

JEE Main January 2024
Question Paper With Text Solution
29 January | Shift-2

MATHEMATICS



JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

**JEE MAIN JANUARY 2024 | 29TH JANUARY SHIFT-2****SECTION – A**

Question ID : 405859760

1. Let $\overrightarrow{OA} = \vec{a}$, $\overrightarrow{OB} = 12\vec{a} + 4\vec{b}$ and $\overrightarrow{OC} = \vec{b}$, where O is the origin. If S is the parallelogram with adjacent sides OA and OC, then $\frac{\text{area of the quadrilateral OABC}}{\text{area of S}}$ is equal to _____.
- (1) 7 (2) 6 (3) 10 (4) 8

Ans. Official answer NTA(4)**Sol.**

Question ID : 405859762

2. If the mean and variance of five observations are $\frac{24}{5}$ and $\frac{194}{25}$ respectively and the mean of the first four observations is $\frac{7}{2}$, then the variance of the first four observations is equal to :
- (1) $\frac{77}{12}$ (2) $\frac{5}{4}$ (3) $\frac{4}{5}$ (4) $\frac{105}{4}$

Ans. Official answer NTA(2)**Sol.**

Question ID : 405859759

3. Let a unit vector $\vec{u} = x\hat{i} + y\hat{j} + z\hat{k}$ make angles $\frac{\pi}{2}$, $\frac{\pi}{3}$ and $\frac{2\pi}{3}$ with the vectors $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{k}$, $\frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ and $\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}$ respectively. If $\vec{v} = \frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$ then $|\vec{u} - \vec{v}|^2$ is equal to :
- (1) $\frac{5}{2}$ (2) 7 (3) $\frac{11}{2}$ (4) 9

Ans. Official answer NTA(1)**Sol.**



Question ID : 405859764

4. The sum of the solutions $x \in \mathbb{R}$ of the equation $\frac{3 \cos 2x + \cos^3 2x}{\cos^6 x - \sin^6 x} = x^3 - x^2 + 6$ is :
- (1) 1 (2) 0 (3) -1 (4) 3

Ans. Official answer NTA(1)**Sol.**

Question ID : 405859753

5. Let $y = \log_e \left(\frac{1-x^2}{1+x^2} \right)$, $-1 < x < 1$. Then at $x = \frac{1}{2}$, the value of $225(y' - y'')$ is equal to :
- (1) 736 (2) 746 (3) 732 (4) 742

Ans. Official answer NTA(1)**Sol.**

Question ID : 405859750

6. If each term of a geometric progression a_1, a_2, a_3, \dots with $a_1 = \frac{1}{8}$ and $a_2 \neq a_1$, is the arithmetic mean of the next two terms and $S_n = a_1 + a_2 + \dots + a_n$, then $S_{20} - S_{18}$ is equal to :
- (1) 2^{15} (2) 2^{18} (3) -2^{15} (4) -2^{18}

Ans. Official answer NTA(3)**Sol.**

Question ID : 405859752

7. The function $f(x) = 2x + 3\left(\frac{x}{3}\right)^{\frac{2}{3}}$, $x \in \mathbb{R}$, has :
- (1) exactly one point of local minima and no point of local maxima
(2) exactly two points of local maxima and exactky one point of local minima
(3) exactly one point of local maxima and exactly one point of local minima
(4) exactly one point of local maxima and no point of local minima

Ans. Official answer NTA(2)

**Sol.**

Question ID : 405859757

8. The distance of the point (2,3) from the line $2x - 3y + 28 = 0$, measured parallel to the line $\sqrt{3}x - y + 1 = 0$, is equal to :

- (1) $4 + 6\sqrt{3}$ (2) $4\sqrt{2}$ (3) $6\sqrt{3}$ (4) $3 + 4\sqrt{2}$

Ans. Official answer NTA(1)**Sol.**

Question ID : 405859748

9. Number of ways of arranging 8 identical books into 4 identical shelves where any number of shelves may retain empty is equal to :

- (1) 16 (2) 15 (3) 12 (4) 18

Ans. Official answer NTA(2)**Sol.**

Question ID : 405859758

10. Let P(3,2,3), Q(4,6,2) and R(7,3,2) be the vertices of ΔPQR . Then, the angle $\angle QPR$ is :

- (1) $\cos^{-1}\left(\frac{1}{18}\right)$ (2) $\frac{\pi}{6}$ (3) $\cos^{-1}\left(\frac{7}{18}\right)$ (4) $\frac{\pi}{3}$

Ans.**Ans.** Official answer NTA(4)

