JEE Main June 2022

Question Paper With Text Solution

29 June | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JUNE 2022 | 29TH JUNE SHIFT-1

SECTION – A

Question ID : 101761

Probability

1. The probability that a randomly chosen 2 × 2 matrix with all the entries from the set of first 10 primes, is singular, is equal to :

एक यादृच्छया चुने गए 2 × 2 के आव्यूह, जिसके सभी अवयव प्रथम 10 अभाज्य संख्याओं के समुच्चय में से हैं, के अव्युत्क्रमणीय होने की प्रायिकता है :

(1) $\frac{133}{10^4}$ (2) $\frac{18}{10^3}$ (3) $\frac{19}{10^3}$ (4) $\frac{271}{10^4}$

Ans. Official Answer NTA (3)

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ a,b,c $\in$ {2, 3, 5, 7, 11, 13, 17, 19, 23, 29}

Total no. of possible cases fro matrix A = 10^4 = n

When |A| = 0 → singular matrix

⇒ ad – bc = 0

Case -1 a = b and c = d i.e. only two prime no. used 10 × 9 × 2

a ≠ c

Case - 2 a = b = c = d ie only one prime is used 10

Total cases for |A| = 0 ⇒ 10 × 9 + 10 × 10 = 10 × 19 = m

Probability that matrix A is singular = $\frac{m}{n} = \frac{10 \times 19}{10^4} = \frac{19}{10^3}$

Question ID : 101762

Differential Equation

2. Let the solution curve of the differential equation $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, y(1) = 3 be y = y(x). Then y(2) is equal to :

माना अवकल समीकरण $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, y(1) = 3 का हल वक्र y = y(x) है। तो y(2) का मान है :

(1) 15 (2) 11 (3) 13 (4) 17

Ans. Official Answer NTA (1)

Sol. $x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$

$$\frac{dy}{dx}=\frac{y}{x}+\sqrt{\frac{y^2}{x^2}+16} \quad ....(1)$$

Put in (1) $\frac{y}{x}=\mu \Rightarrow \frac{dy}{dx}=\mu+\frac{xd\mu}{dx}$

$$\mu+\frac{xd\mu}{dx}=\mu+\sqrt{\mu^2+16}$$

$$\frac{d\mu}{\sqrt{\mu^2+16}}=\frac{dx}{x}$$

integrate

$$\ln\left(\mu+\sqrt{\mu^2+16}\right)=\ln x+\ln c$$

$$\frac{y}{x}+\frac{\sqrt{y^2+16x^2}}{x}=cx$$

$$y+\sqrt{y^2+16x^2}=cx^2 \quad ....(2)$$

given that x = 1 y = 3

$3+5=c \Rightarrow c=8$

from (2) $y+\sqrt{y^2+16x^2}=8x^2$

put x = 2

then y = 15

Question ID : 101763

3D Geometry

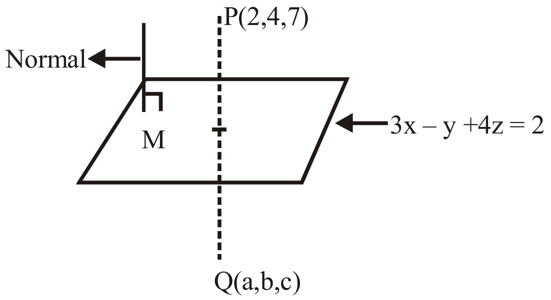
3. If the mirror image of the point (2, 4, 7) in the plane $3x-y+4z=2$ is (a, b, c), then $2a+b+2c$ is equal to :

यदि बिन्दु (2, 4, 7) का समतल $3x-y+4z=2$ में दर्पण प्रतिबिंब (a, b, c) है, तो $2a+b+2c$ बराबर है:

(1) 54 (2) 50 (3) –6 (4) –42

Ans. Official Answer NTA (3)

Sol.



$M\left(\frac{a+2}{2},\frac{b+4}{2},\frac{c+7}{2}\right)$

lies on plane

$3a - b + 4c = -26$ ....(1)

drs of PQ $a-2, b-4, c-7$

drs of normal plane $3, -1, 4$

$\frac{a-2}{3} = \frac{b-4}{-1} = \frac{c-7}{4} = k$ (say)

$a = 3k + 2, b = 4 - k, c = 4k + 7$

Put in (1)

$13k = -28$ ....(2)

Now $2a + b + 2c = 13k + 22 = -28 + 22 = -6$ Ans.

Question ID : 101764

Definite Integration

4. Let $f : R \to R$ be a function by :

माना एक फलन $f : R \to R$

$$f(x) = \begin{cases} \max\limits_{t \le x}\{t^3 - 3t\} & ; \quad x \le 2 \\ x^2 + 2x - 6 & ; \quad 2 < x < 3 \\ [x-3] + 9 & ; \quad 3 \le x \le 5 \\ 2x + 1 & ; \quad x > 5 \end{cases}$$

where [t] is the greatest integer less than or equal to t. Let m be the number of points where f is not differentiable and $I = \int_{-2}^{2} f(x)\,dx$. Then the ordered pair (m, I) is equal to :

द्वारा परिभाषित हैं, जहाँ [t] महत्तम पूर्णांक $\le t$ है। माना m उन बिन्दुओं की संख्या हैं, जहाँ f अवकलनीय नहीं हैं तथा $I = \int_{-2}^{2} f(x)\,dx$ है। तो क्रमित युग्म (m, I) बराबर है :

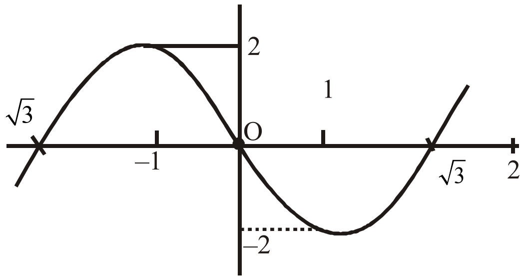
(1) $\left(3, \frac{27}{4}\right)$ (2) $\left(3, \frac{23}{4}\right)$ (3) $\left(4, \frac{27}{4}\right)$ (4) $\left(4, \frac{23}{4}\right)$

Ans. Official Answer NTA (3)

Sol. $\max\{t^3 - 3t\}; x \le 2$

$t \le x$

$g(t) = t^3 - 3t \Rightarrow g'(t) = 3(t^2 - 1)$



clearly $f(x)\begin{cases} x^3 - 3x & x < -1 \\ 2 & -1 \le x \le 2 \\ x^2 + 2x - 6 & 2 < x < 3 \\ 9 & 3 \le x < 4 \\ 10 & 4 \le x < 5 \\ 11 & x = 5 \\ 2x + 1 & x > 5 \end{cases}$

