JEE Main January 2023

Question Paper With Text Solution

30 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2023 | 30TH JANUARY SHIFT-2

SECTION – A

Question ID : 3666942452

1. Let q be the maximum integral value of p in [0,10] for which the roots of the equation $x^2 - px + \frac{5}{4}p = 0$ are rational. Then the area of the region $\{(x, y): 0 \le y \le (x-q)^2, 0 \le x \le q\}$ is

माना [0,10] में p का अधिकतम पूर्णांक मान, जिसके लिए समीकरण $x^2 - px + \frac{5}{4}p = 0$ के मूल परिमेय q है। तब क्षेत्र $\{(x, y): 0 \le y \le (x-q)^2, 0 \le x \le q\}$ का क्षेत्रफल है :

(1) 25 (2) $\frac{125}{3}$ (3) 164 (4) 243

Ans. Official Answer NTA (4)

Sol. $D = p^2 - 5p$ = Perfect square of a rational number

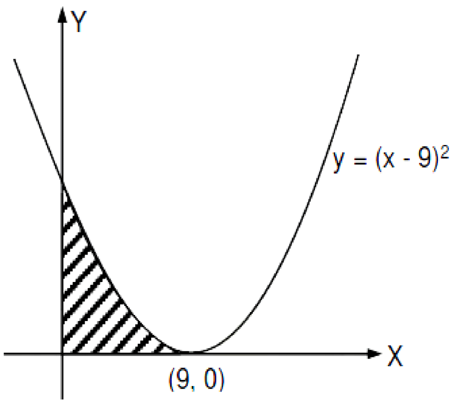
$\Rightarrow D = p(p-5)$

Noe put values in reverse orde as q is maximum value of p

p = 10 reject

p = 9 select

$\Rightarrow q = 9$



So area of shaded region = $\int_0^9 (x-9)^2 dx$

$$= \left[\frac{(x-9)^3}{3}\right]_0^9$$

$\Rightarrow$ 243 sq. unit

Question ID : 3666942469

2. Let $\lambda \in IR$, $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$.

If $\left(\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)\right)\times\left(\vec{a}-\vec{b}\right) = 8\hat{i} - 40\hat{j} - 24\hat{k}$, then $\left|\lambda\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|^2$ is equal to

माना $\lambda \in IR$, $\vec{a} = \lambda\hat{i} + 2\hat{j} - 3\hat{k}$, $\vec{b} = \hat{i} - \lambda\hat{j} + 2\hat{k}$ है। यदि

$\left(\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)\right)\times\left(\vec{a}-\vec{b}\right)=8\hat{i}-40\hat{j}-24\hat{k}$ है, तब $\left|\lambda\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)\right|^2$ बराबर है :

(1) 136 (2) 140 (3) 144 (4) 132

Ans. Official Answer NTA (2)

Sol. $\vec{a}=\lambda\hat{i}+2\hat{j}-3\hat{k}$

$\vec{b}=\hat{i}-\lambda\hat{j}+2\hat{k}$

$\Rightarrow\left(\vec{b}-\vec{a}\right)\times\left(\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}\times\vec{b}\right)\right)=8\hat{i}-40\hat{j}-24\hat{k}$

$\Rightarrow\left(\left(\vec{a}-\vec{b}\right)\cdot\left(\vec{a}+\vec{b}\right)\right)\left(\vec{a}\times\vec{b}\right)=8\hat{i}-40\hat{j}-24\hat{k}$

$\Rightarrow 8\left(\vec{a}\times\vec{b}\right)=8\hat{i}-40\hat{j}-24\hat{k}$

Now, $\vec{a}\times\vec{b}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ \lambda & 2 & -3\\ 1 & -\lambda & 2\end{vmatrix}$

$=(4-3\lambda)\hat{i}-(2\lambda+3)\hat{j}+\left(-\lambda^2-2\right)\hat{k}$

$\Rightarrow \lambda = 1$

$\therefore \vec{a}=\hat{i}+2\hat{j}-3\hat{k}$

$\vec{b}=\hat{i}-\hat{j}+2\hat{k}$

$\Rightarrow \vec{a}+\vec{b}=2\hat{i}+\hat{j}-\hat{k}, \vec{a}-\vec{b}=3\hat{j}-5\hat{k}$

$\Rightarrow\left(\vec{a}+\vec{b}\right)\times\left(\vec{a}-\vec{b}\right)=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & 1 & -1\\ 0 & 3 & -5\end{vmatrix}=2\hat{i}+10\hat{j}+6\hat{k}$

$\therefore$ required answer = 4 + 100 + 36 = 140

Question ID : 3666942461

3. The solution of the differential equation $\frac{dy}{dx}=-\left(\frac{x^2+3y^2}{3x^2+y^2}\right), y(1)=0$ is

अवकल समीकरण $\frac{dy}{dx}=-\left(\frac{x^2+3y^2}{3x^2+y^2}\right), y(1)=0$ का हल है :

(1) $\log_e|x+y|-\frac{xy}{(x+y)^2}=0$ (2) $\log_e|x+y|+\frac{xy}{(x+y)^2}=0$

(3) $\log_e|x+y|+\frac{2xy}{(x+y)^2}=0$ (4) $\log_e|x+y|-\frac{2xy}{(x+y)^2}=0$

Ans. Official Answer NTA (3)

