

JEE Main January 2024
Question Paper With Text Solution
30 January | Shift-2

MATHEMATICS



JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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**JEE MAIN JANUARY 2024 | 30TH JANUARY SHIFT-2****SECTION – A**

Question ID : 4058591024

1. Let $f: \mathbb{R} \rightarrow \mathbb{R}$ be defined as $f(x) = ae^{2x} + be^x + cx$. If $f(0) = -1$, $f'(\log_e 2) = 21$ and $\int_0^{\log_e 4} (f(x) - cx)dx = \frac{39}{2}$, then the value of $|a + b + c|$ equals :
- (1) 12 (2) 16 (3) 8 (4) 10

Ans. Official answer NTA(3)**Sol.**

Question ID : 4058591018

2. Consider the system of linear equations $x + y + z = 5$, $x + 2y + \lambda^2 z = 9$, $x + 3y + \lambda z = \mu$, where $\lambda, \mu \in \mathbb{R}$. Then, which of the following statement is NOT correct?
- (1) System is consistent if $\lambda \neq 1$ and $\mu = 13$
(2) System has unique solution if $\lambda \neq 1$ and $\mu \neq 13$
(3) System is inconsistent if $\lambda = 1$ and $\mu \neq 13$
(4) System has infinite number of solutions if $\lambda = 1$ and $\mu = 13$

Ans. Official answer NTA(2)**Sol.**

Question ID : 4058591034

3. Bag A contains 3 white, 7 red balls and Bag B contains 3 white, 2 red balls. One bag is selected at random and a ball is drawn from it. The probability of drawing the ball from the bag A, if the ball drawn is white, is :
- (1) $\frac{1}{3}$ (2) $\frac{1}{4}$ (3) $\frac{3}{10}$ (4) $\frac{1}{9}$

Ans. Official answer NTA(1)**Sol.**



Question ID : 4058591020

4. Let a and b be two distinct positive real numbers. Let 11^{th} term of a GP, whose first term is a and third term is b , is equal to p^{th} term of another GP, whose first term is a and fifth term is b . Then p is equal to :

- (1) 21 (2) 20 (3) 25 (4) 24

Ans. Official answer NTA(1)

Sol.

Question ID : 4058591017

5. Let $R = \begin{pmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{pmatrix}$ be a non-zero 3×3 matrix, where

$x \sin \theta = y \sin \left(\theta + \frac{2\pi}{3} \right) = z \sin \left(\theta + \frac{4\pi}{3} \right) \neq 0, \theta \in (0, 2\pi)$. For a square matrix M , let trace (M) denote the sum of all the diagonal entries of M . Then, among the statements :

(I) Trace (R) = 0

(II) If trace ($\text{adj}(\text{adj}(R))$) = 0, then R has exactly one non-zero entry.

- (1) Both (I) and (II) are true (2) Only (II) is true
(3) Only (I) is true (4) Neither (I) nor (II) is true

Ans. Official answer NTA(2)

Sol.

Question ID : 4058591022

6. Let a and b be real constants such that the function f defined by $f(x) = \begin{cases} x^2 + 3x + a & , x \leq 1 \\ bx + 2 & , x > 1 \end{cases}$ be differentiable

on \mathbb{R} . Then, the value of $\int_{-2}^2 f(x) dx$ equal :

- (1) 21 (2) $\frac{15}{6}$ (3) 17 (4) $\frac{19}{6}$

Ans. Official answer NTA(3)

Sol.

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Question ID : 4058591026

7. Let $y = f(x)$ be a thrice differentiable function in $(-5, 5)$. Let the tangents to the curve $y = f(x)$ at $(1, f(1))$ and $(3, f(3))$ make angles $\frac{\pi}{6}$ and $\frac{\pi}{4}$, respectively with positive x-axis. If $27 \int_1^3 \left((f'(t))^2 + 1 \right) f''(t) dt = \alpha + \beta\sqrt{3}$ where α, β are integers, then the value of $\alpha + \beta$ equals :
- (1) 26 (2) -16 (3) -14 (4) 36

Ans. Official answer NTA(1)**Sol.**

Question ID : 4058591016

8. If z is a complex number, then the number of common roots of the equations $z^{1985} + z^{100} + 1 = 0$ and $z^3 + 2z^2 + 2z + 1 = 0$, is equal to :
- (1) 1 (2) 3 (3) 1 (4) 2

Ans. Official answer NTA(4)**Sol.**

Question ID : 4058591029

9. Let P be a point on the hyperbola $H: \frac{x^2}{9} - \frac{y^2}{4} = 1$, in the first quadrant such that the area of triangle formed by P and the two foci of H is $2\sqrt{13}$. Then, the square of the distance of P from the origin is :
- (1) 18 (2) 22 (3) 26 (4) 20

