} = \frac{y\left(5x^2 - 3y^2\right)}{2x\left(x^2 - y^2\right)}$$

This is homogeneous

$$\text{Let } y = tx \Rightarrow \frac{dy}{dx} = t + x\frac{dt}{dx}$$

$$t + x\frac{dt}{dx} = \frac{(tx)\left(5x^2 - 3t^2x^2\right)}{2x\left(x^2 - t^2x^2\right)} = \frac{t\left(5 - 3t^2\right)}{2\left(1 - t^2\right)}$$

$$x\frac{dt}{dx} = \frac{5t - 3t^3}{2 - 2t^2} - t = \frac{3t - t^3}{2 - 2t^2}$$

$$\int \frac{\left(1 - t^2\right)}{3t - t^3}dt = \int \frac{1}{x}dx$$

Let $3t - t^3 = z \Rightarrow = 3(1 - t^2)dt = dz$

Question ID : 7155051778

3. The absolute minimum value, of the function $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$, where [t] denotes the greatest integer function, in the interval [-1,2], is :

अंतराल [–1, 2] में फलन $f(x) = |x^2 - x + 1| + [x^2 - x + 1]$ जहाँ [t] महत्तम पूर्णांक फलन है, का निरपेक्ष न्यूनतम मान है :

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$ (3) $\frac{1}{4}$ (4) $\frac{5}{4}$

Ans. Official Answer NTA (2)

Sol. $f(x) = |x^2 - x + 1| + [x^2 - x + 1]; x \in [-1, 2]$

Let $g(x) = x^2 - x + 1$

$= \left(x - \frac{1}{2}\right)^2 + \frac{3}{4}$

$\because |x^2 - x + 1|$ and $[x^2 - x + 2]$

Both have minimum value at $x = \frac{1}{2}$

$\Rightarrow$ Minimum $f(x) = \frac{3}{4} + 0$

$= \frac{3}{4}$

Question ID : 7155051774

4. If a point $P(\alpha, \beta, \gamma)$ satisfying

$$(\alpha \quad \beta \quad \gamma)\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = (0 \quad 0 \quad 0)$$

lies on the plane $2x + 4y + 3z = 5$, then $6\alpha + 9\beta + 7\gamma$ is equal to :

यदि $(\alpha \quad \beta \quad \gamma)\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = (0 \quad 0 \quad 0)$

को संतुष्ट करने वाला एक बिंदु $P(\alpha, \beta, \gamma)$ समतल $2x + 4y + 3z = 5$ पर है, तो $6\alpha + 9\beta + 7\gamma$ बराबर है :

(1) -1 (2) 11 (3) $\frac{11}{5}$ (4) $\frac{5}{4}$

Ans. Official Answer NTA (2)

Sol. $$(\alpha\, \beta\, \gamma)\begin{pmatrix} 2 & 10 & 8 \\ 9 & 3 & 8 \\ 8 & 4 & 8 \end{pmatrix} = (0\,0\,0)$$

$2\alpha + 9\beta + 8\gamma = 0$ ____________(1)
$10\alpha + 3\beta + 4\gamma = 0$ ____________(2)
$8\alpha + 8\beta + 8\gamma = 0$ ____________(3)

from (1) and (3) by cross multiplication

$$\frac{\alpha}{1}=\frac{\beta}{6}=\frac{\gamma}{-7}=k \Rightarrow \alpha=k,\ \beta=6k,\ \gamma=-7k$$

Point P (α, β, γ) lie on the plane $2x + 4y + 3z = 5 \Rightarrow 2\alpha + 4\beta + 3\gamma = 5$

$\Rightarrow 2k + 24k - 21k = 5$

$\Rightarrow k = 1$

Hence $6\alpha + 9\beta + 7\gamma = 6k + 54k - 49k = 11k = 11$

Question ID : 7155051786

5. The foot of perpendicular from the origin O to a plane P which meets the co-ordinate axes at the points A, B, C is (2,a,4), $a \in N$. If the volume of the tetrahedron OABC is 144 unit^3, then which of the following points is **NOT** on P?

मूल बिंदु से एक समतल P, जो निर्देशांक अक्षों को बिंदुओं A,B,C पर मिलता है, पर डाले गये लंब का पाद (2,a,4), $a \in N$ है। यदि चतुष्फलक OABC का आयतन 144 घन इकाई है, तो निम्न बिंदुओं में से कौन सा P पर नहीं है ?

(1) (0,4,4) (2) (2,2,4) (3) (0,6,3) (4) (3,0,4)

Ans. Official Answer NTA (4)

Sol. Equation of Plane :

$$(2\hat{i}+a\hat{j}+4\hat{k})\cdot[(x-2)\hat{i}+(y-a)\hat{j}+(z-4)\hat{k}]=0$$

$$\Rightarrow 2x+ay+4z=20+a^2$$

$$\Rightarrow A \equiv \left(\frac{20+a^2}{2},0,0\right)$$

$$B \equiv \left(0,\frac{20+a^2}{a},0\right)$$

$$C \equiv \left(0,0,\frac{20+a^2}{4}\right)$$

$\Rightarrow$ Volume of tetrahedron

$$=\frac{1}{6}[\vec{a}\cdot\vec{b}\vec{c}]$$

$$=\frac{1}{6}\vec{a}(\vec{b}\times\vec{c})$$

$$\Rightarrow \frac{1}{6}\left(\frac{20+a^2}{2}\right)\cdot\left(\frac{20+a^2}{a}\right)\cdot\left(\frac{20+a^2}{4}\right)=144$$

$\Rightarrow \left(20+a^2\right)^3 = 144\times 48\times a$

$\Rightarrow a = 2$

$\Rightarrow$ Equation of plane is $2x + 2y + 4z = 24$

Or $x + y + 2z = 12$

$\Rightarrow$ (3, 0, 4) Not lies on the Plane $x + y + 2z = 12$

Question ID : 7155051772

6. The complex number $z = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ is equal to :

सम्मिश्र संख्या $z = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}}$ बराबर है :