Sol. Let $y = vx$

$$v + x\frac{dv}{dx} = -\left(\frac{1+3v^2}{3+v^2}\right)$$

$$\Rightarrow \frac{v^2+3}{(v+1)^3}dv + \frac{1}{x}dx = 0$$

$$\Rightarrow \int \frac{4}{(v+1)^3}dv + \int \frac{1}{v+1}dv - \int \frac{2}{(v+1)^2}dv + \int \frac{1}{x}dx = 0$$

$$\Rightarrow \frac{-2}{(v+1)^2} + \ln|v+1| + \frac{2}{v+1} + \ln|x| = c$$

$$\Rightarrow \frac{-2x^2}{(x+y)^2} + \ln\left|\frac{x+y}{x}\right| + \frac{2x}{x+y} + \ln|x| = c$$

$$\Rightarrow \frac{2xy}{(x+y)^2} + \ln|x+y| = c$$

$\because$ $y(1) = 0$

$\Rightarrow c = 0$

$$\Rightarrow \frac{2xy}{(x+y)^2} + \ln|x+y| = 0$$

Question ID : 3666942459

4. If the functions $f(x) = \frac{x^3}{3} + 2bx + \frac{ax^2}{2}$ and $g(x) = \frac{x^3}{3} + ax + bx^2$, $a \neq 2b$ have a common extreme point, then $a + 2b + 7$ is equal to :

यदि फलनों $f(x) = \frac{x^3}{3} + 2bx + \frac{ax^2}{2}$ तथा $g(x) = \frac{x^3}{3} + ax + bx^2$, $a \neq 2b$ का एक उभयनिष्ठ चरम बिंदु है, तब $a + 2b + 7$ बराबर है :

(1) 4 (2) $\frac{3}{2}$ (3) 6 (4) 3

Ans. Official Answer NTA (3)

Sol. $f'(x) = x^2 + 2b + ax$

$g'(x) = x^2 + a + 2bx$

$(2b - a) - x(2b - a) = 0$

$\therefore$ x = 1 is the common root

Put $x = 1$ in $f'(x) = 0$ or $g'(x) = 0$

$1 + 2b + a = 0$

$7 + 2b + a = 6$

Question ID : 3666942467

5. Let S be the set of all values of a_1 for which the mean deviation about the mean of 100 consecutive positive integers $a_1, a_2, a_3, \dots\dots\dots, a_{100}$ is 25. Then S is

माना a_1 के सभी मानों, जिनके लिए 100 क्रमागत धनात्मक पूर्णांकों $a_1, a_2, a_3, \dots\dots\dots, a_{100}$ का माध्य के सापेक्ष विचलन 25 है, का समुच्चय S है, तब S बराबर है।

(1) ϕ (2) IN (3) {99} (4) {9}

Ans. Official Answer NTA (2)

Sol. $X : a_1, a_1 + 1, a_1 + 2, \dots\dots\dots\dots, a_1 + 99$

$$\bar{x} = \frac{a_1 + (a_1 + 1) + \dots\dots + (a_1 + 99)}{100}$$

$\bar{x} = a_1 + 49.5$

$$\therefore \text{M.D.} = \frac{\sum |x_i - \bar{x}|}{n} = 25$$

$\Rightarrow 2\,(49.5 + 48.5 + \dots\dots + 0.5) = 2500$

$\Rightarrow 2500 = 2500$

$\Rightarrow$ It is true for all $a_1 \in N$

Question ID : 3666942456

6. Let $x=\left(8\sqrt{3}+13\right)^{13}$ and $y=\left(7\sqrt{2}+9\right)^{9}$. If [t] denotes the greatest integer $\le t$, then

(1) [x] is even but [y] is odd (2) [x] + [y] is even

(3) [x] and [y] are both odd (4) [x] is odd but [y] is even

माना $x=\left(8\sqrt{3}+13\right)^{13}$ और $y=\left(7\sqrt{2}+9\right)^{9}$ है। यदि [t] महत्तम पूर्णांक $\le t$ है, तब

(1) [x] सम है परन्तु [y] विषम है (2) [x] + [y] सम है

(3) [x] एवं [y] दोनों विषम है (4) [x] विषम है परन्तु [y] सम है

Ans. Official Answer NTA (2)

Sol. $x=\left(a\sqrt{3}+13\right)={}^{13}C_0\cdot\left(8\sqrt{3}\right)^{13}+{}^{13}C_1\left(8\sqrt{3}\right)^{12}+....$

$x'=\left(8\sqrt{3}-13\right)^{13}={}^{13}C_0\left(8\sqrt{3}\right)^{13}-{}^{13}C_1\left(8\sqrt{3}\right)^{12}(13)^1+...$

$x-x'=2\left[{}^{13}C_1\cdot\left(8\sqrt{3}\right)^{12}(13)^1+{}^{13}C_3\left(8\sqrt{3}\right)^{10}\cdot(13)^3....\right]$

therefore, $x-x'$ is even integer, hence [x] is evem

Now, $y=\left(7\sqrt{2}+9\right)^{9}={}^{9}C_0\left(7\sqrt{2}\right)^{9}+{}^{9}C_1\left(7\sqrt{2}\right)^{8}(9)^1+{}^{9}C_2\left(7\sqrt{2}\right)^{7}(9)^2........$

$y'=\left(7\sqrt{2}-9\right)^{9}={}^{9}C_0\left(7\sqrt{2}\right)^{9}-{}^{9}C_1\left(7\sqrt{2}\right)^{8}(9)^1+{}^{9}C_2\left(7\sqrt{2}\right)^{7}(9)^2.....$