Non diff. at x = 2, 3, 4, 5

m = 4

$I = \int_{-2}^{2} f(x)dx = \int_{-2}^{-1}\left(x^3 - 3x\right)dx + \int_{-1}^{2} 2dx$

$= \frac{27}{4}$

Ans. $\left(4, \frac{27}{4}\right)$

Question ID : 101765

Vectors

5. Let $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ where $\alpha, \beta \in R$, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$, then the value of $\alpha + \beta$ is equal to :

माना तीन सदिश $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ तथा $\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$ हैं, जहाँ $\alpha, \beta \in R$ हैं। यदि $\vec{a}$ का $\vec{c}$ पर प्रक्षेप $\frac{10}{3}$ है तथा $\vec{b} \times \vec{c} = -6\hat{i} + 10\hat{j} + 7\hat{k}$ है, तो $\alpha + \beta$ का मान बराबर है :

(1) 3 (2) 4 (3) 5 (4) 6

Ans. Official Answer NTA (1)

Sol. $\vec{a} = \alpha\hat{i} + 3\hat{j} - \hat{k}$, $\vec{b} = 3\hat{i} - \beta\hat{j} + 4\hat{k}$ $\alpha, \beta \in R$

$\vec{c} = \hat{i} + 2\hat{j} - 2\hat{k}$

Projection of $\vec{a}$ on $\vec{c}$ = $\frac{\vec{a}.\vec{c}}{|\vec{c}|} = \frac{10}{3}$

$\alpha + 6 + 2 = 10$

$$\vec{b} \times \vec{c} = \begin{vmatrix} i & j & k \\ 3 & -\beta & 4 \\ 1 & 2 & -2 \end{vmatrix} = i(2\beta - 8) + j(4 + 6) + k(6 + \beta) = -6i + 10j + 7k$$

$2\beta - 8 = -6$

$\beta = 1$

$\alpha + \beta = 3$

Question ID : 101766

Area Under Curve

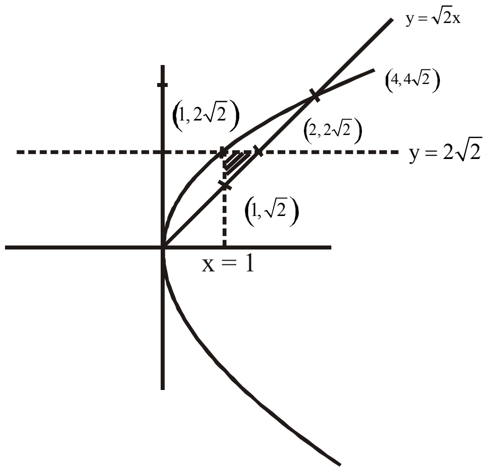
6. The area enclosed by $y^2 = 8x$ and $y = \sqrt{2}x$ that lies outside the triangle formed by $y = \sqrt{2}x$, $x = 1$, $y = 2\sqrt{2}$, is equal to :

वक्रों $y^2 = 8x$ तथा $y = \sqrt{2}x$ से घिरे तथा रेखाओं $y = \sqrt{2}x$, $x = 1$, $y = 2\sqrt{2}$ से बने त्रिभुज के बाहर के क्षेत्र का क्षेत्रफल है :

(1) $\frac{16\sqrt{2}}{6}$ (2) $\frac{11\sqrt{2}}{6}$ (3) $\frac{13\sqrt{2}}{6}$ (4) $\frac{5\sqrt{2}}{6}$

Ans. Official Answer NTA (3)

Sol.



Area of triangle $=\frac{1}{2}\times\sqrt{2}\times 1=\frac{1}{\sqrt{2}}$

req. area $=\int_0^4\left(2\sqrt{2}\sqrt{x}-\sqrt{2}x\right)dx-\frac{1}{\sqrt{2}}$

$$=\left(2\sqrt{2}\frac{x^{3/2}}{\frac{3}{2}}-\sqrt{2}\frac{x^2}{2}\right)_0^4-\frac{1}{\sqrt{2}}$$

$$=\frac{4\sqrt{2}(4)^{3/2}}{3}-\frac{16\sqrt{2}}{2}-\frac{1}{\sqrt{2}}$$

$$=\frac{32\sqrt{2}}{3}-\frac{16\sqrt{2}+\sqrt{2}}{2}$$

$$=\frac{64\sqrt{2}-51\sqrt{2}}{6}=\frac{13\sqrt{2}}{6}$$ Ans.

Question ID : 101767

Determinant

7. If the system of linear equations

$2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$, where $\delta, k \in R$

has infinitely many solutions, then $\delta + k$ is equal to :

यदि रैखिक समीकरण निकाय

$2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$, जहाँ $\delta, k \in R$

हैं, के अनंत हल हैं, तो $\delta + k$ बराबर है :

(1) –3 (2) 3 (3) 6 (4) 9

Ans. Official Answer NTA (2)

Sol. System of equation $2x + y - z = 7$

$x - 3y + 2z = 1$

$x + 4y + \delta z = k$

for infinitely many $\Delta = \begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix} = 0 \Rightarrow \delta = -3$

$\Delta_y = \begin{vmatrix} 2 & 7 & -1 \\ 1 & 1 & 2 \\ 1 & k & -3 \end{vmatrix} = 0 \Rightarrow k = 6$

$\delta + k = 3$ Ans.

Question ID : 101768

Quadratic Equation

8. Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to :

माना समीकरण $x^2 + (2i - 1) = 0$ के मूल α तथा β हैं। तो $|\alpha^8 + \beta^8|$ का मान बराबर है :

(1) 50 (2) 250 (3) 1250 (4) 1500

Ans. Official Answer NTA (1)

Sol. α, β are the roots of equation

$x^2 + 2i - 1 = 0$

$\alpha = +\sqrt{-2i+1} \quad \alpha^2 = 1 - 2i$

$|\alpha|^2 = \sqrt{5} \quad \alpha^4 = 5 \quad \alpha^8 = 25$

Similarly $\beta^8 = 25$

$\alpha^8 + \beta^8 = 50$

Question ID : 101769

Mathematical Reasoning

9. Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q)\Delta((p \vee q) \Rightarrow q)$ is a tautology. Then Δ is equal to :

माना $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ इस प्रकार है कि $(p \wedge q)\Delta((p \vee q) \Rightarrow q)$ एक पुनरूक्ति है। Δ बराबर है :

(1) $\wedge$ (2) $\vee$ (3) $\Rightarrow$ (4) $\Leftrightarrow$

Ans. Official Answer NTA (3)

Sol.