(1) $\cos\frac{\pi}{12} - i\sin\frac{\pi}{12}$ (2) $\sqrt{2}i\left(\cos\frac{5\pi}{12} - i\sin\frac{5\pi}{12}\right)$

(3) $\sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$ (4) $\sqrt{2}\left(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12}\right)$

Ans. Official Answer NTA (3)

Sol. $Z = \dfrac{i-1}{\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}} = \dfrac{\sqrt{2}\,e^{i3\pi/4}}{e^{i\pi/3}} = \sqrt{2}\,e^{i5\pi/12} = \sqrt{2}\left(\cos\frac{5\pi}{12} + i\sin\frac{5\pi}{12}\right)$

Question ID : 7155051777

7. Let : $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ and $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ be there vectors. If $\vec{r}$ is a vector such that, $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ and $\vec{r}\cdot\vec{a} = 0$, then $25|\vec{r}|^2$ is equal to

माना तीन सदिश $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$ तथा $\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$ है। यदि एक सदिश $\vec{r}$ के लिए $\vec{r}\times\vec{b} = \vec{c}\times\vec{b}$ तथा $\vec{r}\cdot\vec{a} = 0$ है, तो $25|\vec{r}|^2$ बराबर है।

(1) 336   (2) 560   (3) 339   (4) 449

Ans. Official Answer NTA (3)

Sol. $\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$

$\vec{b} = \hat{i} - \hat{j} + 2\hat{k}$

$\vec{c} = 5\hat{i} - 3\hat{j} + 3\hat{k}$

$(\vec{r} - \vec{c}) \times \vec{b} = 0, \vec{r} \cdot \vec{a} = 0$

$\Rightarrow \vec{r} - \vec{c} = \lambda\vec{b}$

Also, $(\vec{c} + \lambda\vec{b}) \cdot \vec{a} = 0$

$\Rightarrow \vec{a} \cdot \vec{c} + \lambda(\vec{a} \cdot \vec{b}) = 0$

$\therefore \lambda = \dfrac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{b}} = \dfrac{-8}{5}$

$\vec{\mathbf{r}} = \dfrac{5(5\hat{i} - 3\hat{i} + 3\hat{k}) - 8(\hat{i} - \hat{j} + 2\hat{k})}{5}$

$\vec{r} = \dfrac{17\hat{i} - 7\hat{j} + \hat{k}}{5}$

$|\vec{r}|^2 = \dfrac{1}{25}(289 + 50)$

$25|\vec{r}|^2 = 339$

Question ID : 7155051790

8. The number of values of $r \in \{p, q, \sim p, \sim q\}$ for which $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$ is a tautology, is :

$r \in \{p, q, \sim p, \sim q\}$ के मानों, जिनके लिए $((p \wedge q) \Rightarrow (r \vee q)) \wedge ((p \wedge r) \Rightarrow q)$ एक पुनरूक्ति है, की संख्या है :

(1) 3 (2) 2 (3) 4 (4) 1

Ans. Official Answer NTA (2)

Sol. $p \vee q \rightarrow (r \vee q) \wedge ((p \wedge r) \rightarrow q))$

we know $p \rightarrow q \equiv \sim p \vee q$

Hence $(p \wedge r) \rightarrow q \equiv \sim (p \vee r) \vee q$

$\equiv (\sim p \vee \sim r) \vee q$

$\Rightarrow p \wedge q \rightarrow (r \vee q) \wedge ((\sim p \vee \sim r) \vee q))$

$\Rightarrow \sim (p \wedge q) \vee (r \vee q) \wedge ((\sim p \vee \sim r) \vee q))$

$\Rightarrow (\sim p \vee \sim q) \vee (r \vee q) \wedge ((\sim p \vee \sim r) \vee q))$

$\Rightarrow (\sim p \vee (\sim q \vee r \vee q) \wedge ((\sim p \vee \sim r) \vee q))$

$\Rightarrow (\sim p \vee t) \wedge ((\sim p \vee \sim r) \vee q))$

$\Rightarrow \sim p \vee \sim r \vee q$

when $r = \sim p$ or $r = q \Rightarrow \sim p \vee \sim r \vee q = t$

Question ID : 7155051773

9. The equation $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0, x \in R$ has :

(1) four solutions two of which are negative

(2) two solutions and only one of them is negative

(3) two solutions and both are negative

(4) no solution

समीकरण $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0, x \in R$:

(1) के चार हल हैं जिनमें से दो ऋणात्मक है

(2) के दो हल हैं तथा उनमें से केवल एक ऋणात्मक है

(3) के दो हल हैं जिनमें से दो ऋणात्मक है

(4) का कोई हल नहीं है

Ans. Official Answer NTA (3)

Sol. $e^{4x} + 8e^{3x} + 13e^{2x} - 8e^{x} + 1 = 0$

Let $e^{x} = t$

Now $t^4 + 8t^3 + 13t^2 - 8t + 1 = 0$

Dividing equation by t^2,

$$t^2 + 8t + 13 - \frac{8}{t} + \frac{1}{t^2} = 0$$

$$t^2 + \frac{1}{t^2} + 8\left(t - \frac{1}{t}\right) + 13 = 0$$

$$\left(t - \frac{1}{t}\right)^2 + 2 + 8\left(t - \frac{1}{t}\right) + 13 = 0$$

Let $t - \frac{1}{t} = z$

$$z^2 + 8z + 15 = 0$$

$(z+3)(z+5)=0$

$z=-3$ ro $z=-5$

So, $t-\frac{1}{t}=-3$ or $t-\frac{1}{t}=-5$

$t^2+3t-1=0$ or $t^2+5t-1=0$

$t=\frac{-3\pm\sqrt{13}}{2}$ or $t=\frac{-5\pm\sqrt{29}}{2}$

as $t=e^x$ so t must be positive,

$t=\frac{\sqrt{13}-3}{2}$ or $\frac{\sqrt{29}-5}{2}$

So, $x=\ln\left(\frac{\sqrt{13}-3}{2}\right)$ or $x=\ln\left(\frac{\sqrt{29}-5}{2}\right)$

Hence two solution and both are negative.