$y-y'=2\left[{}^{9}C_1\left(7\sqrt{2}\right)^{8}(9)^1+{}^{9}C_3\left(7\sqrt{2}\right)^{6}(9)^3+...\right]$

$y-y'$ = Even integer, hence [y] is even

Question ID : 3666942464

7. A vector $\vec{v}$ in the first octant is inclined to the x-axis at 60°, to the y-axis at 45° and to the z-axis at an acute angle. If a plane passing through the points $\left(\sqrt{2},-1,1\right)$ and (a,b,c), is normal to $\vec{v}$, then

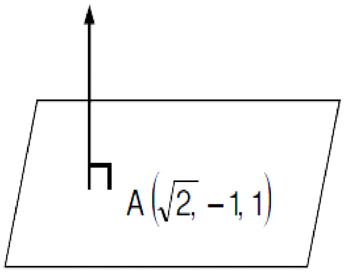
प्रथम अष्टांशक में एक सदिश $\vec{v}$ का झुकाव x अक्ष से 60°, y-अक्ष से 45° तथा z-अक्ष से एक न्यून कोण है। यदि बिंदु $\left(\sqrt{2},-1,1\right)$ व (a,b,c) से होकर जाने वाला समतल $\vec{v}$ पर लंबवत् है, तो

(1) $a+\sqrt{2}b+c=1$ (2) $\sqrt{2}a+b+c=1$ (3) $a+b+\sqrt{2}c=1$ (4) $\sqrt{2}a-b+c=1$

Ans. Official Answer NTA (1)

Sol. $\therefore l = \cos 60^o = \frac{1}{2}$

Let $\vec{v} = l\hat{i} + m\hat{j} + n\hat{k}$



$m = \cos 45^o = \frac{1}{\sqrt{2}}$

$\therefore l^2 + m^2 + n^2 = 1$

$\Rightarrow n^2 = \frac{1}{4}$

$\Rightarrow n = \frac{1}{2}, -\frac{1}{2}$ reject as $\vec{v}$ is in first octant

So, dr's of normal to the plane can be taken as 1, $\sqrt{2}$, 1

Hence

$1\left(a-\sqrt{2}\right)+\sqrt{2}\left(b+1\right)+1\left(c-1\right)=0$ $\Rightarrow a+\sqrt{2}b+c=1$

Question ID : 3666942458

8. Let *f*, *g* and *h* be the real values functions defined on IR as

$$f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 1, & x=0\end{cases},\quad g(x)=\begin{cases}\frac{\sin(x+1)}{(x+1)}, & x\neq -1\\ 1, & x=-1\end{cases}$$

and $h(x) = 2[x] - f(x)$, where $[x]$ is the greatest integer $\leq x$.

Then the value fo $\lim_{x\to 1} g\left(h(x-1)\right)$ is

माना *f*, *g* तथा *h* वास्तविक मान फलन है जो IR पर इस प्रकार परिभाषित है

$f(x)=\begin{cases}\frac{x}{|x|}, & x\neq 0\\ 1, & x=0\end{cases}$, $g(x)=\begin{cases}\frac{\sin(x+1)}{(x+1)}, & x\neq -1\\ 1, & x=-1\end{cases}$

एवं $h(x) = 2[x] - f(x)$, जहाँ $[x]$ महत्तम पूर्णांक $\le x$ है। तब $\lim_{x\to 1} g(h(x-1))$ का मान है :

(1) $\sin(1)$ (2) 0 (3) -1 (4) 1

Ans. Official Answer NTA (4)

Sol. $\text{LHL} = \lim_{k\to 0}\left(4(-k)\right),\ k>0$

$= \lim_{k\to 0} g(-2+1) \quad \because f(x) = -1 \forall x<0$

$= g(-1) = 1$

$\text{RHL} = \lim_{k\to 0} g\left(h(k)\right),\ k>0$

$== \lim_{k\to 0} g(-1), \quad \because f(x) = 1, \forall x>0$

$= 1$

Question ID : 3666942457

9. Let $a, b, c > 1$, a^3, b^3 and c^3 be in A.P., $\log_a b$, $\log_c a$ and $\log_b c$ be in G.P. If the sum of first 20 terms of an A.P., whose first term is $\frac{a+4b+c}{3}$ and the common difference is $\frac{a-8b+c}{10}$ is -444, then abc is equal to

माना $a, b, c > 1$ है, a^3, b^3 व c^3 समान्तर श्रेढी में तथा $\log_a b$, $\log_c a$ व $\log_b c$ गुणोत्तर श्रेढी में है। यदि समान्तर श्रेढी के प्रथम 20 पदों का योग, जिसका प्रथम पद $\frac{a+4b+c}{3}$ है तथा सार्वअंतर $\frac{a-8b+c}{10}$, -444 है। तब abc बराबर है :

(1) 216 (2) 343 (3) $\frac{125}{8}$ (4) $\frac{343}{8}$

Ans. Official Answer NTA (1)

Sol. $2b^3 = a^3 + c^3$

$\left(\frac{\log a}{\log c}\right)^2 = \frac{\log b}{\log a}\cdot\frac{\log c}{\log b}$

$\Rightarrow (\log a)3 = (\log c)3$

$\Rightarrow a = c$

$\Rightarrow 2b^3 = 2c^3 \quad \Rightarrow \quad b = c$

$\Rightarrow a = b = c$

$T_1 = 2a,$ $\qquad d = -\frac{3a}{5}$

$\therefore -444 = 10\left(4a - \frac{57a}{5}\right)$

$-444 = -\frac{370a}{5} \quad \Rightarrow \quad a = \frac{444}{74}$

$\Rightarrow a = 6$

$\Rightarrow abc = 6.6.6 = 216$

Question ID : 3666942470

10. Consider the following statements :

P : I have fever

Q : I will not take medicine

R : I will take rest

The statements "If I have fever, then I will take medicine and I will take rest" is equivalent to :

निम्नलिखित कथनों का विचार कीजिए :

P : मुझे बुखार है

Q : मैं दवा नहीं लूंगा

R : मैं आराम करूंगा

कथन "यदि मुझे बुखार है, तब में दवा लूँगा और मैं आराम करूँगा" के तुल्य है :

(1) $((\sim P)\vee\sim Q)\wedge((\sim P)\vee\sim R)$ (2) $(P\vee\sim Q)\wedge(P\vee\sim R)$

(3) $((\sim P)\vee\sim Q)\wedge((\sim P)\vee R)$ (4) $(P\vee Q)\wedge((\sim P)\vee R)$

Ans. Official Answer NTA (3)

Sol. $P \to (\sim Q \wedge R)$

$\sim P \vee (\sim Q \wedge R)$

$(\sim P \vee \sim Q) \wedge (\sim P \vee R)$

Question ID : 3666942454

11. If P is 3×3 real matrix such that $P^T = aP + (a-1)I$, where $a > 1$, then

(1) $|\text{Adj } P| > 1$ (2) $|\text{Adj } P| = \frac{1}{2}$

(3) $|\text{Adj } P| = 1$ (4) P is a singular matrix

यदि P एक 3×3 का वास्तविक आव्यूह इस प्रकार है कि $P^T = aP + (a-1)I$ है, जहाँ $a > 1$ है, तब :

(1) $|\text{Adj } P| > 1$ (2) $|\text{Adj } P| = \frac{1}{2}$

(3) $|\text{Adj } P| = 1$ (4) P एक singular matrix है

Ans. Official Answer NTA (3)

Sol. $(P^T)^T = (aP + (a-1)I)^T$

$\Rightarrow P = aP^T + (a-1)I$

$\Rightarrow P = a(ap + (a-1)I) + (a-1)I$

$\Rightarrow P = a^2P + a(a-1)I + (a-1)I$

$\Rightarrow P = a^2P + (a^2-1)I$

$\Rightarrow (a^2-1)P + (a^2-1)I = 0$

$\Rightarrow (a^2-1)(P+I) = 0$

$\therefore\ a > 1$

$\Rightarrow P + I = 0$

$\Rightarrow P = -I$

$\text{adj } P = \text{adj }(-I)$

$\Rightarrow |\text{adj } P| = |\text{adj }(-I)| = |-I|^2$

$\Rightarrow |\text{adj } P| = 1$

Question ID : 3666942460

12. $\lim\limits_{n\to\infty} \frac{3}{n}\left\{4+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+.....+\left(3-\frac{1}{n}\right)^2\right\}$ is equal to :

$\lim\limits_{n\to\infty} \frac{3}{n}\left\{4+\left(2+\frac{1}{n}\right)^2+\left(2+\frac{2}{n}\right)^2+.....+\left(3-\frac{1}{n}\right)^2\right\}$ बराबर है :

(1) 12 (2) 19 (3) $\frac{19}{3}$ (4) 0

Ans. Official Answer NTA (2)

Sol. $\lim\limits_{n\to\infty} \frac{3}{n}\sum_{r=0}^{n-1}\left(2+\frac{r}{n}\right)^2$

$= 3\int_0^1 (2+x)^2 dx = 27 - 8 = 19$

Question ID : 3666942455

13. The number of ways of selecting two numbers *a* and *b*, $a \in \{2, 4, 6, ..., 100\}$ and $b \in \{1, 3, 5, ..., 99\}$ such that 2 is the remainder when $a + b$ is divided by 23 is

दो संख्याओं a व b, $a \in \{2, 4, 6, ..., 100\}$ व $b \in \{1, 3, 5, ..., 99\}$ को इस प्रकार चुनने के तरीकों, कि a + b को 23 से विभाजित करने पर शेषफल 2 प्राप्त हो, की संख्या है :

(1) 54 (2) 108 (3) 268 (4) 186

Ans. Official Answer NTA (2)

Sol. Clearly $\frac{a+b}{23} = \lambda + \frac{2}{23}, \lambda \in N$

$\Rightarrow a + b = 23\lambda + 2$

$\Rightarrow$ a + b can be 25, 71, 117, 163

When a + b = 25 $\Rightarrow$ Number of ways = 12

When a + b = 71 $\Rightarrow$ Number of ways = 35

When a + b = 117 $\Rightarrow$ Number of ways = 42

When a + b = 163 $\Rightarrow$ Number of ways = 19

So total number of ways = 108

Question ID : 3666942463

14. Let A be a point on the x-axis. Common tangents are drawn from A to the curves $x^2 + y^2 = 8$ and $y^2 = 16x$. If one of these tangents touches the two curves at Q and R, then $(QR)^2$ is equal to

माना A, x-अक्ष पर एक बिंदु है। A से वक्रों $x^2 + y^2 = 8$ व $y^2 = 16x$ पर उभयनिष्ठ स्पर्श रेखाएं खींची जाती है। यदि इनमें से एक स्पर्श रेखा दोनों वक्रों को Q तथा R पर स्पर्श करती हो, तब $(QR)^2$ बराबर है :