P	q	$P \vee q$	$(P \vee q) \Rightarrow q$	$P \wedge q$	$(p \wedge q) \Rightarrow ((p \vee q) \Rightarrow q)$
T	T	T	T	T	T
T	F	T	F	F	T
F	T	T	T	F	T
F	F	F	T	F	T

Δ is equal to ⇒

Question ID : 101770

Matrices

10. Let $A = \left[a_{ij}\right]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all i, j = 1, 2, 3. Then, the matrix $A^2 + A^3 + ... + A^{10}$ is equal to :

माना कोटि 3 का एक वर्ग आव्यूह $A = \left[a_{ij}\right]$ है, जिसमें $a_{ij} = 2^{j-i}$, $\forall$ i, j = 1, 2, 3 हैं। तो आव्यूह $A^2 + A^3 + ... + A^{10}$ बराबर है :

(1) $\left(\frac{3^{10}-3}{2}\right)A$ (2) $\left(\frac{3^{10}-1}{2}\right)A$ (3) $\left(\frac{3^{10}+1}{2}\right)A$ (4) $\left(\frac{3^{10}+3}{2}\right)A$

Ans. Official Answer NTA (1)

Sol. $$A = \begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 4 & 8 & 16 \\ 2 & 4 & 8 \\ 1 & 2 & 4 \end{bmatrix}$$

$$A^2 = \frac{1}{16}\begin{bmatrix} 4 & 8 & 16 \\ 2 & 4 & 8 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 4 & 8 & 16 \\ 2 & 9 & 8 \\ 1 & 2 & 4 \end{bmatrix} = \frac{1}{16}\begin{bmatrix} 48 & 96 & 192 \\ 24 & 98 & 96 \\ 12 & 24 & 48 \end{bmatrix} = 3\begin{bmatrix} 1 & 2 & 4 \\ \frac{1}{2} & 1 & 2 \\ \frac{1}{4} & \frac{1}{2} & 1 \end{bmatrix}$$

$A^2 = 3A$

$A^3 = A.A^2 = A(3A) = 3A^2 = 33A = 3^2A$

$A^4 = 3^3A + ... + A^{10} = 3^9A$

$A^2 + ... + A^{10} = (3 + 3^2 + ... + 3^9) = 3.\frac{3^9-1}{2}A$

$= \frac{3^{10}-3}{2}A$

Question ID : 101771

Set & Relations

11. Let a set $A = A_1 \cup A_2 \cup .... \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j$, $1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i, 1 \leq i \leq k\}$. Then, R is :

(1) reflexive, symmetric but not transitive

(2) reflexive, transitive but not symmetric

(3) reflexive but not symmetric and transitive

(4) an equivalence relation

माना एक समुच्चय $A = A_1 \cup A_2 \cup ....\cup A_k$ हैं। जहाँ $A_i \cap A_j = \phi$, $i \neq j, 1 \leq i, j \leq k$ है। A से में संबंध R,

$R = \{(x, y) : y \in A_i$ यदि और केवल यदि $x \in A_i, 1 \leq i \leq k\}$:

(1) स्वतुल्य तथा सममित है परन्तु संक्रामक नहीं है

(2) स्वतुल्य तथा संक्रामक है परन्तु सममित नहीं है

(3) स्वतुल्य है परन्तु सममित तथा संक्रामक नहीं है

(4) एक तुल्यता संबंध है

Ans. Official Answer NTA (4)

Sol. $A = A_1 \cup A_2 \cup A_3 A \_\_\_ \cup A_k$

$A_i \cap A_i = \phi \quad i \neq j$

$R : R \rightarrow A \qquad 1 \leq i_j j \leq k$

1. $R \equiv \{(x, x)\ \forall x \in A\}$ R is an equivdence relation

2. Symmetric if possible let $(x, y) \in R\ \ x, y \in A$;

$\Rightarrow y \in A_i$ iff $x \in A_i\ \ 1 \leq i \leq k$

$\Rightarrow x_f \in A_i$ iff $y \in A_i\ \ 1 \leq i \leq k$

$\Rightarrow (y, x) \in R$

$(x, y) \in R \Rightarrow (y, x)\ \lambda x, y \in A$

$\Rightarrow$ R is symmetric

3. Transitive if possible let $(x, y), (y, z) \in R\ x, y, z \in A_i$

$\Rightarrow (x, y) \in R$ and $(y, z) \in R$

$\Rightarrow y \in A_i$ iff $x_i \in A_i$ and $z \in A_i$ iff $y \in A_i$

$\Rightarrow x_i \in A_i$ iff $y_i \in A_i$ and $z \in A_i$ iff $y \in A_i$

$\Rightarrow Z \in A_i$ iff $x \in A_i$

$\Rightarrow (x, z) \in R$

$(x, y), (y, z) \in R \Rightarrow (x, z) \in R \quad \forall\ x, y, z \in A_i$

R is transitive

Hence R is an equivalence redation

Question ID : 101772

Sequence & progression

12. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to :

माना $\{a_n\}_{n=0}^{\infty}$ एक अनुक्रम है जिसमें $a_0 = a_1 = 0$ हैं तथा सभी $n \geq 0$ के लिए $a_{n+2} = 2a_{n+1} - a_n + 1$ हैं। तो $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ बराबर हैं :

(1) $\frac{6}{343}$ (2) $\frac{7}{216}$ (3) $\frac{8}{343}$ (4) $\frac{49}{216}$

Ans. Official Answer NTA (2)

Sol. $a_0 = a_1 = 0$

$a_{n+2} = 2a_{n+1} - a_n + 1$

$a_2 = 2a_1 - a_0 + 1 = 1$

$a_3 = 2a_2 - a_1 + 1 = 2 + 1 = 3$

$a_4 = 2a_3 - a_2 + 1 = 6$

$a_5 = 2a_4 - a_3 + 1 = 10$

$$S_\infty = \sum_{n=2}^{\infty} \frac{a_n}{7^n}$$

$$S_\infty = \frac{1}{7^2} + \frac{3}{7^3} + \frac{6}{7^4} + \frac{10}{7^5} + \quad \text{.............(1)}$$

$$\frac{1}{7} S_\infty = \frac{1}{7^3} + \frac{3}{7^4} + \frac{4}{7^5} + \quad \text{................(2)}$$

(1)-(2)

$$\frac{6}{7} S_\infty = \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \frac{4}{7^5} + \quad \text{..............(3)}$$

$$\frac{6}{7^2} S_\infty = \frac{1}{7^3} + \frac{2}{7^4} + \frac{3}{7^5} + \quad \text{...............(4)}$$

(3)-(4)

$$\frac{6}{7}S_\infty\left(1-\frac{1}{7}\right)=\frac{1}{7^2}+\frac{1}{7^3}+\frac{1}{7^4}+..........$$

$$36S_\infty=1+\frac{1}{7}+\frac{1}{7^2}+.........$$

$$S_\infty=\frac{1}{1-\frac{1}{7}}\cdot\frac{1}{36}=\frac{7}{216}\text{ Ans.}$$

Question ID : 101773

Straight Line

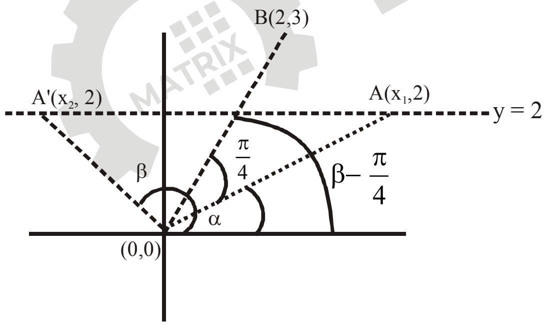
13. The distance between the two points A and A′ which lie on y = 2 such that both the line segments AB and A′ B (where B is the point (2, 3)) subtent angle $\frac{\pi}{4}$ at the origin, is equal to :

माना रेखा y = 2 पर दो बिन्दु A तथा A′ इस प्रकार हैं कि दोनों रेखा खंड AB तथा A′ B (जहाँ B, बिन्दु (2, 3) है), मूल बिन्दु पर $\frac{\pi}{4}$ का कोण बनाते हैं। तो बिन्दुओं A तथा A′ के बीच की दूरी है :

(1) 10 (2) $\frac{48}{5}$ (3) $\frac{52}{5}$ (4) 3

Ans. Official Answer NTA (3)

Sol.



$$\tan\left(\frac{\pi}{4}+\alpha\right)=\frac{3}{2},\tan\alpha=\frac{2}{x_1}$$

$$\frac{1+\frac{2}{x_1}}{1-\frac{2}{x_1}}=\frac{3}{2}\Rightarrow x_1=10$$

And

$$\begin{cases} \tan\left(\beta - \frac{\pi}{4}\right) = \frac{3}{2} \\ \tan\beta = \frac{2}{x_2} \end{cases} \rightarrow \text{solving}$$

$$\Rightarrow \; x_2 = -\frac{2}{5}$$

$$AA^1 = |x_2 - x_1| = \frac{52}{5}$$

Question ID : 101774

Maxima & Minima

14. A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is :

एक 22 मी. लंबे तार को दो टुकड़ों में विभक्त किया जाना है। एक टुकड़े से एक वर्ग तथा दूसरे से एक समबाहु त्रिभुज बनाया जाना है। समबाहु त्रिभुज की भुजा की लंबाई, जिसके लिए वर्ग तथा समबाहु त्रिभुज का सम्मिलित क्षेत्रफल न्यूनतम हो, है :

(1) $\frac{22}{9+4\sqrt{3}}$ (2) $\frac{66}{9+4\sqrt{3}}$ (3) $\frac{22}{4+9\sqrt{3}}$ (4) $\frac{66}{4+9\sqrt{3}}$

Ans. Official Answer NTA (2)

Sol. Let x be the length out from a wire of length 22 m.
and made of wire an equilateral Δ whose side is b.
3b = x and remaining length = 22 – x

$$b = \frac{x}{3}$$

Let a be the side of square

$$4a = 22 - x \Rightarrow a = \frac{22-x}{4}$$

A = combined area of the square and equilateral Δ.

$$= \frac{\sqrt{3}}{4}b^2 + a^2$$

$$=\frac{\sqrt{3}}{4}\frac{x^2}{9}+\frac{(22-x)^2}{16}=\frac{\sqrt{3}}{36}x^2+\frac{1}{16}x^2-\frac{44x}{16}+\frac{(22)^2}{16}$$

$$A=\left(\frac{\sqrt{3}}{36}+\frac{1}{16}\right)x^2-\frac{11}{4}x+\frac{484}{16}$$

$$A_{min}\text{ at }x=\frac{\frac{11}{4}}{2\left(\frac{\sqrt{3}}{36}+\frac{1}{16}\right)}$$

$$b=\frac{x}{3}=\frac{1}{3}.\frac{11.4.4.9}{4.2\left(4\sqrt{3}+9\right)}$$

$$b=\frac{66}{9+4\sqrt{3}}$$

Question ID : 101775

ITF

15. The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is :

फलन $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ का प्रांत है :

(1) $R-\left\{-\frac{1}{2},\frac{1}{2}\right\}$

(2) $(-\infty,-1]\cup[1,\infty)\cup\{0\}$

(3) $\left(-\infty,\frac{-1}{2}\right)\cup\left(\frac{1}{2},\infty\right)\cup\{0\}$

(4) $\left(-\infty,\frac{-1}{\sqrt{2}}\right]\cup\left[\frac{1}{\sqrt{2}},\infty\right)\cup\{0\}$

Ans. Official Answer NTA (4)

Sol. For domain $\cos^{-1}\left(\dfrac{2\sin^{-1}\left(\dfrac{1}{4x^2-1}\right)}{\pi}\right)$

$$-1 \le \frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi} - \le 1$$

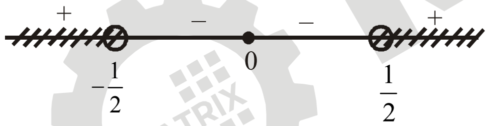
$$\Rightarrow\ -\frac{\pi}{2} \le \sin^{-1}\left(\frac{1}{4x^2-1}\right) \le \frac{\pi}{2}$$

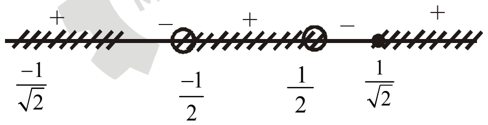
$$\Rightarrow\ -1 \le \frac{1}{4x^2-1} \le 1$$

$$\Rightarrow\ \frac{1}{4x^2-1} \le 1 \text{ and } -1 \le \frac{1}{4x^2-1}$$

$$\Rightarrow\ 0 \le \frac{4x^2-1-1}{4x^2-1} \text{ and } 0 \le \frac{x^2}{(2x-1)(2x+1)}$$

$$\Rightarrow 0 \le \frac{(\sqrt{2}x-1)(\sqrt{2}x+1)}{(2x-1)(2x+1)} \text{ and } \le \frac{x^2}{(2x-1)(2x+1)}$$





$$x \in \left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$$

Question ID : 101776

Binomial Theorem

16. If the constant term in the expansion of $\left(3x^3-2x^2+\frac{5}{x^5}\right)^{10}$ is $2^k . l$, where l is an odd integer, then the value of k is equal to :

