JEE Main January 2024

Question Paper With Text Solution

30 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2024 | 30TH JANUARY SHIFT-1

SECTION – A

Question ID : 533543827

1. If the domain of the function $f(x)=\cos^{-1}\left(\frac{2-|x|}{4}\right)+\{\log_e(3-x)\}^{-1}$ is $[-\alpha,\beta)-\{\gamma\}$, then $\alpha+\beta+\gamma$ is equal to :

(1) 9 (2) 11 (3) 12 (4) 8

Ans. Official answer NTA (2)

Sol.

Question ID : 533543832

2. Let S_n deonte the sum of first n terms of an arithemetic progression. If $S_{20}=790$ and $S_{10}=146$, then $S_{15}-S_5$ is :

(1) 410 (2) 395 (3) 405 (4) 390

Ans. Official answer NTA (2)

Sol.

Question ID : 533543846

3. If $2\sin^3x+\sin 2x\cos x+4\sin x-4=0$ has exactly 3 solutions in the interval $\left[0,\frac{n\pi}{2}\right]$, $n\in N$, then the roots of the equation $x^2+nx+(n-3)=0$ belongs to :

(1) $(-\infty,0)$ (2) $\left(-\frac{\sqrt{17}}{2},\frac{\sqrt{17}}{2}\right)$ (3) $(0,\infty)$ (4) Z

Ans. Official answer NTA (1)

Sol.

Question ID : 533543835

4. Let $g:R\to R$ be a non constant twice differentialbe function such that $g'\left(\frac{1}{2}\right)=g'\left(\frac{3}{2}\right)$. If a real valued function f is defined as , then :

(1) $f''(x) = 0$ for no x in (0, 1)

(2) $f''(x) = 0$ for exactly one x in (0, 1)

(3) $f'\left(\frac{3}{2}\right) + f'\left(\frac{1}{2}\right) = 1$

(4) $f''(x) = 0$ for atleast two x in (0, 2)

Axns. Official answer NTA (4)

Sol.

Question ID : 533543844

5. Let $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$ and $\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$ be two vectors such that $|\vec{a}| = 1$, $\vec{a}.\vec{b} = 2$ and $|\vec{b}| = 4$.

If $\vec{c} = 2\left(\vec{a} \times \vec{b}\right) - 3\vec{b}$, then the angle between $\vec{b}$ and $\vec{c}$ is equal to ::

(1) $\cos^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ (2) $\cos^{-1}\left(\frac{2}{3}\right)$ (3) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ (4) $\cos^{-1}\left(\frac{2}{\sqrt{3}}\right)$

Ans. Official answer NTA (3)

Sol.

Question ID : 533543836

6. The value of $\lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \frac{n^3}{\left(n^2+k^2\right)\left(n^2+3k^2\right)}$ is :

(1) $\frac{\left(2\sqrt{3}+3\right)\pi}{24}$ (2) $\frac{\pi}{8\left(2\sqrt{3}+3\right)}$ (3) $\frac{13\left(2\sqrt{3}-3\right)\pi}{8}$ (4) $\frac{13\pi}{8\left(4\sqrt{3}+3\right)}$

Ans. Official answer NTA (4)

Sol.

Question ID : 533543840

7. If the circles $(x+1)^2 + (y+2)^2 = r^2$ and $x^2 + y^2 - 4x - 4y + 4 = 0$ intersect at exactly two distinct points, then :

(1) $0 < r < 7$ (2) $\frac{1}{2} < r < 7$ (3) $5 < r < 9$ (4) $3 < r < 7$

Ans. Official answer NTA (4)

Sol.

Question ID : 53354383

8. Let $f:\left[-\frac{\pi}{2},\frac{\pi}{2}\right]\to R$ be a differentiable function such that. If the $\frac{x\int_0^x f(t)dt}{e^{x^2}-1}=\alpha$, then $8\alpha^2$ is equal to :

(1) 16 (2) 2 (3) 4 (4) 1

Ans. Official answer NTA (2)

Sol.

Question ID : 533543829

9. Consider then system of linear equations $x+y+z=4\mu, x+2y+2\lambda z=10\mu, x+3y+4\lambda^2 z=\mu^2+15$, where $\lambda, \mu \in R$. Which one of the following statements is NOT correct ?