(1) 81 (2) 72 (3) 64 (4) 76

Ans. Official Answer NTA (2)

Sol. $y = mx + \frac{4}{m}$

$$\frac{\left|\frac{4}{m}\right|}{\sqrt{1+m^2}} = 2\sqrt{2} \qquad \therefore m = \pm 1$$

$y = \pm x \pm 4$. Point of contact on parabola

Let m = 1, $\left(\frac{a}{m^2}, \frac{2a}{m}\right)$

R (4, 8)

Point of contact on circle Q (–2, 2)

$\therefore\ (QR)^2 = 36 + 36 = 72$

Question ID : 3666942451

15. The range of the function $f(x) = \sqrt{3-x} + \sqrt{2+x}$ is

फलन $f(x) = \sqrt{3-x} + \sqrt{2+x}$ का परिसर है :

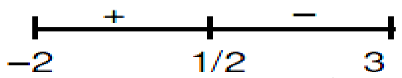
(1) $[2\sqrt{2}, \sqrt{11}]$ (2) $[\sqrt{5}, \sqrt{13}]$ (3) $[\sqrt{2}, \sqrt{7}]$ (4) $[\sqrt{5}, \sqrt{10}]$

Ans. Official Answer NTA (4)

Sol. $f'(x) = \dfrac{1}{2\sqrt{2+x}} - \dfrac{1}{2\sqrt{3-x}}$

$\Rightarrow f'(x) = \dfrac{\sqrt{3-x} - \sqrt{2+x}}{2\sqrt{2+x}\sqrt{3-x}} = 0$

$\Rightarrow x = \dfrac{1}{2}$ (domain is $-2 \le x \le 3$)



So at $x = \dfrac{1}{2}$, maxima

$f(-2) = \sqrt{5}, f\left(\dfrac{1}{2}\right) = \sqrt{10}, f(3) = \sqrt{5}$

range $\in \left[\sqrt{5}, \sqrt{10}\right]$

Question ID : 3666942468

16. Let $a_1 = 1, a_2, a_3, a_4, ...$ be consecutive natural numbers.

Then $\tan^{-1}\left(\dfrac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\dfrac{1}{1+a_2a_3}\right) + ... + \tan^{-1}\left(\dfrac{1}{1+a_{2021}a_{2022}}\right)$ is equal to

माना $a_1 = 1, a_2, a_3, a_4, ...$ क्रमागत धन पूर्णांक है। तब

$\tan^{-1}\left(\dfrac{1}{1+a_1a_2}\right) + \tan^{-1}\left(\dfrac{1}{1+a_2a_3}\right) + ... + \tan^{-1}\left(\dfrac{1}{1+a_{2021}a_{2022}}\right)$ बराबर है :

(1) $\tan^{-1}(2022) - \dfrac{\pi}{4}$ (2) $\dfrac{\pi}{4} - \cot^{-1}(2022)$ (3) $\dfrac{\pi}{4} - \tan^{-1}(2022)$ (4) $\cot^{-1}(2022) - \dfrac{\pi}{4}$

Ans. Official Answer NTA (1, 2)

Sol. $a_2 - a_1 = a_3 - a_2 = ... = a_{2022} - a_{2021} = 1.$

$\therefore \tan^{-1}\left(\frac{a_2 - a_1}{1 + a_1 a_2}\right) + \tan^{-1}\left(\frac{a_3 - a_2}{1 + a_2 a_3}\right) + ... + \tan^{-1}\left(\frac{a_{2022} - a_{2021}}{1 + a_{2021} a_{2022}}\right)$

$\left[\left(\tan^{-1} a_2\right) - \tan^{-1} a_1\right] + \left[\tan^{-1} a_3 - \tan^{-1} a_2\right] + ... + \left[\tan^{-1} a_{2022} - \tan^{-1} a_{2021}\right]$

$= \tan^{-1} a_{2022} - \tan^{-1} a_1$

$= \tan^{-1}(2022) - \tan^{-1} 1 = \tan^{-1} 2022 - \frac{\pi}{4}$ **(Option 3)**

$= \left(\frac{\pi}{2} - \cot^{-1}(2022)\right) - \frac{\pi}{4}$

$= \frac{\pi}{4} - \cot^{-1}(2022)$ **(Option 1)**

Question ID : 3666942466

17. Let $\vec{a}$ and $\vec{b}$ be two vectors, Let $|\vec{a}| = 1$, $|\vec{b}| = 4$ and $\vec{a} \cdot \vec{b} = 2$. If $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$, then the value of $\vec{b} \cdot \vec{c}$ is

माना $\vec{a}$ तथा $\vec{b}$ दो सदिश है। माना $|\vec{a}| = 1$, $|\vec{b}| = 4$ तथा $\vec{a} \cdot \vec{b} = 2$ है। यदि $\vec{c} = (2\vec{a} \times \vec{b}) - 3\vec{b}$ है, तब $\vec{b} \cdot \vec{c}$ का मान है :

(1) -60 (2) -24 (3) -84 (4) -48

Ans. Official Answer NTA (4)

Sol. $\vec{c} = \left(2\vec{a} \times \vec{b}\right) - 3\vec{b}$ ....(1)

$\vec{b}$..(1)

$\Rightarrow \vec{b}.\vec{c} = 0 - 3\vec{b}.\vec{b} = -48$

Question ID : 3666942462

18. The parabolas : $ax^2 + 2bx + cy = 0$ and $dx^2 + 2ex + fy = 0$ intersect on the line $y = 1$. If a, b, c, d, e, f are positive real numbers and a, b, c are in G.P., then

(1) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P.