यदि $\left(3x^3-2x^2+\frac{5}{x^5}\right)^{10}$ के प्रसार में अचर पद $2^k . l$ है, जहाँ l एक विषम पूर्णांक है, तो k का मान बराबर है :

(1) 6 (2) 7 (3) 8 (4) 9

Ans. Official Answer NTA (4)

Sol. $\left(3x^2-2x^2+\frac{5}{x^5}\right)^{10}$ its constant term is $2^k.l$

its general term $=\frac{10!}{r_1!r_2!r_3!}\left(3x^3\right)^{r_1}\left(-2x^2\right)^{r_2}\left(\frac{5}{x^2}\right)^{r_3}$

where $r_1 + r_2 + r_3 = 10$

$=\frac{10!}{r_1!r_2!r_3!}3^{r_1}\left(-2\right)^{r_2}5^{r_2}x^{3r_1+2r_2-5r_3}$

for constant term $= 3r_1 + 2r_2 - 5r_3 = 0$

$\ell.2^k=\frac{10!}{r_1!r_2!r_3!}3^{r_1}(-2)^{r_2}.5^{r_3}$ $\quad r_1=1, r_2=8 \quad r_3=1 \quad r_2 = \text{even}$

$= 10.9.3.5.2^8$

$l.\ 2^k = 5^2.3^3.2^9$ $\quad r_1 = 5r_3 = 0 \quad r_2 = 10$

$l.\ 2^k = 5^2.\ 3^3\ .\ 2^9$ $\quad r_1 = r_3 = 0 \quad r_2 = 10$

$l.\ 2^k = 1.2^{10}$

k = 9 Ans. $\quad$ k = 10

Question ID : 101777

Definite Integration

17. $\int_0^5 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx$ where [t] denotes greatest integer less than or equal to t, is equal to :

$\int_0^5 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx$ जहाँ [t] महत्तम पूर्णांक $\leq$ t है, बराबर है :

(1) – 3 (2) – 2 (3) 2 (4) 0

Ans. Official Answer NTA (4)

Sol. $\int_0^5 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx = \int_0^2 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx$

$+\int_2^4 \cos\left(\pi\left(x-\left[\frac{x}{2}\right]\right)\right)dx$

$+\int_4^5 \cos\left(\pi\left(-.\left[\frac{x}{2}\right]\right)\right)dx$

$= \left.\frac{\sin \pi x}{\pi}\right)_0^2 + \left.\frac{\sin \pi(x-1)}{\pi}\right)_2^4 + \left.\frac{\sin \pi(x-2)}{\pi}\right)_4^5$

$= 0$

Question ID : 101778

Ellipse

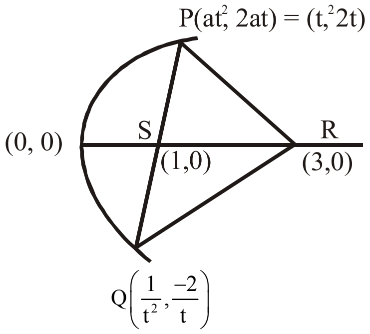
18. Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point (3, 0). Let the line segment PQ be also a focal chord of the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is equal to :

माना परवलय $y^2 = 4x$ की एक नाभीय जीवा PQ, बिंदु (3, 0) पर $\frac{\pi}{2}$ कोण बनाती है माना रेखाखण्ड PQ, दीर्घवृत्त $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a^2 > b^2$, की भी एक नाभीय जीवा है। यदि दीर्घवृत्त E की उत्तकेन्द्रता e है, तो $\frac{1}{e^2}$ का मान बराबर है :

(1) $1+\sqrt{2}$ (2) $3+2\sqrt{2}$ (3) $1+2\sqrt{3}$ (4) $4+5\sqrt{3}$

Ans. Official Answer NTA (2)

Sol.



$M_{PR}.\ M_{QR} = -1$

$$\frac{2t-0}{t^2-3}.\frac{\frac{-2}{t}-0}{\frac{1}{t^2}-3}=-1$$

$$\frac{4t^2}{(t^2-3)(1-3t^2)}=1$$

t = + 1

P(1, 2) Q (1, – 2)

PQ also focal chord of ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ (a > b)

$$\frac{1}{a^2}+\frac{4}{b^2}=1\ ,\ ae=1$$

$$\frac{1}{a^2}+\frac{4}{a^2(1-e^2)}=1$$

$$\frac{1}{1}+\frac{4}{1-e^2}=\frac{1}{e^2}\ \Rightarrow e^4-6e^2-1=0$$

$$\Rightarrow \frac{1}{e^2}=3+2\sqrt{2}$$

Question ID : 101779

Circle

19. Let the tangent to the circle $C_1 : x^2 + y^2 = 2$ at the point M(–1, 1) intersect the circle $C_2 : (x-3)^2 + (y-2)^2 = 5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to :

माना वृत्त $C_1 : x^2 + y^2 = 2$ के बिंदु $M(-1, 1)$ पर स्पर्श रेखा वृत्त $C_2 : (x-3)^2 + (y-2)^2 = 5$ को दो भिन्न बिन्दुओं A तथा B पर काटती है। यदि बिन्दुओं A तथा B पर वृत्त C_2 की स्पर्श रेखाएँ बिन्दु N पर मिलती हैं, तो त्रिभुज ANB का क्षेत्रफल बराबर है :

(1) $\frac{1}{2}$ (2) $\frac{2}{3}$ (3) $\frac{1}{6}$ (4) $\frac{5}{3}$

Ans. Official Answer NTA (3)

Sol. Tangent at M (– 1, 1) of circle $C_1 : x^2 + y^2 = 2$

is $-x + y = 2$

$y - 2 = x$ ....(1)

This tangent intersect circle $(x-3)^2 + x^2 = 5$

$2x^2 - 6x + 4 = 0$

$x^2 - 3x + 2 = 0$

$x = 1, 2$

A (1, 3) B(2, 4)

Let (h, k) AB is chord of contact of circle

$(x-3)^2 + (y-2)^2 = 5$

$\Rightarrow x^2 + y^2 - 6x - 4y + 8 = 0$

COC $hx + ky - 3(x + 4) - 2(y + k) + 8 = 0$

$(h-3)x + (k-2)y - 3h - 2k + 8 = 0$ ....(2)

from (1) and (2) $\frac{h-3}{-1} = \frac{k-2}{1} = \frac{3h+2k-8}{2}$

$(h, k) \equiv \left(\frac{4}{3}, \frac{11}{3}\right)$

Area of NAB $= \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 2 & 4 & 1 \\ \frac{4}{3} & \frac{11}{3} & 1 \end{vmatrix} = \frac{1}{6}$ Units