(1) The system is consistent if $\lambda \neq \frac{1}{2}$

(2) The system is inconsistent if $\lambda=\frac{1}{2}$ and $\mu \neq 1$

(3) The system has infinite number of solutions if $\lambda=\frac{1}{2}$ and $\mu\neq 15$

(4) The system has unique solution if $\lambda \neq \frac{1}{2}$ and $\mu \neq 1, 15$

Ans. Official answer NTA (2)

Sol.

Question ID : 533543830

10. If $f(x)=\begin{vmatrix} 2\cos^4 x & 2\sin^4 x & 3+\sin^2 2x \\ 3+2\cos^4 x & 2\sin^4 x & \sin^2 2x \\ 2\cos^4 x & 3+2\sin^4 x & \sin^2 2x \end{vmatrix}$ then $\frac{1}{5}f'(0)=$ is equal to :

(1) 2 (2) 6 (3) 0 (4) 1

Ans. Official answer NTA (3)

Question ID : 533543838

11. Let $y=y(x)$ be the solution of the differential equation $\sec x dy+\{2(1-x)\tan x+x(2-x)\}dx=0$ such that $y(0)=2$. Then $y(2)$ is equal to :

(1) 1 (2) $2\{\sin(2)+1\}$ (3) 2 (4) $2\{1-\sin(2)\}$

Ans. Official answer NTA (3)

Sol.

Question ID : 533543841

12. if the length of the minor axis of an ellipse is equal to half of the distance the foci, then eccentricity of the ellipse is :

(1) $\frac{\sqrt{3}}{2}$ (2) $\frac{\sqrt{5}}{3}$ (3) $\frac{1}{\sqrt{3}}$ (4) $\frac{2}{\sqrt{5}}$

Ans. Official answer NTA (4)

Sol.

Question ID : 533543839

13. A line passing through the point A (9, 0) makes an angle of 30° with the positive direction of x-axis. If this line is rotated about A through an angle of 15° in the clockwise direction, then its equation in the new position is :

(1) $\frac{x}{\sqrt{3}-2}+y=9$ (2) $\frac{x}{\sqrt{3}+2}+y=9$ (3) $\frac{x}{\sqrt{3}-2}+y=9$ (4) $\frac{y}{\sqrt{3}+2}+x=9$

Ans. Official answer NTA (3)

Sol.

Question ID : 533543828

14. If $z=x+iy$, $xy\neq 0$, satisfies the equation $z^1+i\bar{z}=0$, then $|z^2|$ is equal to :

(1) 1 (2) 1/4 (3) 9 (4) 4

Ans. Official answer NTA (1)

Sol.

Question ID : 533543837

15. The area (in square units) of the region bounded by the parabola $y^2=4(x-2)$ and the line $y=2x-8$, is :

(1) 9 (2) 8 (3) 7 (4) 6

Ans. Official answer NTA (1)

Sol.

Question ID : 533543833

16. The maximum area of a triangle whose one vertex is at (0, 0) and the other two vertices lie on the curve $y = -2x^2 + 54$ at points (x, y) and (– x, y), where $y > 0$, is :

(1) 122 (2) 88 (3) 92 (4) 108

Ans. Official answer NTA (4)

Sol.

Question ID : 533543845

17. Let M denote the median of the following frequence distribution.

Class	0 – 4	4 – 8	8 – 12	12 – 16	16 – 20
Frequency	3	9	10	8	16

Then 20M is equal to :

(1) 416 (2) 208 (3) 104 (4) 52

Ans. Official answer NTA (2)

Sol.

Question ID : 533543831

18. Two integers x and y are chosen with replacement from the set {0, 1, 2, 3,.....,10}. Then the probability that $|x - y| > 5$, is :

(1) 62/121 (2) 60/121 (3) 31/121 (4) 30/131

Ans. Official answer NTA (4)

Sol.

Question ID : 533543842

19. Let (α, β, γ) be the foot of perpendicular from the point (1, 2, 3) on the line $\frac{x+3}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. Then $19(\alpha + \beta + \gamma)$ is equal to :

(1) 102 (2) 101 (3) 99 (4) 100

Ans. Official answer NTA (2)

Sol.