Question ID : 405859749

11. If $\log_e a, \log_e b, \log_e c$ are in A.P. and $\log_e a - \log_e 2b, \log_e 2b - \log_e 3c, \log_e 3c - \log_e a$ are also in an A.P., then $a : b : c$ is equal to:

- (1) 9 : 6 : 4 (2) 25 : 10 : 4 (3) 6 : 3 : 2 (4) 16 : 4 : 1

Ans. Official answer NTA(1)**Sol.****MATRIX JEE ACADEMY**

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911

Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in



Question ID : 405859761

12. An integer is chosen at random from the integers 1,2,3,...,50. The probability that the chosen integer is a multiple of atleast one of 4,6 and 7 is :

- (1) $\frac{14}{25}$ (2) $\frac{21}{50}$ (3) $\frac{9}{50}$ (4) $\frac{8}{25}$

Ans. Official answer NTA(2)

Sol.

Question ID : 405859746

13. Let r and θ respectively be the modulus and amplitude of the complex number $z = 2 - i \left(2 \tan \frac{5\pi}{8} \right)$, then (r, θ) is equal to :

- (1) $\left(2 \sec \frac{11\pi}{8}, \frac{11\pi}{8} \right)$ (2) $\left(2 \sec \frac{3\pi}{8}, \frac{5\pi}{8} \right)$ (3) $\left(2 \sec \frac{3\pi}{8}, \frac{3\pi}{8} \right)$ (4) $\left(2 \sec \frac{5\pi}{8}, \frac{3\pi}{8} \right)$

Ans. Official answer NTA(3)

Sol.

Question ID : 405859763

14. Let $x = \frac{m}{n}$ (m, n are co-prime natural numbers) be a solution of the equation $\cos \left(2 \sin^{-1} x \right) = \frac{1}{9}$ and let α, β ($\alpha > \beta$) be the roots of the equation $mx^2 - nx - m + n = 0$. Then the point (α, β) lies on the line :

- (1) $5x + 8y = 9$ (2) $5x - 8y = -9$ (3) $3x + 2y = 2$ (4) $3x - 2y = -2$

Ans. Official answer NTA(1)

Sol.

Question ID : 405859756

15. Let A be the point of intersection of the lines $3x + 2y = 14$, $5x - y = 6$ and B be the point of intersection of the lines $4x + 3y = 8$, $6x + y = 5$. The distance of the point $P(5, -2)$ from the line AB is :

- (1) $\frac{5}{2}$ (2) 8 (3) $\frac{13}{2}$ (4) 6

Ans. Official answer NTA(4)

MATRIX JEE ACADEMY

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911

Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

**Sol.**

Question ID : 405859745

16. If R is the smallest equivalence relation on the set $\{1,2,3,4\}$ such that $\{(1,2), (1,3)\} \subset R$, then the number of elements in R is _____.

- (1) 10 (2) 12 (3) 8 (4) 15

Ans. Official answer NTA(1)**Sol.**

Question ID : 405859747

17. Let $A = \begin{bmatrix} 2 & 1 & 2 \\ 6 & 2 & 11 \\ 3 & 3 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 2 & 0 \\ 5 & 0 & 2 \\ 7 & 1 & 5 \end{bmatrix}$. The sum of the prime factors of $|P^{-1}AP - 2I|$ is equal to :

- (1) 27 (2) 26 (3) 66 (4) 23

Ans. Official answer NTA(2)**Sol.**

Question ID : 405859755

18. If $\sin\left(\frac{y}{x}\right) = \log_e|x| + \frac{\alpha}{2}$ is the solution of the differential equation $x \cos\left(\frac{y}{x}\right) \frac{dy}{dx} = y \cos\left(\frac{y}{x}\right) + x$ and $y(1) = \frac{\pi}{3}$, then α^2 is equal to :

- (1) 3 (2) 9 (3) 12 (4) 4

Ans. Official answer NTA(1)**Sol.**



Question ID : 405859751

19. The function $f(x) = \frac{x}{x^2 - 6x - 16}$, $x \in \mathbb{R} - \{-2, 8\} ::$

- (1) decreases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$
- (2) increases in $(-\infty, -2) \cup (-2, 8) \cup (8, \infty)$
- (3) decreases in $(-2, 8)$ and increases in $(-\infty, -2) \cup (8, \infty)$
- (4) decreases in $(-\infty, -2)$ and increases in $(8, \infty)$

Ans. Official answer NTA(1)

Sol.