Question ID : 101780

Statistics

20. Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively. If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, the $(4a + x_5)$ is equal to :

माना 5 प्रेक्षणों x_1, x_2, x_3, x_4, x_5 के माध्य तथा प्रसरण क्रमशः $\frac{24}{5}$ तथा $\frac{194}{25}$ हैं। यदि प्रथम 4 प्रेक्षणों के माध्य तथा प्रसरण क्रमशः $\frac{7}{2}$ तथा a हैं, तो $(4a + x_5)$ बराबर है :

(1) 13 (2) 15 (3) 17 (4) 18

Ans. Official Answer NTA (2)

Sol. Mean $= \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5} = \frac{24}{5} \Rightarrow x_1 + x_2 + x_3 + x_4 + x_5 = 24$

$$\Rightarrow \frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{7}{2}$$

$$\Rightarrow x_1 + x_2 + x_3 + x_4 = 14 \qquad \Rightarrow x_5 = 10$$

$$\frac{\sum_{i=1}^{5} x_1^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$$

$$\sum_{i=1}^{5} x_i^2 = 154 \Rightarrow x_1^2 + x_2^2 + x_3^2 + x_4^2 = 154 - 100 = 54$$

$$\frac{\sum_{i=1}^{4} x_1^2}{4} - \left(\frac{7}{2}\right)^2 = a$$

$$\Rightarrow a = \frac{5}{4}$$

$x_5 + 4a = 10 + 5 = 15$

SECTION – B

Question ID : 101781

Complex number

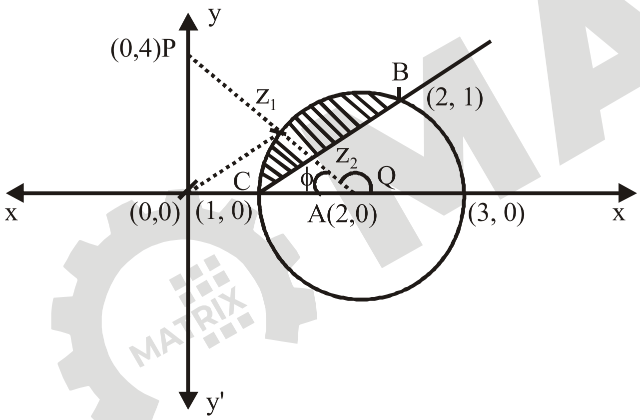
21. Let $S=\{z\in C:|z-2|\le 1, z(1+i)+\bar{z}(1-i)\le 2\}$. Let |z – 4i| attains minimum and maximum values, respectively, at $z_1\in S$ and $z_2\in S$. If $5(|z_1|^2+|z_2|^2)=\alpha+\beta\sqrt{5}$, where α and β are integers, then the value of α + β is equal to ________.

माना $S=\{z\in C:|z-2|\le 1, z(1+i)+\bar{z}(1-i)\le 2\}$ है। माना |z – 4i| के निम्नतम तथा उच्चतम मान क्रमशः $z_1\in S$ तथा $z_2\in S$ पर प्राप्त होते हैं। यदि $5(|z_1|^2+|z_2|^2)=\alpha+\beta\sqrt{5}$ है, जहाँ α तथा β पूर्णांक हैं, तो α + β का मान बराबर है ________

Ans. Official Answer NTA (26)

Sol. $S=\{z\in C:|z-2|\le 1, z(1+i)+\bar{z}(1-i)\le 2\}$

S represent shaded region



S represents the shaded region shown in the diagram.
Clearly z_1 will be the point of intersection PA and circle.
PA : 2x + y = 4
equation of circle $(x-2)^2+y^2=1$

on solving we get $z_1=\left(2-\frac{1}{\sqrt{5}}\right)+\frac{2}{\sqrt{5}}i$

$|z_1|^2=5-\frac{4\sqrt{5}}{5}$

z_2 will will be either B or C

$PB=\sqrt{17}$ and $PC=\sqrt{13}$

Hence $z_2 = 1$

$5\left(|z_1|^2+|z_2|^2\right)=30-4\sqrt{5}$

Clearly $\alpha = 30$ and $\alpha = -4 \Rightarrow \alpha + \beta = 26$

Question ID : 101782

Differential Equation

22. Let $y=y(x)$ be the solution of the differential equation $\frac{dy}{dx}+\frac{\sqrt{2}y}{2\cos^4 x-\cos 2x}=xe^{\tan^{-1}\left(\sqrt{2}\cot 2x\right)},\ 0<x<\frac{\pi}{2}$ with $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{32}$. If $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{18}e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to _______.

माना अवकल समीकरण $\frac{dy}{dx}+\frac{\sqrt{2}y}{2\cos^4 x-\cos 2x}=xe^{\tan^{-1}\left(\sqrt{2}\cot 2x\right)},\ 0<x<\frac{\pi}{2},\ y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{32}$ का हल $y = y(x)$ है।

यदि $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{18}e^{-\tan^{-1}(\alpha)}$ है, तो $3\alpha^2$ का मान बराबर है __________

Ans. Official Answer NTA (2)

Sol. $\frac{dy}{dx}+\frac{\sqrt{2}y}{2\cos^4 x-\cos 2x}=xe^{\tan^{-1}\left(\sqrt{2}\cot 2x\right)},\ 0<x<\frac{\pi}{2}$

$\frac{dy}{dx}+\frac{2\sqrt{2}y}{1+\cos^2 2x}=xe^{\tan^{-1}\left(\sqrt{2}\cot+2x\right)}$ ....(1)

$I.F.=e^{\int\frac{2\sqrt{2}}{1+\cos^2 2x}dx}=e^{\int\frac{2\sqrt{2}\sec^2 2x}{\tan^2 2x+2}dx}=e^{\tan^{-1}\left(\frac{\tan 2x}{\sqrt{2}}\right)}$

Solution of (1)

$y.e^{\tan^{-1}\left(\frac{\tan^2 x}{\sqrt{2}}\right)}=\int x.e^{\tan^{-1}\left(\sqrt{2}\cot 2x\right)+\tan^{-1}\left(\frac{1}{\sqrt{2}\cot 2x}\right)}dx$

$y.e^{\tan^{-1}\left(\frac{\tan^2 x}{\sqrt{2}}\right)}=e^{\frac{\pi}{2}}\frac{x^2}{2}+c$

$\because\ y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{32}\Rightarrow c=0$

$y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{18}e^{-\tan^{-1}\left(\frac{\tan\frac{2\pi}{3}}{\sqrt{2}}\right)}=\frac{\pi}{18}e^{-\tan^{-1}(\alpha)}$

$$\Rightarrow \frac{\pi}{2} - \tan^{-1}\left(-\frac{\sqrt{3}}{\sqrt{2}}\right) = -\tan^{-1}(\alpha)$$

$$\Rightarrow \alpha = +\sqrt{\frac{2}{3}} \Leftarrow 3\alpha^2 = 2 \text{ Ans.}$$

Question ID : 101783

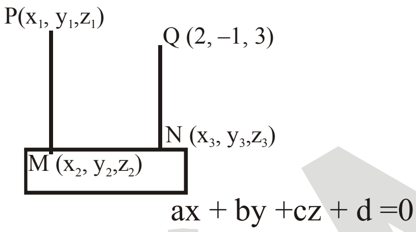
3D Geometry

23. Let d be the distance between the foot of perpendiculars of the points P(1, 2, –1) and Q(2, –1, 3) on the plane $-x + y + z = 1$. Then d^2 is equal to _______.