Question ID : 533543843

20. Let A (2, 3, 5) and C (–3, 4, –2) be opposite vertices of a parallelogram ABCD. If the diagonal $\overrightarrow{BD} = \hat{i} + 2\hat{j} + 3\hat{k}$, then the area of the parallelogram is equal to :

(1) $\frac{1}{2}\sqrt{586}$ (2) $\frac{1}{2}\sqrt{306}$ (3) $\frac{1}{2}\sqrt{410}$ (4) $\frac{1}{2}\sqrt{474}$

Ans. Official answer NTA (4)

Sol.

SECTION – B

Question ID : 533543850

21. Let $\alpha - 1^2 + 4^2 + 8^2 + 13^2 + 19^2 + 26^2 + \ldots$ upto 10 terms and $\beta = \sum_{n=1}^{10} n^4$. If $4\alpha - \beta = 55k + 40$, then k is equal to _____.

Ans. Official answer NTA (353)

Sol.

Question ID : 533543853

22. Let the latus rectum of the hyperbola $\frac{x^2}{9} - \frac{y^2}{b^2} = 1$ subtend an angle of $\frac{\pi}{3}$ at the centre of the hyperbola. If b^2 is equal to $\frac{1}{m}\left(1 + \sqrt{n}\right)$, where *l* and m are co-prime numbers, then $1^2 + m^2 + n^2$ is equal to _____.

Ans. Official answer NTA (182)

Sol.

Question ID : 533543849

23. Number of the integral terms in the expansion of $\left\{7^{\left(\frac{1}{2}\right)} + 11^{\left(\frac{1}{6}\right)}\right\}^{824}$ is equal to _____.

Ans. Official answer NTA (138)

Sol.

Question ID : 533543856

24. If the function $f(x)=\begin{cases}\frac{1}{|x|}, & |x|\geq 2\\ ax^2+2b, & |x|<2\end{cases}$ is differentiable on R, then $48(a+b)$ is equal to _____.

Ans. Official answer NTA (15)

Sol.

Question ID : 533543854

25. If d_1 is the shortest between the lines $x+1=2y=-12z$, $x=y+2=6z-6$ and d_2 is the shortest distance between the lines $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}, \frac{z-4}{5}, \frac{x-1}{1}=\frac{z-6}{-3}$, then the value of $\frac{32\sqrt{3}d_1}{d_2}$ is _____.

Ans. Official answer NTA (16)

Sol.

Question ID : 533543847

26. Let $A=\{1, 2, 3, ..., 7\}$ and let $P(A)$ denote the power of A. If the number of functions $f: A \to P(A)$ such that $a \in f(a)$, $\forall a \in A$, is m^n, m and $n \in N$ and m is least, then $m+n$ is equal to _____.

Ans. Official answer NTA (44)

Sol.

Question ID : 533543855

27. A group of 40 students appeared in an examination of 3 subjects - Mathematics, Physics and Chemistry. It was found that all students passed in atleast one of the subjects, 20 students passed in Mathematics, 25 students passed in Physics, 16 students passed in Chemistry, atmost 11 students passed in both Mathematics and Physics, atmost 15 students passed in both Physics and Chemistry, atmost 15 students passed in both Mathematics and Chemistry. The maximum numbers of students passed in all three subjects is _____.

Ans. Official answer NTA (11)

Sol.

Question ID : 533543852

28. Let y = y(x) be the solution of the differential equation $\left(1-x^2\right)dy=\left[xy+\left(x^3+2\right)\sqrt{3\left(1-x^2\right)}\right]dx$, $-1<x<1, y(0)=0$. If $y\left(\frac{1}{2}\right)=\frac{m}{n}$, m and n are co-prime numbers, then m + n is equal to _____.

Ans. Official answer NTA (97)

Sol.

Question ID : 533543848

29. Let $\alpha, \beta \in N$ be roots of the equation $x^2 - 70x + \lambda = 0$, where $\frac{\lambda}{2}, \frac{\lambda}{3} \notin N$. If λ assuments the minimum possible value, then $\frac{\left(\sqrt{\alpha-1}+\sqrt{\beta-1}\right)(\lambda+35)}{|\alpha-\beta|}$ is equal to _____.

Ans. Official answer NTA (60)

Sol.

Question ID : 533543851

30. The value of $9\int_0^9\left[\sqrt{\frac{10x}{x+1}}\right]dx$, whre [t] denotes the greatest integer less than or equal to t, is _____.

Ans. Official answer NTA (155)

Sol.