Question ID : 7155051789

10. Let the mean and standard deviation of marks of class A of 100 students be respectively 40 and $\alpha(>0)$, and the mean and standard deviation of marks of class B of n students be respectively 55 and $30-\alpha$. If the mean and variance of the marks of the combined class of $100+n$ students are respectively 50 and 350, then the sum of variance of classes A and B is :

माना 100 छात्रों की कक्षा A के छात्रों के अंकों के माध्य तथा मानक विचलन क्रमशः 40 तथा $\alpha(>0)$ हैं तथा n छात्रों की कक्षा B के छात्रों के अंकों के माध्य तथा मानक विचलन क्रमशः 55 तथा $30-\alpha$ है। यदि संयुक्त कक्षा के $100+n$ छात्रों के अंकों के माध्य तथा प्रसरण क्रमशः 50 तथा 350 है, तो कक्षाओं A तथा B के प्रसरणों का योग है :

(1) 650 (2) 500 (3) 450 (4) 900

Ans. Official Answer NTA (2, 4)

Sol.

A	B	A+B
$\bar{x}_1=40$	$\bar{x}_2=55$	$\bar{x}=50$
$\sigma_1=\alpha$	$\sigma_2=30-\alpha$	$\sigma_2=350$
$n_1=100$	$n_2=n$	$100+n$

$$\bar{x}=\frac{100\times 40+55n}{100+n}=50$$

$\Rightarrow n=200$

$$\sigma_1^2=\frac{\sum x_i^2}{100}-40^2=\alpha^2$$

$$\sigma_2^2 = \frac{\sum y_i^2}{200} - 55^2 = (30-\alpha)^2$$

$$\sigma^2 = \frac{\sum x_i^2 + \sum y_i^2}{300} - 50^2 = 350^2$$

$$350^2 = \frac{(1600+\alpha^2)\times 100 + \left[3025 + (30-\alpha)^2\right]\times 200}{300} - 50^2$$

on solving $\alpha^2 - 40\alpha + 300 = 0$

$\Rightarrow \alpha = 10$ or 30

$\alpha = 30$ is not possible

$\sigma_1^2 + \sigma_2^2 = 10^2 + 20^2 = 500$

Question ID : 7155051782

11. Let $f: R - \{2,6\} \to R$ be real valued function defined as $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$. Then range of *f* is

माना $f: R - \{2,6\} \to R$ एक वास्तविक मान फलन है, जो $f(x) = \frac{x^2+2x+1}{x^2-8x+12}$ द्वारा परिभाषित है। तो *f* का परिसर है।

(1) $\left(-\infty, -\frac{21}{4}\right] \cup \left[\frac{21}{4}, \infty\right)$

(2) $\left(-\infty, -\frac{21}{4}\right) \cup [1, \infty)$

(3) $\left(-\infty, -\frac{21}{4}\right] \cup [0, \infty)$

(4) $\left(-\infty, -\frac{21}{4}\right) \cup (0, \infty)$

Ans. Official Answer NTA (3)

Sol. Let $y = \frac{x^2+2x+1}{x^2-8x+12}$

By cross multiplying

$yx^2 - 8xy + 12y - x^2 - 2x - 1 = 0$

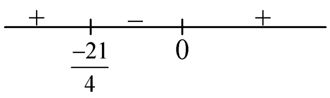
$x^2(y-1) - x(8y+2) + (12y-1) = 0$

Case 1, $y \neq 1$

$D \geq 0$

$\Rightarrow (8y+2)^2 - 4(y-1)(12y-1) \geq 0$

$\Rightarrow y(4y+21) \geq 0$



$$y \in \left(-\infty, \frac{-21}{4}\right] \cup [0, \infty) - \{1\}$$

Case 2, y = 1

$x^2 + 2x + 1 = x^2 - 8x + 12$

$10x = 11$

$x = \frac{11}{10}$ So, y can be 1

Hence $y \in \left(-\infty, \frac{-21}{4}\right] \cup [0, \infty)$

Question ID : 7155051785

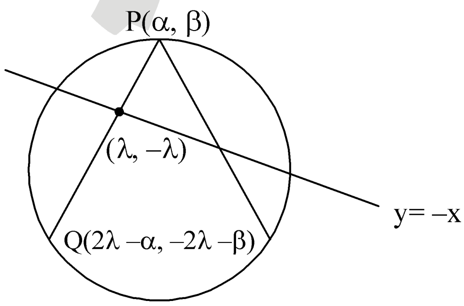
12. The set of all values of a^2 for which the line $x + y = 0$ bisects two distinct chords drawn from a point $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ on the circle $2x^2 + 2y^2 - (1 + a)x - (1 - a)y = 0$, is equal to :

a^2 के सभी मानों, जिनके लिए रेखा $x + y = 0$, वृत्त $2x^2 + 2y^2 - (1 + a)x - (1 - a)y = 0$ के बिंदु $P\left(\frac{1+a}{2}, \frac{1-a}{2}\right)$ से खींची गई दो भिन्न जीवाओं को समद्विभाजित करती है, का समुच्चय बराबर है

(1) (0,4] (2) (8,∞) (3) (4,∞) (4) (2,12]

Ans. Official Answer NTA (2)

Sol.



$$x^2 + y^2 - \left(\frac{1+a}{2}\right)x - \left(\frac{1-a}{2}\right)y = 0$$

Let $\frac{1+a}{2} = \alpha$, $\frac{1-a}{2} = \beta$

circle will be $x^2 + y^2 - \alpha x - \beta y = 0$

Point Q will lie on circle

$(2\lambda-\alpha)^2+(-2\lambda-\beta)^2-\alpha(2\lambda-\alpha)-\beta(-2\lambda-\beta)=0$