(2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

(3) d, e, f are in A.P.

(4) d, e, f are in G.P.

परवलय : $ax^2 + 2bx + cy = 0$ व $dx^2 + 2ex + fy = 0$ रेखा $y = 1$ पर मिलते हैं। यदि a, b, c, d, e, f घनात्मक वास्तविक संख्याएँ है और a,b,c G.P. में है, तब

(1) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$, G.P. में हैं (2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$, A.P. में हैं

(3) d, e, f, A.P. में हैं (4) d, e, f, G.P. में हैं

Ans. Official Answer NTA (2)

Sol. $ax^2 + 2bx + c = 0$

$\Rightarrow ax^2 + 2\sqrt{ac}x + c = 0 (\because b^2 = ac)$

$\Rightarrow \left(x\sqrt{a} + \sqrt{c}\right)^2 = 0$

$x^2 - \frac{\sqrt{c}}{\sqrt{a}}$ ....(1)

Now, $dx^2 + 2ex + f = 0$

$\Rightarrow d\left(\frac{c}{a}\right) + 2e\left[-\frac{\sqrt{c}}{\sqrt{a}}\right] + f = 0$

$\Rightarrow \frac{dc}{a} + f = 2e\sqrt{\frac{c}{a}}$

$\Rightarrow \frac{d}{a} + \frac{f}{c} = 2e\sqrt{\frac{1}{ac}}$

$\Rightarrow \frac{d}{a} + \frac{f}{c} = \frac{2e}{b}\left[\text{as } b = \sqrt{ae}\right]$

$\therefore \frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P.

Question ID : 3666942465

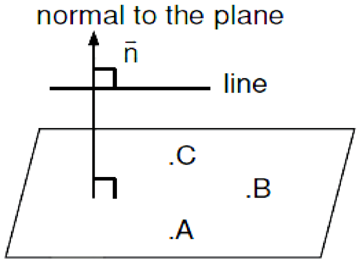
19. If a plane passes through the points (-1, k, 0), (2, k, -1), (1,1,2) and is parallel to the line $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$, then the value of $\frac{k^2+1}{(k-1)(k-2)}$

यदि समतल बिंदुओं (-1, k, 0), (2, k, -1), (1,1,2) से होकर जाता है एवं रेखा $\frac{x-1}{1} = \frac{2y+1}{2} = \frac{z+1}{-1}$ के समान्तर है, तब $\frac{k^2+1}{(k-1)(k-2)}$ का मान है :

(1) $\frac{13}{6}$ (2) $\frac{5}{17}$ (3) $\frac{17}{5}$ (4) $\frac{6}{13}$

Ans. Official Answer NTA (1)

Sol. Line is parallel to the vector $\hat{i}+\hat{j}-\hat{k}$



Let A (–1, k, 0), B (2, k, –1), C (1, 1, 2)

So $$\vec{n}=\overrightarrow{AB}\times\overrightarrow{AC}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 0 & -1\\ 2 & 1-k & 2\end{vmatrix}$$

$\Rightarrow \vec{n}=(1-k)\hat{i}-8\hat{j}+(3-3)\hat{k}$

$\therefore \hat{n}.\left(\hat{i}+\hat{j}-\hat{k}\right)=0$

$\Rightarrow 1 - k - 8 - 3 + 3k = 0$

$\Rightarrow k = 5$

hence $\dfrac{k^2+1}{(k-1)(k-2)}=\dfrac{26}{(4)(3)}=\dfrac{13}{6}$

Question ID : 3666942453

20. For $\alpha,\beta\in IR$, suppose system of linear equations

$x - y + z = 5$

$2a + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

has infinitely many solutions. Then α and β are the roots of

$\alpha,\beta\in IR$ के लिए, माना समीकरण निकाय

$x - y + z = 5$

$2a + 2y + \alpha z = 8$

$3x - y + 4z = \beta$

के अनंत हल है। तब α व β निम्न में से किसके मूल है :

(1) $x^2 - 10x + 16 = 0$ (2) $x^2 + 18x + 56 = 0$

(3) $x^2 + 14x + 24 = 0$ (4) $x^2 - 18x + 56 = 0$

Ans. Official Answer NTA (4)

Sol. $\begin{vmatrix} 1 & -1 & 1 \\ 2 & 2 & \alpha \\ 3 & -1 & 4 \end{vmatrix} = 0$; $8 + \alpha - 2(-4 + 1) + 3(-\alpha - 2) = 0$

$8 + \alpha + 6 - 3\alpha - 6 = 0$

$\alpha = 4$

SECTION – B

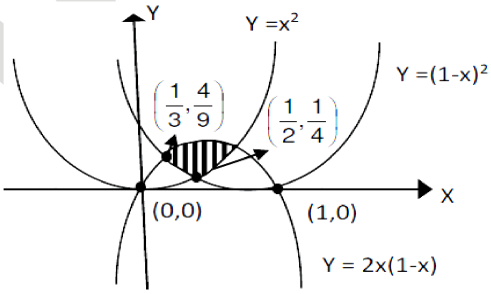
Question ID : 3666942476

21. Let A be the area of the region $\left\{(x, y) : y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\right\}$. Then 540 A is equal to ________.