Ans. Official answer NTA(2)**Sol.**

Question ID : 4058591031

10. Let $L_1 : \vec{r} = (\hat{i} - \hat{j} + 2\hat{k}) + \lambda(\hat{i} - \hat{j} + 2\hat{k}), \lambda \in \mathbb{R}$, $L_2 : \vec{r} = (\hat{j} - \hat{k}) + \mu(3\hat{i} + \hat{j} + \hat{k}), \mu \in \mathbb{R}$ and $L_3 : \vec{r} = \delta(\ell\hat{i} + m\hat{j} + n\hat{k}), \delta \in \mathbb{R}$ be three lines such that L_1 is perpendicular to L_2 and L_3 is perpendicular to both L_1 and L_2 . Then, the point which lies on L_3 is :
- (1) (1, 7, -4) (2) (-1, -7, 4) (3) (-1, 7, 4) (4) (1, -7, 4)

**Ans.****Ans.** Official answer NTA(3)

Question ID : 4058591027

11. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a function defined by $f(x) = \frac{x}{(1+x^4)^{1/4}}$ and $g(x) = f(f(f(f(x))))$. Then,

$18 \int_0^{\sqrt{2\sqrt{5}}} x^2 g(x) dx$ equal to :

(1) 42

(2) 33

(3) 36

(4) 39

Ans. Official answer NTA(4)**Sol.**

Question ID : 4058591028

12. Let $A(\alpha, 0)$ and $B(\beta, 0)$ be the points on the line $5x + 7y = 50$. Let the point P divide the line segment AB internally in the ratio $7:3$. Let $3x - 25 = 0$ be a directrix of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the corresponding focus be S . If from S , the perpendicular on the x -axis passes through P , then the length of the latus rectum of E is equal to :

(1) $\frac{32}{5}$ (2) $\frac{32}{9}$ (3) $\frac{25}{9}$ (4) $\frac{25}{3}$ **Ans.** Official answer NTA(1)**Sol.**

Question ID : 4058591019

13. Suppose $2 - p$, p , $2 - \alpha$, α are the coefficients of four consecutive terms in the expansion of $(1 + x)^n$. Then the value of $p^2 - \alpha^2 + 6\alpha + 2p$ equals :

(1) 8

(2) 10

(3) 6

(4) 4

Ans. Official answer NTA(1)**Sol.****MATRIX JEE ACADEMY**

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Question ID : 4058591032

14. Let \vec{a} and \vec{b} be two vectors such that $|\vec{b}| = 1$ and $|\vec{b} \times \vec{a}| = 2$. Then $|\vec{b} \times \vec{a} - \vec{b}|^2$ is equal to :

- (1) 3 (2) 1 (3) 5 (4) 4

Ans. Official answer NTA(3)**Sol.**

Question ID : 4058591021

15. Let $f : \mathbb{R} - \{0\} \rightarrow \mathbb{R}$ be a function satisfying $f\left(\frac{x}{y}\right) = \frac{f(x)}{f(y)}$ for all $x, y, f(y) \neq 0$. If $f'(1) = 2024$, then :

- (1) $xf'(x) + f(x) = 2024$ (2) $xf'(x) - 2024f(x) = 0$
 (3) $xf'(x) + 2024f(x) = 0$ (4) $xf'(x) - 2023f(x) = 0$

Ans. Official answer NTA(2)**Sol.**

Question ID : 4058591033

16. Let $\vec{a} = \hat{i} + \alpha\hat{j} + \beta\hat{k}, \alpha, \beta \in \mathbb{R}$. Let a vector \vec{b} be such that the angle between \vec{a} and \vec{b} is $\frac{\pi}{4}$ and $|\vec{b}|^2 = 6$. If

$\vec{a} \cdot \vec{b} = 3\sqrt{2}$, then the value of $(\alpha^2 + \beta^2)|\vec{a} \times \vec{b}|^2$ is :

- (1) 75 (2) 95 (3) 85 (4) 90

Ans. Official answer NTA(4)**Sol.**

Question ID : 4058591023

17. Let $f(x) = (x+3)^2(x-2)^3, x \in [-4, 4]$. If M and m are the maximum and minimum values of f , respectively in $[-4, 4]$, then the value of $M - m$ is :

- (1) 392 (2) 600 (3) 108 (4) 608

Ans. Official answer NTA(4)**Sol.****MATRIX JEE ACADEMY****Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911****Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in**



Question ID : 4058591025

18. For $\alpha, \beta \in (0, \pi/2)$, let $3 \sin(\alpha + \beta) = 2 \sin(\alpha - \beta)$ and a real number k be such that $\tan \alpha = k \tan \beta$. Then the value of k is equal to:

- (1) $-\frac{2}{3}$ (2) 5 (3) -5 (4) $\frac{2}{3}$

Ans. Official answer NTA(3)

Sol.