माना बिन्दुओं P(1, 2, –1) तथा Q(2, –1, 3) से समतल $-x + y + z = 1$ पर डाले गए लंबों के पादों के बीच की दूरी d है, तो d^2 बराबर है ________

Ans. Official Answer NTA (26)

Sol.



for $M \equiv \frac{x_2 - x_1}{a} = \frac{y_2 - y_1}{b} = \frac{z_2 - z_1}{c} = -\frac{ax_1 + by_1 + cz_1 + d}{a^2 + b^2 + c^2}$

P(1, 2, –1)

$M \equiv \frac{x_2 - 1}{1} = \frac{y_2 - 2}{-1} = -\frac{z_2 + 1}{-1} = -\frac{1 - 2 + 1 + 1}{3} = \frac{-1}{3}$

plane $x - y - z + 1 = 0$

$M\ (x_2 y_2 z_2) \equiv \left(\frac{2}{3}, \frac{7}{3}, \frac{-2}{3}\right)$

Q (2, – 1, 3)

$N \rightarrow$ foot of $\perp N(x_3, y_3, z_3)$ when $\frac{x_3 - 2}{1} = \frac{y_3 + 1}{-1} = \frac{z_3 - 3}{-1} = -\frac{1}{3}$

$N\left(\frac{5}{3}, \frac{-2}{3}, \frac{10}{3}\right)$

$MN = d = \sqrt{1^2 + 3^2 + 4^2} = \sqrt{26}$

$d^2 = 26$

Question ID : 101784

Trigonometric Equation

24. The number of elements in the set S = {$\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$} is ______.

समुच्चय S = {$\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$} में अवयवों की संख्या ________

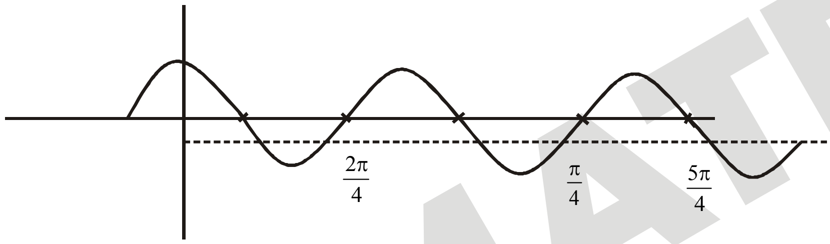
Ans. Official Answer NTA (32)

Sol. $S = \left\{\theta \in [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\right\}$

$3\cos^2 2\theta + 6\cos 2\theta - 5(1+\cos 2\theta) + 5 = 0$

$\cos 2\theta(3\cos 2\theta + 1) = 0$

16 sol. $\cos 2\theta = 0$ $\quad \theta \in [-4\pi, 4\pi]$, $\cos 2\theta = \frac{-1}{3}$ 16 sol.



Total No. of elements in S = 32

Question ID : 101785

Trigonometric Equation

25. The number of solutions of the equation $2\theta - \cos^2\theta + \sqrt{2} = 0$ in R is equal to ________.

R में $2\theta - \cos^2\theta + \sqrt{2} = 0$ के हलों की संख्या है ________

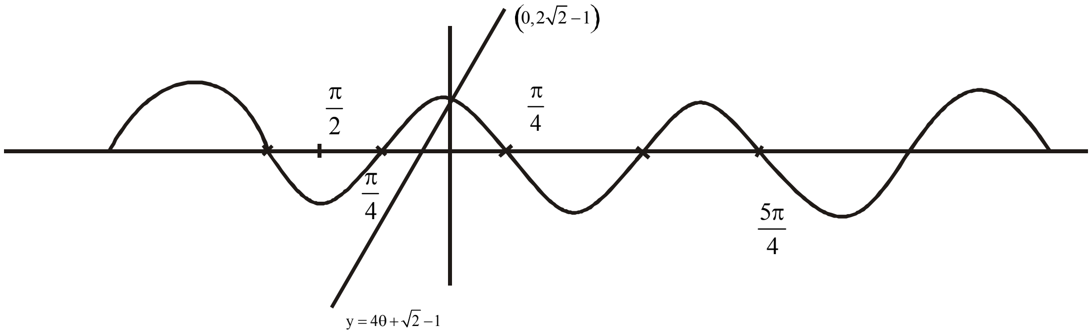
Ans. Official Answer NTA (1)

Sol. $2\theta - \cos^2\theta + \sqrt{2} = 0$

$2\theta + \sqrt{2} = \frac{1+\cos 2\theta}{2}$

$4\theta + 2\sqrt{2} - 1 = \cos 2\theta$

No. of sol. = No. of intersection parts of y = cos 2θ and $y = 4\theta + 2\sqrt{2} - 1$



No. of sol. = 1

Question ID : 101786

ITF

26. $50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right)+2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$ is equal to ________.

$50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right)+2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$ का मान बराबर है ________

Ans. Official Answer NTA (29)

Sol. $50\tan\left(3\tan^{-1}\left(\frac{1}{2}\right)+2\cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right)+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}\left(2\sqrt{2}\right)\right)$

$=50\tan\left(\tan^{-1}\left(\frac{1}{2}\right)+2\tan^{-1}\left(\frac{1}{2}\right)+2\tan^{-1}(2)\right)+4\sqrt{2}\left(\frac{1}{\sqrt{2}}\right)$

$=50\tan\left(\pi+\tan^{-1}\left(\frac{1}{2}\right)\right)+4$

= 25 + 4
= 29

Question ID : 101787

Function

27. Let c, k $\in$ R. If $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ and $f(x + y) = f(x) + f(y) - xy$, for all x, y $\in$ R, then the value of $|2(f(1) + f(2) + f(3) + ... + f(20))|$ is equal to ________.

माना c, k $\in$ R हैं। यदि सभी x, y $\in$ R के लिए $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ तथा $f(x + y) = f(x) + f(y) - xy$, हैं, तो $|2(f(1) + f(2) + f(3) + ... + f(20))|$ का मान बराबर है ________

Ans. Official Answer NTA (3395)

Sol. f(x) is a polynomial

s.t. $f(x)=(c+1)x^2+(1-c^2)x+2k$ ....(i)

and $f(x+y)=f(x)+f(y)-xy \quad x,y\in R$ ....(ii)

put x =y = 0 in (2)

f(0) = 0 ⇒ from (1) k = 0

put x = y

$2f(x) - f(2x) = x^2$

$2(c+1)x^2+2(1+c^2)x-(c+1)4x^2-(1-c^2)2x=x^2$

Compair coefficient of x^2 ⇒ $-2(1+c)=1$

$$C=\frac{-3}{2}$$

from (1) $2f(x)=-x^2-\frac{5}{2}x$

$$\left|2[f(1)+f(2)+....+f(20)]\right|=\left\{\frac{20(21)(41)}{6}+\frac{5}{2}\frac{20(21)}{2}\right\}=3395$$

Question ID : 101788

Hyperbola

28. Let $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$, a > 0, b > 0, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4(2\sqrt{2}+\sqrt{14})$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then the value of $a^2 + b^2$ is equal to ________.

माना अतिपरवलय $H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, a>0, b>0,$ के लिए अनुप्रस्थ तथा संयुग्मी अक्षों की लंबाईयों का योग $4(2\sqrt{2}+\sqrt{14})$ हैं यदि H की उत्केन्द्रता $\frac{\sqrt{11}}{2}$ है, तो $a^2 + b^2$ का मान बराबर है ________

Ans. Official Answer NTA (88)

Sol. $2a+2b=4(2\sqrt{2}+\sqrt{14})$ $\quad H:\frac{x^2}{a^2}-\frac{y^2}{b^2}=1, a>0,\ b>0$

$a+b=4\sqrt{2}+2\sqrt{14}$ ....(1)

$e^2 = 1 + \frac{b^2}{a^2} = \frac{11}{4}$

$\frac{b^2}{a^2} = \frac{7}{4}$

$\frac{b}{a} = \frac{\sqrt{7}}{2}$

$\frac{a}{b} = \frac{2}{\sqrt{7}} \Rightarrow \frac{a+b}{a-b} = \frac{2+\sqrt{7}}{2-\sqrt{7}} \Rightarrow \frac{a+b}{2+\sqrt{7}} = \frac{a-b}{2-\sqrt{7}}$

$a - b = 2\sqrt{2}\left(2-\sqrt{7}\right)$

$b - a = 2\sqrt{14} - 4\sqrt{2}$

$2b = 4\sqrt{14} \Rightarrow b = 2\sqrt{14}$

$\Rightarrow a^2 + b^2 = 32 + 56 = 88$

Question ID : 101789

Vectors

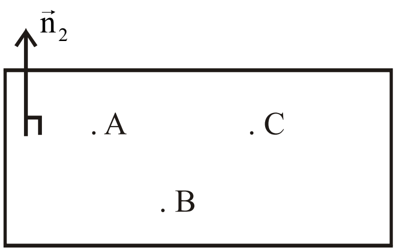
29. Let $P_1 : \vec{r}.\left(2\hat{i} + \hat{j} - 3\hat{k}\right) = 4$ be a plane. Let P_2 be another plane which passes through the points (2, –3, 2), (2, –2, –3) and (1, –4, 2). If the direction ratios of the line of intersection of P_1 and P_2 be 16, α, β, then the value of $\alpha + \beta$ is equal to ________.

माना एक समतल $P_1 : \vec{r}.\left(2\hat{i} + \hat{j} - 3\hat{k}\right) = 4$ है। माना P_2 एक अन्य समतल है जो बिन्दुओं (2, –3, 2), (2, –2, –3) तथा (1, –4, 2) से होकर जाता है। यदि P_1 तथा P_2 की प्रतिच्छेदन रेखा के दिक् अनुपात 16, α, β हैं, तो $\alpha + \beta$ का मान बराबर है ________

Ans. Official Answer NTA (28)

Sol. $P_1 : \vec{r}.\left(2\hat{i} + \hat{j} - 3\hat{k}\right) = 4 \Rightarrow \vec{n}_1 = 2i + j - 3k$

P_2 :



A (2, –3, 2), B (2, –2, –3), C (1, –4, 2)

$$\Rightarrow \vec{n}_2 = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 0 & 1 & -5 \\ -1 & -1 & 0 \end{vmatrix} = -5i + 5j + k(1)$$

direction vector of line of intersection of planes P_1 and P_2

$$\vec{n}_1 \times \vec{n}_2 \begin{vmatrix} i & j & k \\ 2 & 1 & -3 \\ -5 & 5 & 1 \end{vmatrix} = +16i + 13j + 15k$$

$$drs \equiv 16, \alpha, \beta \equiv 16, 13, 15 \;\Rightarrow \alpha + \beta = 28 \text{ Ans.}$$

Question ID : 101790

P & C

30. Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i \in \{1, 2, 3, ..., 100\}$ for $1 \le i \le 4$ and $b_i \ne b_j$ for $i \ne j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers. Then the number of such permutations $b_1b_2b_3b_4$ is equal to ________.

माना $b_i \in \{1, 2, 3, ..., 100\}$, $1 \le i \le 4$ तथा $b_i \ne b_j$, $i \ne j$ के साथ एक 4-अवयव क्रमचय $b_1b_2b_3b_4$ इस प्रकार है किया तो b_1, b_2, b_3 क्रमागत पूर्णांक हैं या b_2, b_3, b_4 क्रमागत पूर्णांक हैं। तो इस प्रकार के क्रमचयों $b_1b_2b_3b_4$ की संख्या है ________

Ans. Official Answer NTA (18915)

Sol. There are 98 sets of three consecutive integer.

i.e. [123] [234] [98.99.100]
(1) (2) (98)

and 97 sets of four consecutive integers.

i.e. [1234] [1235] .... [97.98.99.100]
(1) (2) (97)

Using principle of inclusion and exclution,

Number of permutation of $b_1b_2b_3b_4$ = No. of permutation

$b_1b_2b_3$ are consecutive = 97 × 98

No. of permutation

when $b_2b_3b_4$ are consecutive = 97 × 98

No. of permutation when

$b_1b_2b_3b_4$ are consective

$= 97 \times 98 + 97 \times 98 - 97$

= 97 (195) = 18915 Ans.