माना क्षेत्र $\left\{(x, y) : y \geq x^2, y \geq (1-x)^2, y \leq 2x(1-x)\right\}$ का क्षेत्रफल A है। तब 540 A का मान बराबर है।

Ans. Official Answer NTA (25)

Sol. $A = 2\int_{\frac{1}{3}}^{\frac{1}{2}} \left(2x - 2x^2 - (1-x)^2\right) dx$



$= 2\left[2x^2 - x^3 - x\right]_{1/3}^{1/2}$

So $A = \frac{5}{108} \Rightarrow 540A = \frac{5}{108} \times 540 = 25$

Question ID : 3666942480

22. A bag contains six balls of different colours. Two balls are drawn in succession with replacement. The probability that both are of the same colour is p. Next four balls are drawn in succession with replacement and the probability that exactly three balls are of the same colour is q. If p : q = m : n, where m and n are coprime, then m + n is equal to ________ .

एक थैले में भिन्न रंगों की छः गेंद है। माना एक–एक कर प्रतिस्थापन सहित दो गेंद निकाली जाती है तथा दोनों गेंदों के एक ही रंग के होने की प्रायिकता p है। फिर एक–एक कर प्रतिस्थापन सहित चार गेंद निकाली जाती है तथा ठीक तीन गेंदों का एक ही रंग के होने की प्रायिकता q है। यदि p : q = m : n है, जहाँ m व n सहअभाज्य है तब m + n बराबर है।

Ans. Official Answer NTA (14)

Sol. $p = \frac{{}^6C_1}{6 \times 6} = \frac{1}{6}$

$q = \frac{{}^6C_1 \times {}^5C_1 \times 4}{6 \times 6 \times 6 \times 6} = \frac{5}{54}$

$\therefore p : q = 9 : 5 \Rightarrow m + n = 14$

Question ID : 3666942477

23. If $\int \sqrt{\sec 2x - 1}\,dx = \alpha \log_e \left| \cos 2x + \beta + \sqrt{\cos 2x \left(1 + \cos \frac{1}{\beta} x\right)} \right| + \text{constant}$, then $\beta - \alpha$ is equal to __________.

यदि $\int \sqrt{\sec 2x - 1}\,dx = \alpha \log_e \left| \cos 2x + \beta + \sqrt{\cos 2x \left(1 + \cos \frac{1}{\beta} x\right)} \right| + \text{constant}$, तो $\beta - \alpha$ बराबर है।

Ans. Official Answer NTA (1)

Sol. $\int \sqrt{\sec 2x - 1}\,dx = \int \sqrt{\frac{1 - \cos 2x}{\cos 2x}}\,dx = \sqrt{2} \int \frac{\sin x}{\sqrt{2\cos^2 x - 1}}$

put $\cos x = t \Rightarrow -\sin x\,dx = dt$

$= -\sqrt{2} \int \frac{dt}{\sqrt{2t^2 - 1}} = -\ln\left|\sqrt{2}\cos x + \sqrt{\cos 2x}\right| + c = -\frac{1}{2}\ln\left|\cos 2x + \frac{1}{2} + \sqrt{\cos 2x}.\sqrt{1 + \cos 2x}\right| + c$

$\because \beta = \frac{1}{2}, \alpha = -\frac{1}{2} \Rightarrow \beta - \alpha = 1$

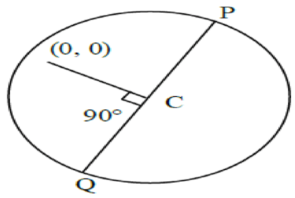
Question ID : 3666942478

24. Let $P(a_1, b_1)$ and $Q(a_2, b_2)$ be two distinct points on a circle with center $C(\sqrt{2}, \sqrt{3})$. Let O be the origin and OC be perpendicular to both CP and CQ. If the area of the triangle OCP is $\frac{\sqrt{35}}{2}$, then $a_1^2 + a_2^2 + b_1^2 + b_2^2$ is equal to __________.

माना वृत्त $C(\sqrt{2}, \sqrt{3})$ केन्द्र के एक वृत्त पर दो भिन्न बिंदु $P(a_1, b_1)$ एवं $Q(a_2, b_2)$ है। माना O मूल बिंदु है तथा OC, रेखाओं CP एवं CQ दोनों पर लम्बवत् है। यदि त्रिभुज OCP का क्षेत्रफल $\frac{\sqrt{35}}{2}$ है, तो $a_1^2 + a_2^2 + b_1^2 + b_2^2$ बराबर है।

Ans. Official Answer NTA (24)

Sol. $\frac{1}{2} \times PC \times \sqrt{5} = \frac{\sqrt{35}}{2}; PC = \sqrt{7}$



$a_1^2 + b_1^2 + a_2^2 + b_2^2 = OP^2 + OQ^2$

$= 2\ (5 + 7) = 24$

Question ID : 3666942474

25. 50^{th} root of a number x is 12 and 50^{th} root of another number y is 18. Then the remainder obtained on dividing (x + y) by 25 is __________ .

एक संख्या x का 50वाँ मूल 12 है एवं दूसरी संख्या y का 50वाँ मूल 18 है। तब (x + y) को 25 से विभाजित करने पर प्राप्त शेषफल है।

Ans. Official Answer NTA (23)

Sol. $(x)^{\frac{1}{50}} = 12$ and $(y)^{\frac{1}{50}} = 18$

$\Rightarrow x = 12^{50} \Rightarrow y = 18^{50}$

$x + y = 12^{50} + 18^{50}$

$(2^2 \times 3^1)^{50} + (3^2 \times 2)^{50}$

$= 6^{50}\ (2^{50} + 3^{50})$

$= (5 + 1)^{50}\ (4^{25} + 9^{25})$

$= (25\lambda + 1)\ [(5 - 1)^{25} + (10 - 1)^{25}]$

$= (25\lambda + 1)\ (25k - 1 + 25\mu - 1)$

$= (25\lambda + 1)\ (25t + 23)$

$= 25\lambda + 23$

so remiander is 23, when x + y is divided by 25.