Question ID : 4058591030

19. If $x^2 - y^2 + 2hxy + 2gx + 2fy + c = 0$ is the locus of a point, which moves such that it is always equidistant from the lines $x + 2y + 7 = 0$ and $2x - y + 8 = 0$, then the value of $g + c + h - f$ equals :

- (1) 6 (2) 8 (3) 29 (4) 14

Ans. Official answer NTA(4)

Sol.

Question ID : 4058591015

20. If the domain of the function $f(x) = \log_e \left(\frac{2x+3}{4x^2+x-3} \right) + \cos^{-1} \left(\frac{2x-1}{x+2} \right)$ is $(\alpha, \beta]$ then the value of $5\beta - 4\alpha$ is equal to :

- (1) 10 (2) 9 (3) 12 (4) 11

Ans. Official answer NTA(3)

**Sol.****SECTION - B**

Question ID : 4058591036

21. The number of real solutions of the equation $x(x^2 + 3|x| + 5|x-1| + 6|x-2|) = 0$ is _____.**Ans.** Official answer NTA(1)**Sol.**

Question ID : 4058591043

22. Let a line passing through the point $(-1, 2, 3)$ intersect the lines $L_1 : \frac{x-1}{3} = \frac{y-2}{2} = \frac{z+1}{-2}$ at $M(\alpha, \beta, \gamma)$ and $L_2 : \frac{x+2}{-3} = \frac{y-2}{-2} = \frac{z-1}{4}$ at $N(a, b, c)$. Then the value of $\frac{(\alpha + \beta + \gamma)^2}{(a + b + c)^2}$ equal _____.**Ans.** Official answer NTA(196)**Sol.**

Question ID : 4058591040

23. The area of the region enclosed by the parabola $(y-2)^2 = x-1$, the line $x-2y+4=0$ and the positive coordinate axes is _____.**Ans.** Official answer NTA(5)**Sol.**

Question ID : 4058591042

24. Consider two circles $C_1 : x^2 + y^2 = 25$ and $C_2 : (x-\alpha)^2 + y^2 = 16$, where $\alpha \in (5, 9)$. Let the angle between the two radii (one to each circle) drawn from one of the intersection points of C_1 and C_2 be $\sin^{-1}\left(\frac{\sqrt{63}}{8}\right)$. If the length of common chord of C_1 and C_2 is β , then the value of $(\alpha\beta)^2$ equals _____.**Ans.** Official answer NTA(1575)**Sol.****MATRIX JEE ACADEMY****Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911****Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in**



Question ID : 4058591038

25. Let $\alpha = \sum_{k=0}^n \left(\frac{{}^n C_k}{k+1} \right)^2$ and $\beta = \sum_{k=0}^{n-1} \left(\frac{{}^n C_k {}^n C_{k+1}}{k+2} \right)$. If $5\alpha = 6\beta$, then n equals _____.

Ans. Official answer NTA(10)**Sol.**

Question ID : 4058591039

26. Let S_n be the sum to n-terms of an arithmetic progression 3, 7, 11, If $40 < \left(\frac{6}{n(n+1)} \sum_{k=1}^n S_k \right) < 42$, then n equals _____.

Ans. Official answer NTA(9)**Sol.**

Question ID : 4058591037

27. In an examination of Mathematics paper, there are 20 questions of equal marks and the question paper is divided into three sections : A, B and C. A student is required to attempt total 15 questions taking at least 4 questions from each section. If section A has 8 questions, section B has 6 questions and section C has 6 questions, then the total number of ways a student can select 15 questions is _____.

Ans. Official answer NTA(11376)**Sol.**

Question ID : 4058591041

28. Let $Y=Y(X)$ be a curve lying in the first quadrant such that the area enclosed by the line $Y - y = Y'(x)(X - x)$ and the co-ordinate axes, where (x, y) is any point on the curve, is always $\frac{-y^2}{2Y'(x)} + 1$, $Y'(x) \neq 0$. If $Y(1) = 1$, then $12Y(2)$ equals _____.

Ans. Official answer NTA(20)**Sol.****MATRIX JEE ACADEMY****Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911****Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in**



Question ID : 4058591035

29. The number of symmetric relations defined on the set $\{1, 2, 3, 4\}$ which are not reflexive is _____.**Ans.** Official answer NTA (960)**Sol.**

Question ID : 4058591044

30. The variance σ^2 of the data

x_i	0	1	5	6	10	12	17
f_i	3	2	3	2	6	3	3

is _____.

Ans. Official answer NTA (29)**Sol.**