$\Rightarrow 4\lambda^2+3\lambda(\beta-\alpha)+(\alpha^2+\beta^2)=0$

Now $D>0 \Rightarrow 9(\beta-\alpha)^2-4(4)(\alpha^2+\beta^2)>0$

$\Rightarrow 9a^2-16\left(\frac{1+a^2}{2}\right)>0$

$\Rightarrow a^2>8$

Question ID : 7155051780

13. Let $\alpha>0$. If $\int_0^\alpha \frac{x}{\sqrt{x+a}-\sqrt{x}}dx=\frac{16+20\sqrt{2}}{15}$, then α is equal to :

माना $\alpha>0$ है। यदि $\int_0^\alpha \frac{x}{\sqrt{x+a}-\sqrt{x}}dx=\frac{16+20\sqrt{2}}{15}$ है, तो α बराबर है :

(1) $\sqrt{2}$ (2) 4 (3) $2\sqrt{2}$ (4) 2

Ans. Official Answer NTA (4)

Sol. After rationalising

$\int_0^\alpha \frac{x}{\alpha}(\sqrt{x+\alpha}+\sqrt{x})$

$\int_0^\alpha \frac{1}{\alpha}\left[(x+\alpha)^{3/2}-\alpha(x+\alpha)^{1/2}+x^{3/2}\right]$

$\frac{1}{\alpha}\left[\frac{2}{5}(x+\alpha)^{5/2}-\alpha\frac{2}{3}(x+\alpha)^{3/2}+\frac{2}{5}x^{5/2}\right]\Big|_0^\alpha$

$=\frac{1}{\alpha}\left(\frac{2}{5}(2\alpha)^{5/2}-\frac{2\alpha}{3}(2\alpha)^{3/2}+\frac{2}{5}\alpha^{5/2}-\frac{2}{5}\alpha^{5/2}+\frac{2}{3}\alpha^{5/2}\right)$

$=\frac{1}{\alpha}\left(\frac{2^{7/2}\alpha^{5/2}}{5}-\frac{2^{5/2}\alpha^{5/2}}{3}+\frac{2}{3}\alpha^{5/2}\right)$

$=\alpha^{3/2}\left(\frac{2^{7/2}}{5}-\frac{2^{5/2}}{3}+\frac{2}{3}\right)$

$$=\frac{\alpha^{3/2}}{15}(24\sqrt{2}-20\sqrt{2}+10)=\frac{\alpha^{3/2}}{15}(4\sqrt{2}+10)$$

Now, $\frac{\alpha^{\frac{3}{2}}}{15}\left(4\sqrt{2}+10\right)=\frac{16+20\sqrt{2}}{15}$

$\Rightarrow \alpha = 2$

Question ID : 7155051787

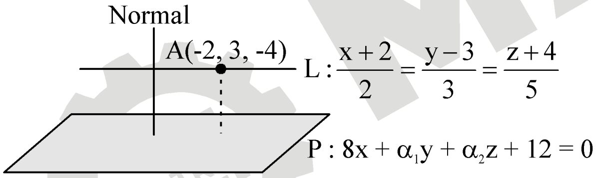
14. Let the plane $P: 8x+\alpha_1 y+\alpha_2 z+12=0$ be parallel to the line $L:\frac{x+2}{2}=\frac{y-3}{3}=\frac{z+4}{5}$. If the intercept of P on the y-axis is 1, then the distance between P and L is :

माना समतल $P: 8x+\alpha_1 y+\alpha_2 z+12=0$ रेखा $L:\frac{x+2}{2}=\frac{y-3}{3}=\frac{z+4}{5}$ के समांतर है। यदि P का y-अक्ष पर अंतःखंड 1 है, तो P तथा L के बीच दूरी है :

(1) $\frac{6}{\sqrt{14}}$ (2) $\sqrt{\frac{2}{7}}$ (3) $\sqrt{\frac{7}{2}}$ (4) $\sqrt{14}$

Ans. Official Answer NTA (4)

Sol.



Normal $\perp$ L $\Rightarrow 8\times 2+3\alpha_1+5\alpha_2=0$

For Y intercept of plane put x = z = 0 in equation of plane

$$\Rightarrow y=\frac{-12}{\alpha_1}=1\Rightarrow \alpha_1=-12$$

Put α_1 in equation (1) we get $\alpha_2 = 4$

Hence equation of plane P : $2x-3y+z+3=0$

Now distance of point A (–2, 3, –4) from the plane P is $=\left|\frac{2(-2)-3(3)-4+3}{\sqrt{(2)^2+(-3)^2+(1)^2}}\right|=\sqrt{14}$

Question ID : 7155051771

15. Among the relations

$S=\left\{(a,b):a,b\in R-\{0\},2+\frac{a}{b}>0\right\}$ and $T=\left\{(a,b):a,b\in R,a^2-b^2\in Z\right\}$

(1) S is transitive but T is not
(2) T is symmetridc but S is not
(3) neither S nor T is transitive
(4) both S and T are symmetric

संबंधों

$S=\left\{(a,b):a,b\in R-\{0\},2+\frac{a}{b}>0\right\}$ तथा $T=\left\{(a,b):a,b\in R,a^2-b^2\in Z\right\}$ में

(1) S संक्रामक है परन्तु T नहीं है
(2) T सममित है परन्तु S नहीं है
(3) न तो S न हीं T संक्रामक है
(4) S तथा T दोनों सममित है

Ans. Official Answer NTA (2)

Sol. For relation $T=a^2-b^2=-I$

Then, (b, a) on relation R

$\Rightarrow b^2-a^2=-I$

$\therefore$ T is symmetric

$S=\left\{(a,b):a,b\in R-\{0\},2+\frac{a}{b}>0\right\}$

$2+\frac{a}{b}>0\Rightarrow\frac{a}{b}>-2,\Rightarrow\frac{b}{a}<\frac{-1}{2}$

If (b, a) $\in$ S then $2+\frac{b}{a}$ not necessarily positive

$\therefore$ S is not symmetric

Question ID : 7155051776

16. $\lim\limits_{x\to\infty}\frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$

(1) does not exist
(2) is equal to 27
(3) is equal to 9
(4) is equal to $\frac{27}{2}$

$\lim\limits_{x\to\infty}\frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}x^3$

(1) का अस्तित्व नहीं है (2) 27 के बराबर है

(3) 9 के बराबर है (4) $\frac{27}{2}$ के बराबर है

Ans. Official Answer NTA (2)

Sol. $$\lim_{x\to\infty} x^3 \frac{\left(\sqrt{3x+1}+\sqrt{3x-1}\right)^6+\left(\sqrt{3x+1}-\sqrt{3x-1}\right)^6}{\left(x+\sqrt{x^2-1}\right)^6+\left(x-\sqrt{x^2-1}\right)^6}$$

$(a+b)^6 + (a-b)^6 = 2\,(T_0 + T_2 + T_4 + T_6)$
$= 2\,({}^6C_0\, a^6 + {}^6C_2\, a^4b^2 + {}^6c_4\, a^2\, b^4 + {}^6C_6\, b^6)$

$$\lim_{x\to\infty}\frac{\left(2(3x+1)^3+30(3x+1)^2(3x-1)+30(3x+1)(3x-1)^2+2(3x-1)^3\right)x^3}{2x^6+30x^4\left(x^2-1\right)+30x^2\left(x^2-1\right)^2+\left(x^2-1\right)^3}$$

$$\lim_{x\to\infty}\frac{x^6\left[2\left(3+\frac{1}{x}\right)^3+30\left(3+\frac{1}{x}\right)^2\left(3-\frac{1}{x}\right)+30\left(3+\frac{1}{x}\right)\left(3-\frac{1}{x}\right)^2+2\left(3-\frac{1}{x}\right)^3\right]}{x^6\left[2+30\left(1-\frac{1}{x^2}\right)+30\left(1-\frac{1}{x^2}\right)^2+2\left(1-\frac{1}{x^2}\right)^3\right]}$$

$$=\frac{2(3)^3+30(3)^3+30(3)^3+2(3)^3}{2+30+30+2}=(3)^3=27$$

Question ID : 7155051779

17. If $\phi(x)=\frac{1}{\sqrt{x}}\int_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t-3\phi'(t)\right)dt$, $x>0$, then $\phi'\left(\frac{\pi}{4}\right)$ is equal to :

यदि $\phi(x)=\frac{1}{\sqrt{x}}\int_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t-3\phi'(t)\right)dt$, $x>0$ है, तो $\phi'\left(\frac{\pi}{4}\right)$ बराबर है :

(1) $\frac{8}{6+\sqrt{\pi}}$ (2) $\frac{4}{6+\sqrt{\pi}}$ (3) $\frac{8}{\sqrt{\pi}}$ (4) $\frac{4}{6-\sqrt{\pi}}$

Ans. Official Answer NTA (1)

Sol. $$\phi'(x)=\frac{1}{\sqrt{x}}\left[\left(4\sqrt{2}\sin x-3\phi'(x)\right)\cdot 1-0\right]-\frac{1}{2}x^{-3/2}$$

$\int_{\frac{\pi}{4}}^{x}\left(4\sqrt{2}\sin t-3\phi'(t)\right)dt$

$\phi'\left(\frac{\pi}{4}\right)=\frac{2}{\sqrt{\pi}}\left[4-3\phi'\left(\frac{\pi}{4}\right)\right]+0$

$\left(1+\frac{6}{\sqrt{\pi}}\right)\phi'\left(\frac{\pi}{4}\right)=\frac{8}{\sqrt{\pi}}$

$\phi'\left(\frac{\pi}{4}\right)=\frac{8}{\sqrt{\pi}+6}$

Question ID : 7155051783

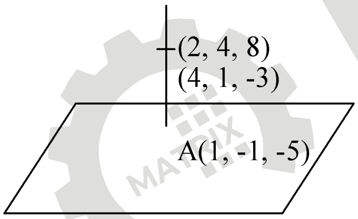
18. Let P be the plane, passing through the point (1,-1,-5) and perpedicular to the line joining the points (4,1,-3) and (2,4,3). Then the distance of P from the point (3,-2,2) is

माना बिंदु (1,-1,-5) से होकर जाने वाले तथा बिंदुओं (4,1,-3) तथा (2,4,3) को मिलाने वाली रेखा के लंबवत समतल P है। तो P की बिंदु (3,-2,2) से दूरी है।

(1) 7 (2) 6 (3) 5 (4) 4

Ans. Official Answer NTA (3)

Sol.



D'r of Plane Normal : 2, –3, –6
Equation of Plane : 2 (x – 1) –3(j + 1) – 6 (z + 5) = 0
$2x-3y-6z-35=0$
Hence ⊥ distance of plane from (3, –2, 2)

$$\perp \text{distance} = \left|\frac{6+6-12-35}{\sqrt{2^2+(-3)^2+(-6)^2}}\right| = 5$$

Question ID : 7155051784

19. Let H be the hyperbola, whose foci are $\left(1\pm\sqrt{2},0\right)$ and eccentricity is $\sqrt{2}$. Then the length of its latus rectum is ____________ .

माना अतिपरवलय H की नाभियाँ $\left(1\pm\sqrt{2},0\right)$ तथा उत्केन्द्रता $\sqrt{2}$ है। तो H की नाभिलंब जीवा की लंबाई है :

(1) $\frac{5}{2}$ (2) 3 (3) $\frac{3}{2}$ (4) 2

Ans. Official Answer NTA (4)

Sol. $2ae=\left|\left(1+\sqrt{2}\right)-\left(1+\sqrt{2}\right)\right|=2\sqrt{2}$

$ae=\sqrt{2}$

$a = 1$

$\Rightarrow b=1\ \because e=\sqrt{2}\Rightarrow$ Hyperbola is rectangular

$\Rightarrow \text{L.R}=\frac{2b^2}{a}=2$

Question ID : 7155051775

20. Let $a_1, a_2, a_3, ...$ be an A.P. If $a_7 = 3$, the product a_1a_4 is minimum and the sum of its first n terms is zero, then $n! - 4a_{n(n+2)}$ is equal to :

माना $a_1, a_2, a_3, ...$ एक A.P. है। यदि $a_7 = 3$ है, गुणनफल a_1a_4 न्यूनतम है, तथा इसके प्रथम पदों का योग शून्य है, तो $n! - 4a_{n(n+2)}$ बराबर है :

(1) $\frac{33}{4}$ (2) 24 (3) 9 (4) $\frac{381}{4}$

Ans. Official Answer NTA (2)

Sol. $a_7=3=a+6d$

$\Rightarrow a=3-6d$

$a_1a_4 = a(a + 3d)$

$= (3 - 6d)(3 - 3d)$

$= 18d^2 - 27d + 9$

$d_{min}=\frac{27}{2\times18}=\frac{3}{4}$

$a=\frac{-3}{2}=\frac{-3}{2}$

$S_n=\frac{n}{2}[2a+(n-1)d]=0$

$-3+(n-1)\frac{3}{4}=0$

$\Rightarrow n = 5$

Now $n! - 4a_{n(n+2)} = 5! - 4a_{35}$

$= 120 - 4(a + 34d)$

$= 120 - 4\left(\frac{-3}{2} + 34 \times \frac{3}{4}\right)$

$= 120 + 6 - 102 = 24$

SECTION – B

Question ID : 7155051799

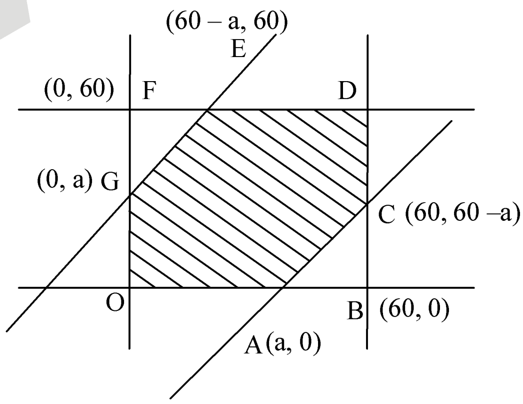
21. Let A be the event that the absolute difference between two randomly choosen real numbers in the sample space [0, 60] is less than or equal to a. If $P(A) = \frac{11}{36}$, then a is equal to ________ .

माना प्रतिदर्श समष्टि [0, 60] से यादृच्छया चुनी गई दो वास्तविक संख्याओं का निरपेक्ष अंतर a से कम या इसके बराबर होने की घटना A है। यदि $P(A) = \frac{11}{36}$ है, तो a बराबर है।

Ans. Official Answer NTA (10)

Sol. $|x - y| < a \Rightarrow -a < x - y < a$

$\Rightarrow x - y < a$ and $x - y > -a$



$P(A) = \frac{ar(OACDEG)}{(OBDF)}$

$= \frac{ar(OBDF) - ar(ABC) - ar(EFG)}{ar(OBDF)}$

$\Rightarrow \frac{11}{36} = \frac{(60)^2 - \frac{1}{2}(60-a)^2 - \frac{1}{2}(60-a)^2}{3600}$

$\Rightarrow \quad 1100 = 3600 - (60-a)^2$

$\Rightarrow (60-a)^2 = 2500 \Rightarrow 60 - a = 50$

$\Rightarrow \quad a = 10$

Question ID : 7155051798

22. Let $\vec{a}, \vec{b}, \vec{c}$ be three vectors such that $|\vec{a}| = \sqrt{31}$, $4|\vec{b}| = |\vec{c}| = 2$ and $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$. If the angle between $\vec{b}$ and $\vec{c}$ is $\frac{2\pi}{3}$, then $\left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}}\right)^2$ is equal to ____________ .

माना तीन सदिशों $\vec{a}, \vec{b}, \vec{c}$ के लिए $|\vec{a}| = \sqrt{31}$, $4|\vec{b}| = |\vec{c}| = 2$ तथा $2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a})$ है। यदि $\vec{b}$ तथा $\vec{c}$ के बीच कोण $\frac{2\pi}{3}$ है, तो $\left(\frac{\vec{a} \times \vec{c}}{\vec{a} \cdot \vec{b}}\right)^2$ बराबर है।

Ans. Official Answer NTA (3)

Sol. Given $|\vec{a}| = \sqrt{31}, 4|\vec{b}| = |\vec{c}| = 2$

$2(\vec{a} \times \vec{b}) = 3(\vec{c} \times \vec{a}) \qquad \vec{b} \wedge \vec{c} = \frac{2\pi}{3}$

$3(\vec{c} \times \vec{a}) + 2(\vec{b} \times \vec{a}) = 0$

$(3\vec{c} + 2\vec{b}) \times \vec{a} = 0$

$3\vec{c} + 2\vec{b} = \lambda\vec{a}$

Squaring both sides

$9|\vec{c}|^2 + 4|\vec{b}|^2 + 12(\vec{b}.\vec{c}) = \lambda^2 |\vec{a}|^2$

$36 + 1 + 12 \times \frac{1}{2} \times 2\left(\cos\left(\frac{2\pi}{3}\right)\right) = \lambda^2 (31)$

$\lambda^2 = 1$

$\lambda^2 = \pm 1$

$3\vec{c} + 2\vec{b} = \pm\vec{a}$ __________(1)

Dot with $\vec{b} = 3(\vec{b}.\vec{c}) + 2(\vec{b}.\vec{b}) = \pm\vec{a}.\vec{b}$

$\vec{a}.\vec{b} = \pm\left(-\frac{3}{2} + \frac{1}{2}\right) = \pm(-1)$

$(\vec{a}.\vec{b})^2 = 1$

$3(\vec{c} \times \vec{a}) = 2(\vec{a} \times \vec{b})$

$(\vec{c} \times \vec{a})^2 = \frac{4}{9}(\vec{a} \times \vec{b})^2$

$$=\frac{4}{9}\left[|\vec{a}|^2|\vec{b}|^2-(\vec{a}.\vec{b})^2\right]$$

$$=\frac{4}{9}\left[\frac{31}{4}-(1)\right]$$

$$=\frac{4}{9}\times\frac{27}{4}=3$$

Question ID : 7155051795

23. If ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11:21$, then n^2+n+15 is equal to :

यदि ${}^{2n+1}P_{n-1} : {}^{2n-1}P_n = 11:21$ है, तो n^2+n+15 बराबर है :

Ans. Official Answer NTA (45)

Sol. $\frac{(2n+1)!(n-1)!}{(n+2)!(2n-1)!}=\frac{11}{21}$

$\Rightarrow \frac{(2n+1)(2n)}{(n+2)(n+1)n}=\frac{11}{21}$

$\Rightarrow \frac{2n+1}{(n+1)(n+2)}=\frac{11}{42}$

$\Rightarrow n=5$

$\Rightarrow n^2+n+15=25+5+15=45$

Question ID : 7155051800

24. The coefficient of x^{-6}, in the expansion of $\left(\frac{4x}{5}+\frac{5}{2x^2}\right)^9$, is __________ .

$\left(\frac{4x}{5}+\frac{5}{2x^2}\right)^9$ के प्रसार में x^{-6} का गुणांक है।

Ans. Official Answer NTA (5040)

Sol. $T_{r+1}={}^9C_r\left(\frac{5}{2x^2}\right)^r.\left(\frac{4x}{5}\right)^{9-r}={}^9C_r\frac{2^{18-3r}}{5^{9-2r}}x^{9-3r}$

for coefficient of $x^{-6} \Rightarrow 9-3r=-6 \Rightarrow r=5$

coff. of $x^{-6}=\frac{{}^9C_5.2^3}{5^{-1}}=\frac{9.8.7.5}{4.3.2.1}.(8\times5)$

$= 5040$

Question ID : 7155051793

25. If the constant term in the binomial expansion of $\left(\frac{x^{\frac{5}{2}}}{2}-\frac{4}{x^{\ell}}\right)^9$ is -84 and the coefficient of x^{-3l} is $2^{\alpha}\beta$, where $\beta < 0$ is an odd number, then $|\alpha l-\beta|$ is equal to ________ .

यदि $\left(\frac{x^{\frac{5}{2}}}{2}-\frac{4}{x^{\ell}}\right)^9$ के द्विपद प्रसार में अचर पद –84 है तथा x^{-3l} का गुणांक $2^{\alpha}\beta$ है, जहाँ $\beta < 0$ एक विषम संख्या है, तो $|\alpha l-\beta|$ बराबर है।

Ans. Official Answer NTA (98)

Sol. In, $\left(\frac{x^{\frac{5}{2}}}{2}-\frac{4}{x^{\ell}}\right)^9$

$T_{r+1} = {}^9C_r \frac{\left(x^{5/2}\right)^{9-r}}{2^{9-r}}\left(\frac{-4}{x^{\ell}}\right)^r$

$= (-1)^r \frac{{}^9C_r}{2^{9-r}} 4^r x^{\frac{45}{2}-\frac{5r}{2}-lr}$

$= 45 - 5r - 21r = 0$

$r = \frac{45}{5+21}$ ________(1)

Now, according to the question, $(-1)^r \frac{{}^9C_r}{2^{9-r}} 4^r = -84$

$= (-1)^r \, {}^9C_r \, 2^{3r-9} = 21 \times 4$

Only natural value of r possible if $3r - 9 = 0$

$r = 3$ and ${}^9C_3 = 84$

$\therefore 1 = 5$ from equation (1)

Now, coefficient of $x^{-31} = x^{\frac{45}{2}-\frac{5r}{2}-1r}$ at $1 = 5$, gives

$r = 5$

$\therefore {}^9C_5(-1)\frac{4^5}{2^4} = 2^\alpha \times \beta$

$= -63 \times 2^7$

$\Rightarrow \alpha = 7, \beta = -63$

$\therefore$ value of $|\alpha\ell - \beta| = 98$

Question ID : 7155051791

26. Let A be a $n \times n$ matrix such that $|A| = 2$. If the determinant of the matrix Adj $(2\cdot \text{Adj}(2A^{-1}))\cdot$ is 2^{84}, then n is equal to ___________ .

माना A एक $n \times n$ आव्यूह है तथा $|A| = 2$ है। यदि आव्यूह Adj $(2\cdot \text{Adj}(2A^{-1}))$ का सारणिक 2^{84} है, तो n बराबर है।

Ans. Official Answer NTA (5)

Sol. $|A| = 2$

Hence Adj $(2.\text{Adj}(2A^{-1})) = |2.\text{Adj}(2A^{-1})|^{n-1}$

$= \left(2^n |\text{Adj}(2A^{-1})|\right)^{n-1}$

$= \left(2^n |2A^{-1}|^{n-1}\right)^{n-1}$

$= 2^{n(n-1)}\left(\left(2^n |A^{-1}|\right)^{(n-1)}\right)^{(n-1)} \quad \therefore |A^{-1}| = \frac{1}{|A|} = \frac{1}{2}$

$= 2^{n(n-1)}\left(\left(2^{(n-1)}\right)^{(n-1)}\right)^{(n-1)}$

$= 2^{n(n-1)+(n-1)^3} = 2^{84}$

$\Rightarrow n = 5$

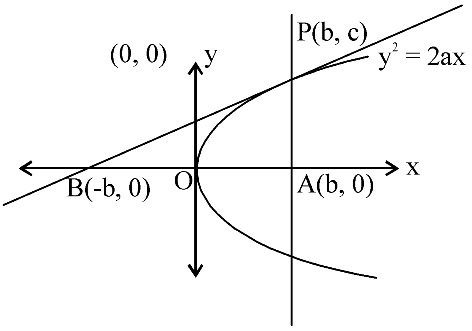
Question ID : 7155051797

27. Let S be the set of all $a \in N$ such that the area of the triangle formed by the tangent at the point $P(b,c), b, c \in N$, on the parabola $y^2 = 2ax$ and the lines $x = b, y = 0$ is 16 unit², then $\sum_{a \in S} a$ is equal to

माना सभी $a \in N$ जिनके लिए परवलय $y^2 = 2ax$ के बिंदु $P(b,c), b, c \in N$ पर स्पर्श रेखा तथा रेखाओं $x = b, y = 0$ से बने त्रिभुज का क्षेत्रफल 16 वर्ग इकाई है, का समुच्चय S है, तो $\sum_{a \in S} a$ बराबर है।

Ans. Official Answer NTA (146)

Sol.



As P (b, c) lies on parabola so $c^2 = 2ab$ ________(1)

Now equation of tangent to parabola $y^2 = 2ax$ in point form is $yy_1 = 2a\frac{(x+x_1)}{2}, (x_1, y_1) = (b, c)$

$\Rightarrow yc = a(x + b)$

For point B, put y = 0, now x= –b

So, area of $\Delta PBA, \frac{1}{2} \times AB \times AP = 16$

$\Rightarrow \frac{1}{2} \times 2b \times c = 16$

$\Rightarrow bc = 16$

As b and c are natural number so possible values of (b, c) are (1, 16), (2, 8), (4,4), (8, 2) and (16,1)

Now from equation (1) $a = \frac{c^2}{2b}$ and $a \in N$, so value of (b, c) are (1, 16), (2, 8) and (4, 4) now values of are 128, 16 and 2.

Hence sum of values of a is 146

Question ID : 7155051792

28. Let $A = [a_{ij}], a_{ij} \in Z \cap [0, 4], 1 \le i, j \le 2$. The number of matrices A such that the sum of all entries is a prime number $p \in (2, 13)$ is ____________ .

माना $A = [a_{ij}], a_{ij} \in Z \cap [0, 4], 1 \le i, j \le 2$ है। ऐसे आव्यूहों A जिनके सभी अवयवों का योग एक अभाज्य संख्या $p \in (2, 13)$ है, की संख्या है।

Ans. Official Answer NTA (196)

Sol. As given a + b + c + d = 3 or 5 or 7 or 11

if sum = 3

$\left(1+x+x^2+........+x^4\right)^4 \rightarrow x^3$

$\left(1-x^5\right)^4 (1-x)^{-4} \rightarrow x^3$

$\therefore\ {}^{4+3-1}C_3 = {}^6C_3 = 20$

If sum = 5

$\left(1-4x^5\right)(1-x)^{-4} \rightarrow x^5$

$\Rightarrow {}^{4+5-1}C_5 - 4x\ {}^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 7

$\left(1-4x^5\right)(1-x)^{-4} \rightarrow x^7$

$\Rightarrow {}^{4+5-1}C_4 - {}^{4.4+0-1}C_0 = {}^8C_5 - 4 = 52$

If sum = 11

$\left(1-4x^5+6x^{10}\right)(1-x)^{-4} \rightarrow x^{11}$

$\Rightarrow {}^{4+11-1}C_{11} - 4.\ {}^{4+6-4}C_6 + 6.\ {}^{4+1-1}C_1$

$= {}^{14}C_{11} - 4.\ {}^9C_6 + 6.4 = 364 - 336 + 24 = 52$

$\therefore$ Total matrices = 20 + 52 + 80 + 52 = 204

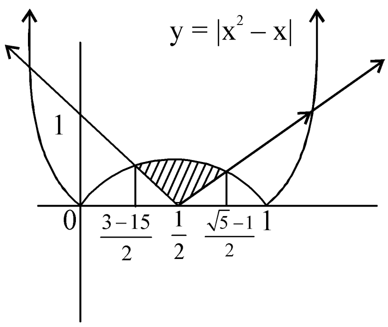
Question ID : 7155051796

29. Let the area of the region $\left\{(x,y) : |2x-1| \le y \le \left|x^2 - x\right|, 0 \le x \le 1\right\}$ be A. Then $(6A + 11)^2$ is equal to ________.

माना क्षेत्र $\left\{(x,y) : |2x-1| \le y \le \left|x^2 - x\right|, 0 \le x \le 1\right\}$ का क्षेत्रफल A है। तो $(6A+11)^2$ बराबर है।

Ans. Official Answer NTA (125)

Sol. $y \ge |2x-1|$ $\qquad$ $y \le |x^2 - x|$



Both curve are symmetric about $x = \frac{1}{2}$ Hence

$$A = 2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}} \left(\left(x - x^2\right) - \left(1 - 2x\right)\right) dx$$

$$A = 2\int_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}} \left(-x^2 + 3x - 1\right) dx = 2\left(\frac{-x^3}{3} + \frac{3}{2}x^2 - x\right)_{\frac{3-\sqrt{5}}{2}}^{\frac{1}{2}}$$

on solving $\quad 6A + = 5\sqrt{5}$

$$(6A + 11)^2 = 125$$

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30. The sum $1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + 5 \cdot 9^2 - \ldots + 15 \cdot 29^2$ is ___________.

$1^2 - 2 \cdot 3^2 + 3 \cdot 5^2 - 4 \cdot 7^2 + 5 \cdot 9^2 - \ldots + 15 \cdot 29^2$ बराबर है।

Ans. Official Answer NTA (6952)

Sol. Separating odd placed and even placed terms we get

$$S = \left(1.1^2 + 3.5^2 + \ldots..15.(29)^2\right) - \left(2.3^2 + 4.7^2 + \ldots\ldots + 14.(27)^2\right.$$

$$S = \sum_{n=1}^{8} (2n-1)(4n-3)^2 - \sum_{n=1}^{7} (2n)(4n-1)^2$$

Applying summation formula we get = 29856 – 22904 = 6952