Question ID : 405859754

20. If $\int \frac{\sin^{\frac{3}{2}} x + \cos^{\frac{3}{2}} x}{\sqrt{\sin^3 x \cos^3 x \sin(x-\theta)}} dx = A\sqrt{\cos \theta \tan x - \sin \theta} + b\sqrt{\cos \theta - \sin \theta \cot x} + C$, where C is the

integration constant, then AB is equal to :

- (1) $4 \sec \theta$
- (2) $4 \operatorname{cosec} (2\theta)$
- (3) $2 \sec \theta$
- (4) $8 \operatorname{cosec} (2\theta)$

Ans. Official answer NTA(4)

Sol.

**SECTION - B**

Question ID : 405859766

21. Let α, β be the roots of the equation $x^2 - \sqrt{6}x + 3 = 0$ such that $\text{Im}(\alpha) > \text{Im}(\beta)$. Let a, b be integers not divisible by 3 and n be a natural number such that $\frac{\alpha^{99}}{\beta} + \alpha^{98} = 3^n (a + ib)$, $i = \sqrt{-1}$. Then $n + a + b$ is equal to _____.

Ans. Official answer NTA (49)**Sol.**

Question ID : 405859769

22. Let the slope of the line $45x + 5y + 3 = 0$ be $27r_1 + \frac{9r_2}{2}$ for some $r_1, r_2 \in \mathbb{R}$. Then

$$\lim_{x \rightarrow 3} \left(\int \frac{8t^2}{\frac{3r_2x}{2} - r_2x^2 - r_1x^3 - 3x} dt \right) \text{ is equal to } \underline{\hspace{2cm}}.$$

Ans. Official answer NTA (12)**Sol.**

Question ID : 405859765

23. Let the set $C = \{(x, y) | x^2 - 2y = 2023, x, y \in \mathbb{N}\}$. Then $\sum_{(x,y) \in C} (x+y)$ is equal to _____.

Ans. Official answer NTA (46)**Sol.**

Question ID : 405859774

24. Let O be the origin and M and N be the points on the lines $\frac{x-5}{4} = \frac{y-4}{1} = \frac{z-5}{3}$ and $\frac{x+8}{12} = \frac{y+2}{5} = \frac{z+11}{9}$ respectively such that MN is the shortest distance between the given lines. Then $\overrightarrow{OM} \cdot \overrightarrow{ON}$ is equal to _____.

Ans. Official answer NTA (9)**MATRIX JEE ACADEMY**

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911

Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

**Sol.**

Question ID : 405859770

25. If $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \sqrt{1 - \sin 2x} dx = \alpha + \beta\sqrt{2} + \gamma\sqrt{3}$, where α , β and γ are rational numbers, then $3\alpha + 4\beta - \gamma$ is equal to

Ans. Official answer NTA(6)**Sol.**

Question ID : 405859772

26. Let $f(x) = \lim_{r \rightarrow x} \left\{ \frac{2r^2 \left[(f(r))^2 - f(x)f(r) \right]}{r^2 - x^2} - r^3 e^{\frac{f(r)}{r}} \right\}$ be differentiable in $(-\infty, 0) \cup (0, \infty)$ and $f(1) = 1$.

Then the value of ea, such that $f(a) = 0$, is equal to _____.**Ans.** Official answer NTA(2)**Sol.**

Question ID : 405859768

27. Remainder when $64^{32^{32}}$ is divided by 9 is equal to _____.**Ans.** Official answer NTA(1)**Sol.**

Question ID : 405859767

28. Let for any three distinct consecutive terms a, b, c of an A.P., the lines $ax + by + c = 0$ be concurrent at the point P and Q (α, β) be a point such that the system of equations

$$x + y + z = 6,$$

$$2x + 5y + \alpha z = \beta \text{ and}$$

$$x + 2y + 3z = 4, \text{ has infinitely many solutions. Then } (PQ)^2 \text{ is equal to } \underline{\hspace{2cm}}.$$

MATRIX JEE ACADEMY**Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911****Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in**



Ans. Official answer NTA(113)

Sol.

Question ID : 405859773

29. Let $P(\alpha, \beta)$ be a point on the parabola $y^2 = 4x$. If P also lies on the chord of the parabola $x^2 = 8y$ whose mid point is $\left(1, \frac{5}{4}\right)$, then $(\alpha - 28)(\beta - 8)$ is equal to _____ .

Ans. Official answer NTA(192)

Sol.

Question ID : 405859771

30. Let the area of the region $\{(x, y) : 0 \leq x \leq 3, 0 \leq y \leq \min\{x^2 + 2, 2x + 2\}\}$ be A . Then $12A$ is equal to :

Ans. Official answer NTA(164)

Sol.