Question ID : 3666942475

26. The 8^{th} common term of the series

$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots\ldots\ldots,$

$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots\ldots\ldots$ is ______________ .

श्रेणियों

$S_1 = 3 + 7 + 11 + 15 + 19 + \ldots\ldots\ldots.,$

$S_2 = 1 + 6 + 11 + 16 + 21 + \ldots\ldots\ldots$

का 8वाँ उभयनिष्ठ पद है।

Ans. Official Answer NTA (151)

Sol. $T_8 = 11 + (8 - 1) \times 20$

$= 11 + 140 = 151$

Question ID : 3666942473

27. The number of seven digits odd numbers, that can be formed using all the seven digits 1, 2, 2, 2, 3, 3, 5 is ___________ .

सभी सात अंकों 1, 2, 2, 2, 3, 3, 5 के प्रयोग से बनाई जा सकने वाली सात अंको की विषम संख्याओं की संख्या है।

Ans. Official Answer NTA (240)

Sol. $\_\,\_\,\_\,\_\,\_\,\_\,\underline{1} = \frac{6!}{2!3!} = 60$

$\_\,\_\,\_\,\_\,\_\,\_\,\underline{3} = \frac{6!}{3!} = 120$

$\_\,\_\,\_\,\_\,\_\,\_\,\underline{5} = \frac{6!}{3!2!} = 60$

Total = 60 + 120 + 60 = 240

Question ID : 3666942479

28. Let a line L pass through the point P (2,3,1) and be parallel to the line $x + 3y - 2z - 2 = 0 = x - y + 2z$. If the distance of L of the point (5,3,8) is α, then $3\alpha^2$ is equal to __________ .

माना रेखा L बिंदु P (2,3,1) से होकर जाती है तथा रेखा $x + 3y - 2z - 2 = 0 = x - y + 2z$ के समान्तर है। यदि बिंदु (5,3,8) की रेखा L से दूरी α है, तो $3\alpha^2$ बराबर है।

Ans. Official Answer NTA (158)

Sol. $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & -2 \\ 1 & -1 & 2 \end{vmatrix} = 4\hat{i} - 4\hat{j} - 4\hat{k}$

$\therefore$ Equation of line is $\frac{x-2}{1} = \frac{y-3}{-1} = \frac{z-1}{-1}$

Let Q be (5, 3, 8) and foot of $\perp$ from Q on this line be R.

Now, $R \equiv (k + 2, -k + 3, -k + 1)$

DR of QR are $(k - 3, -k, -k - 7)$

$\therefore (1)(k-3) + (-1)(-k) + (-1)(-k-7) = 0$

$\Rightarrow k = -\frac{4}{3}$

$\therefore \alpha^2 = \left(\frac{13}{3}\right)^2 + \left(\frac{4}{3}\right)^2 + \left(\frac{17}{3}\right)^2 = \frac{474}{9}$

$\therefore 3\alpha^2 = 158$

Question ID : 3666942472

29. If the value of real number $a > 0$ for which $x^2 - 5ax + 1 = 0$ and $x^2 - ax - 5 = 0$ have a common real root is $\frac{3}{\sqrt{2\beta}}$ then β is equal to ______________.

यदि वास्तविक संख्या $a > 0$ जिसके लिए $x^2 - 5ax + 1 = 0$ तथा $x^2 - ax - 5 = 0$ का उभयनिष्ठ वास्तविक मूल है, का मान $\frac{3}{\sqrt{2\beta}}$ है, तब β बराबर है।

Ans. Official Answer NTA (13)

Sol. Let α be the common root

so $\alpha^2 - 5a\alpha + 1 = 0$

$\alpha^2 - a\alpha - 5 = 0$

$\Rightarrow \frac{\alpha^2}{26a} = \frac{\alpha}{6} = \frac{1}{4a}$

$\Rightarrow \alpha^2 = \frac{13}{2} \Rightarrow \alpha = \sqrt{\frac{13}{2}} \Rightarrow a = \frac{3}{2\alpha} = \frac{3}{\sqrt{2\beta}}$

$\Rightarrow \beta = 13$

Question ID : 3666942471

30. Let $A = \{1,2,3,5,8,9\}$. Then the number of possible functions $f : A \to A$ such that $f(m \cdot n) = f(m) \cdot f(n)$ for every $m, n \in A$ with $m \cdot n \in A$ is equal to ___________ .

माना $A = \{1,2,3,5,8,9\}$ है। तब संभव फलनों $f : A \to A$, जब कि $m, n \in A$ तथा $m \cdot n \in A$ को संतुष्ट करने वाले सभी m, n के लिए $f(m \cdot n) = f(m) \cdot f(n)$ है, की संख्या है।

Ans. Official Answer NTA (432)

Sol. $f(1) = 1$; $f(9) = f(3) \times f(3)$

i.e., $f(3) = 1$ or 3

Total function $= 1 \times 6 \times 2 \times 6 \times 6 \times 1 = 432$