JEE Main June 2022

Question Paper With Text Solution

29 June | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JUNE 2022 | 29TH JUNE SHIFT-2

SECTION – A

Question ID : 50111

Complex Number

1. Let α be a root of the equation $1 + x^2 + x^4 = 0$. Then the value of $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ is equal to :

माना समीकरण $1 + x^2 + x^4 = 0$ का एक मूल α है। तो $\alpha^{1011} + \alpha^{2022} - \alpha^{3033}$ का मान बराबर है :

(1) 1 (2) α (3) $1 + \alpha$ (4) $1 + 2\alpha$

Ans. Official Answer NTA (1)

Sol. $1+x^2+x^4 \Rightarrow (1+x+x^2)(1-x+x^2)=0$

one root is ω & other is ω^2

$\therefore \omega^3 = 1 \Rightarrow \alpha^3 = 1$

then $\alpha^{1011} + \alpha^{2022} - \alpha^{3033} = (\alpha^3)^{337} + (\alpha^3)^{674} - (\alpha^3)^{1011}$

$= 1 + 1 - 1 = 1$

Question ID : 50112

Complex Number

2. Let arg(z) represent the principal argument of the complex number z. Then, $|z| = 3$ and $\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$ intersect :

(1) exactly at one point (2) exactly at two points

(3) nowhere (4) at infinitely many points

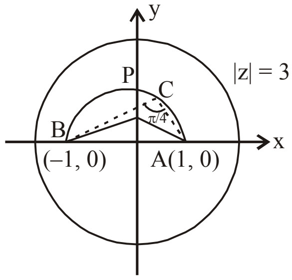
माना arg(z) सम्मिश्र संख्या z के मुख्य आयाम को निरूपित करता है। तो $|z| = 3$ तथा $\arg(z-1) - \arg(z+1) = \frac{\pi}{4}$:

(1) मात्रा एक बिन्दु पर काटते हैं (2) मात्रा दो बिन्दुओं पर काटते हैं

(3) किसी भी बिन्दु पर नहीं काटते हैं (4) असंख्य बिन्दुओं पर काटते है

Ans. Official Answer NTA (3)

Sol. $|z| = 3, \arg(z-1) - \arg(z+1) = \frac{\pi}{4}$



$\angle AKL = \angle ACB = \frac{\pi}{4}$

$\Rightarrow LK = AL = \alpha = 1$

$\Rightarrow K(0, 1)$

Radius = $\sqrt{2}$

Now PL = PK + KL = $\sqrt{2}+1$

So, point P(0, 1 + $\sqrt{2}$)

$\therefore$ number of intersection = 0

Question ID : 50113

Matrices

3. Let $A=\begin{pmatrix}2 & -1\\ 0 & 2\end{pmatrix}$. If $B = I - {}^5C_1\,(\text{adj }A) + {}^5C_2\,(\text{adj }A)^2 - \ldots\ldots\ldots - {}^5C_5\,(\text{adj }A)^5$ then the sum of all elements of the matrix B is :

माना $A=\begin{pmatrix}2 & -1\\ 0 & 2\end{pmatrix}$ है। यदि $B = I - {}^5C_1\,(\text{adj }A) + {}^5C_2\,(\text{adj }A)^2 - \ldots\ldots\ldots - {}^5C_5\,(\text{adj }A)^5$ है, तो आव्यूह B के सभी अवयवों का योगफल है :

(1) –5 (2) –6 (3) –7 (4) –8

Ans. Official Answer NTA (3)

Sol. Given $A=\begin{bmatrix}2 & -1\\ 0 & 2\end{bmatrix}$

$B = I - {}^5C_1\,(\text{adj }A) + {}^5C_2\,(\text{adj }A)^2 - \ldots\ldots\ldots - {}^5C_5\,(\text{adj }A)^5$

Then $B=\left[I-\text{adj}A\right]^5$

$=\left[\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}-\begin{bmatrix}2 & 0\\ 1 & 2\end{bmatrix}\right]^5$

$$B=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix}^5 \equiv$$

$$B=\begin{bmatrix}-1 & 0\\ -5 & -1\end{bmatrix}^5 \Rightarrow$$

Sum if element = – 1 –5 – 1= – 7

Question ID : 50114

Sequence & progression

4. The sum of the infinite series $1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+\frac{35}{6^4}+\frac{51}{6^5}+\frac{70}{6^6}+.....$ is equal to :

अपरिमित श्रेणी $1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+\frac{35}{6^4}+\frac{51}{6^5}+\frac{70}{6^6}+.....$ का योगफल बराबर है :

(1) $\frac{425}{216}$ (2) $\frac{429}{216}$ (3) $\frac{288}{125}$ (4) $\frac{280}{125}$

Ans. Official Answer NTA (3)

Sol. $S=1+\frac{5}{6}+\frac{12}{6^2}+\frac{22}{6^3}+................$

$$\frac{1}{6}S=\quad \frac{1}{6}+\frac{5}{6^2}+\frac{12}{6^3}+\frac{22}{6^4}+\frac{35}{6^5}+\frac{51}{6^6}+......$$

$$\frac{5}{6}S=1+\frac{4}{6}+\frac{7}{6^2}+\frac{10}{6^3}+\frac{13}{6^4}+\frac{16}{6^5}+\frac{19}{6^6}+......$$

$$\frac{5}{36}S=\quad \frac{1}{6}+\frac{4}{6^2}+\frac{7}{6^3}+\frac{10}{6^4}+\frac{13}{6^5}+\frac{16}{6^6}+......$$

$$\frac{25}{36}S=1+\frac{3}{6}+\frac{3}{6^2}+\frac{3}{6^3}+\frac{3}{6^4}+\frac{3}{6^5}+\frac{3}{6^6}+......$$

$$\frac{25}{36}S=1+\frac{\frac{3}{6}}{1-\frac{1}{6}}=1+\frac{3}{5}=\frac{8}{5}$$

$$S=\frac{288}{125}$$

Question ID : 50115

Limit

5. The value of $\lim_{x\to1}\frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ is equal to :

$\lim_{x\to1}\frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$ का मान बराबर है :

(1) $\frac{\pi^2}{6}$ (2) $\frac{\pi^2}{3}$ (3) $\frac{\pi^2}{2}$ (4) π^2

Ans. Official Answer NTA (4)

Sol. $\lim_{x\to1}\frac{(x^2-1)\sin^2(\pi x)}{x^4-2x^3+2x-1}$

$\lim_{x\to1}\frac{(x-1)(x+1)\sin^2(\pi x)}{(x-1)^3(x+1)}$

$\lim_{x\to1}\frac{\sin^2(\pi x)}{(x-1)^2}$

Let $x-1\to t$ $\lim_{x\to0}\frac{\sin^2(\pi(\pi x)}{t^2}$

$x-1\to0$

$\Rightarrow\lim_{x\to0}\frac{\sin^2(t)}{n^2t^2}\pi^2=\pi^2$

Question ID : 50116

Maxima & Minima

6. Let $f:R\to R$ be a function defined by $f(x)=(x-3)^{n_1}(x-5)^{n_2}$, $n_1,n_2\in N$. Then, which of the following is NOT true?

(1) For $n_1=3$, $n_2=4$, there exists $\alpha\in(3, 5)$ where f attains local maxima

(2) For $n_1=4$, $n_2=3$, there exists $\alpha\in(3, 5)$ where f attains local minima

(3) For $n_1=3$, $n_2=5$, there exists $\alpha\in(3, 5)$ where f attains local maxima

(4) For $n_1=4$, $n_2=6$, there exists $\alpha\in(3, 5)$ where f attains local maxima

Ans. Official Answer NTA (3)

माना फलन $f : R \to R, f(x) = (x-3)^{n_1}(x-5)^{n_2}, n_1, n_2 \in N$ द्वारा परिभाषित है। तो निम्न में से कौनसा सत्य नहीं है ?

(1) $n_1 = 3, n_2 = 4$, के लिए ