# JEE Main January 2024

## Question Paper With Text Solution

## 29 January | Shift-1

# MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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# JEE MAIN JANUARY 2024 | 29TH JANUARY SHIFT-1

## SECTION – A

Question ID : 405859835

1. If $f(x)=\begin{cases}2+2x, & -1\le x>0\\ 1-\frac{x}{3}, & 0\le x\le 3\end{cases}$; $g(x)=\begin{cases}-x, & -3\le x\le 0\\ x, & 0<x\le 1\end{cases}$, then range of (fog) (x) is :

(1) [0, 1) (2) [0, 3) (3) (0, 1] (4) [0, 1]

**Ans.** Official answer NTA (4)

**Sol.**

Question ID : 405859848

2. Let $\left(5,\frac{a}{4}\right)$ be the circumcenter of a triangle with vertices A(a, –2), B(a, 6) and. Let a denote the circumradius, $\beta$ denote the area and $\gamma$ denote the perimeter of the triangle. Then $\alpha+\beta+\gamma$ is :

(1) 53 (2) 60 (3) 62 (4) 30

**Ans.** Official answer NTA (1)

**Sol.**

Question ID : 405859852

3. Let O be the origin and the position vectors of A and B be $2\hat{i}+2\hat{j}+\hat{k}$ and $2\hat{i}+4\hat{j}+4\hat{k}$ respectively. If the internal bisector of meets the line AB at C, then the length of OC is :

(1) $\frac{2}{3}\sqrt{34}$ (2) $\frac{3}{2}\sqrt{31}$ (3) $\frac{2}{3}\sqrt{31}$ (4) $\frac{3}{2}\sqrt{34}$

**Ans.** Official answer NTA (1)

**Sol.**

Question ID : 405859846

4. For $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$, if $y(x) = \int \frac{\operatorname{cosec} x \sin x}{\operatorname{cosec} x \sec x + \tan x \sin^2 x} dx$, and $\lim\limits_{x \to \left(\frac{\pi}{2}\right)^-} y(x) = 0$ then $y\left(\frac{\pi}{4}\right)$ is equal to :

(1) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (2) $\frac{1}{\sqrt{2}}\tan^{-1}\left(-\frac{1}{2}\right)$ (3) $-\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$ (4) $\frac{1}{2}\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)$

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 405859838

5. Let A be a square matrix such that $AA^T = I$. Then $\frac{1}{2}A\left[\left(A + A^T\right)^2 + \left(A - A^T\right)^2\right]$ is equal to :

(1) $A^2 + I$ (2) $A^2 + A^T$ (3) $A^3 + I$ (4) $A^3 + A^T$

**Ans.** Official answer NTA (4)

**Sol.**

Question ID : 405859840

6. In an A.P., the sixth term $a_6 = 2$. If the product $a_1 a_4 a_5$ is the greatest, then the common difference of the A.P. is equal to :

(1) 5/8 (2) 8/5 (3) 2/3 (4) 3/2

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 405859854

7. If $a, -\frac{\pi}{2} < a < \frac{\pi}{2}$ is the solution of $\cos\theta + 5\sin\theta = 1$, then the value of $\tan\alpha$ is :

(1) $\frac{10 - \sqrt{10}}{12}$ (2) $\frac{\sqrt{10} - 10}{6}$ (3) $\frac{10 - \sqrt{10}}{6}$ (4) $\frac{\sqrt{10} - 10}{12}$

**Ans.** Official answer NTA (4)

**Sol.**

Question ID : 405859845

8. If the value of the integral $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\frac{x^2\cos x}{1+\pi^x}+\frac{1+\sin^2 x}{1+e^{\sin x^{2023}}}\right)dx=\frac{\pi}{4}(\pi+a)-2$, then the value of a is :

(1) 3/2 (2) 3 (3) 2 (4) $-\frac{3}{2}$

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 405859851

9. Let $\vec{a}, \vec{b}$ and $\vec{c}$ be three non-zero vectors such that $\vec{b}$ and $\vec{c}$ are non-collinear. If $\vec{a}+5\vec{b}$ is collinear with $\vec{c}, \vec{b}+6\vec{c}$, is collinear with $\vec{a}$ and $\vec{a}+a\vec{b}+\beta\vec{c}=\vec{0}$ then $\alpha+\beta$ is equal to :

(1) – 25 (2) – 30 (3) 35 (4) 30

**Ans.** Official answer NTA (3)

**Sol.**

Question ID : 405859843

10. Suppose $f(x)=\frac{\left(2^x+2^{-x}\right)\tan x\sqrt{\tan^{-1}\left(x^2-x+1\right)}}{\left(7x^2+3x+1\right)^3}$. Then the value of f'(0) is equal to :

(1) $\frac{\pi}{2}$ (2) $\pi$ (3) 0 (4) $\sqrt{\pi}$

**Ans.**

**Ans.** Official answer NTA (4)

Question ID : 405859842

11. $\lim\limits_{x\to\frac{\pi}{2}}\left(\dfrac{1}{\left(x-\frac{\pi}{2}\right)^2}\int\limits_{x^3}^{\left(\frac{\pi}{3}\right)^3}\cos\left(t^{\frac{t}{3}}\right)dt\right)$ is equal to :

(1) $\dfrac{3\pi}{4}$ (2) $\dfrac{3\pi^2}{8}$ (3) $\dfrac{3\pi^2}{4}$ (4) $\dfrac{3\pi}{8}$

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 405859847

12. A function $y = f(x)$ satisfies $f(x)\sin 2x + \sin x - (1+\cos^2 x)f'(x) = 0$ with condition $f(0) = 0$. Then, $f\left(\dfrac{\pi}{2}\right)$ is equal to :

(1) 2 (2) 1 (3) 0 (4) – 1

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 405859836

13. Let R be a relation on $Z \times Z$ defined by (a, b) R (c, d) if and only if ad – bc is divisible by 5. Then R is :

(1) Reflexive but neither symmetric nor transitive

(2) Reflexive and symmetric but not transitive

(3) Reflexive and transitive but not stmmetric

(4) Reflexive, symmetric and transitive

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 405859850

14. Let PQR be a triangle with R (–1, 4, 2). Suppose M (2, 1, 2) is the mid point of PQ. The distance of the centroid of $\Delta PQR$ from the point of intersection of the lines $\frac{x-2}{0}=\frac{y}{2}=\frac{z+3}{-1}$ and $\frac{x-1}{1}=\frac{y+3}{-3}=\frac{z+1}{1}$ is :

(1) $\sqrt{99}$ (2) $\sqrt{69}$ (3) 69 (4) 9

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 405859849

15. In a $\Delta ABC$, suppose $y = x$ is the equation of the bisector of the angle B and the equation of the side AC is $2x - y = 2$. If $2AB = BC$ and the points A and B are respectively (4, 6) and $(\alpha, \beta)$, then $\alpha + 2\beta$ is equal to :

(1) 48 (2) 39 (3) 45 (4) 42

**Ans.** Official answer NTA (4)

**Sol.**

Question ID : 405859837

16. If $z=\frac{1}{2}-2i$ is such that $|z+1|=\alpha z+\beta(1+i), i=\sqrt{-1}$ and $\alpha,\beta\in R$, then $\alpha+\beta$ is :

(1) 2 (2) – 4 (3) 3 (4) –1

**Ans.** Official answer NTA (3)

**Sol.**

Question ID : 405859841

17. If in a G.P. of 64 terms, the sum of all the terms is 7 times sum of the odd terms the G.P., then the common ratio of the G.P. is equal to :

(1) 7 (2) 6 (3) 4 (4) 5

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 405859844

18. Consider the function $f:\left[\frac{1}{2},1\right]\to\mathbb{R}$ defined by $f(x)=4\sqrt{2}x^3-3\sqrt{2}x-1$. Consider the statements :

(I) The curve y = f(x) intersects the x-axis exactly at one point

(II) The curve y = f(x) intersects the x-axis at $x=\cos\frac{\pi}{12}$.

Then :

(1) Only (I) is correct

(2) Both (I) and (II) are the correct

(3) Both (I) and (II) are incorrect

(4) Only (II) is correct

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 405859839

19. Let $A=\begin{bmatrix}1&0&0\\0&\alpha&\beta\\0&\beta&\alpha\end{bmatrix}$ and $|2A|^3=2^{21}$ where $\alpha,\beta\in Z$. Then a value of $\alpha$ is :

(1) 5 (2) 17 (3) 3 (4) 9

**Ans.** Official answer NTA (1)

**Sol.**

Question ID : 405859853

20. A fair die is thrown until 2 appears. Then the probability, then 2 appears in even number of throws, is :

(1) 5/11 (2) 1/6 (3) 6/11 (4) 5/6

**Ans.** Official answer NTA (1)

**Sol.**

**SECTION – B**

Question ID : 405859857

21. If $\frac{^{11}C_1}{2}+\frac{^{11}C_2}{3}+\ldots+\frac{^{11}C_9}{10}=\frac{n}{m}$ with gcd(n, m) = 1, then n + m is equal to _____.

**Ans.** Official answer NTA (2041)

**Sol.**

Question ID : 405859856

22. All the letters of the word "GTWENTY" are written in all possible ways with or without meaning and these words are written as in a dictionary. The serial number of the word "GTWENTY" is _____.

**Ans.** Official answer NTA (553)

**Sol.**

Question ID : 405859862

23. If the points of intersection of two distinct conics $x^2+y^2=4b$ and $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ lie on the curve $y^2=3x^2$, then $3\sqrt{3}$ times the area of the rectangle formed by the intersection points _____.

**Ans.** Official answer NTA (432)

**Sol.**

Question ID : 405859860

24. If the solution curve y = y(x) of the differential equation $(1+y^2)(1+\log_e x)\,dx + x\,dy = 0$, $x>0$ passes through the point (1, 1) and $y(e)=\frac{\alpha-\tan\left(\frac{3}{2}\right)}{\beta+\tan\left(\frac{3}{2}\right)}$, then $\alpha+2\beta$ is _____.

**Ans.** Official answer NTA (3)

**Sol.**

Question ID : 405859863

25. A line with direction ratios 2, 1, 2 meets the lines $x = y + 2 = z$ and $x + 2 = 2y = 2z$ respectively at the points P and Q. If the length of the perpendicular from the point (1, 2, 12) to the line PQ is $l$, then $l^2$ is _____.

**Ans.** Official answer NTA (65)

**Sol.**

Question ID : 405859861

26. Equation of two diameters of a circle are $2x - 3y = 5$ and $3x - 4y = 7$. The line joining the points $\left(-\frac{22}{7}, -4\right)$ and $\left(-\frac{1}{7}, 3\right)$ intersects the circle at only one point $P(\alpha, \beta)$. Then $17\beta - \alpha$ is equal to _____.

**Ans.** Official answer NTA (2)

**Sol.**

Question ID : 405859858

27. Let $f(x) = 2^x - x^2$, $x \in R$. If m and n are respectively the number of points at which the curves $y = f(x)$ and $y = f'(x)$ intersects the x-axis, then the value of $m + n$ is _____.

**Ans.** Official answer NTA (5)

**Sol.**

Question ID : 405859859

28. The area (in sq. units) of the part of the circle $x^2 + y^2 = 169$ which is below the line $5x - y = 13$ is $\frac{\pi\alpha}{2\beta} - \frac{65}{2} + \frac{\alpha}{\beta}\sin^{-1}\left(\frac{12}{13}\right)$, where $\alpha$, $\beta$ are coprime number. Then $\alpha + \beta$ is equal to _____.

**Ans.** Official answer NTA (171)

**Sol.**

Question ID : 405859864

29. If the mean and variance of the data 65, 68,58, 44, 48, 45, 60, $\alpha$, $\beta$, 60 where $\alpha > \beta$, are 56 and 66.2 respectively, then $\alpha^2 + \beta^2$ is equal to _____.

**Ans.** Official answer NTA (6344)

**Sol.**

Question ID : 405859855

30. Let $\alpha$, $\beta$ the roots of the equation $x^2 - x + 2 = 0$ with $Im(\alpha) > Im(\beta)$. Then $\alpha^6 + \alpha^4 + \beta^4 - 5\alpha^2$ is equal to _____.

**Ans.** Official answer NTA (13)

**Sol.**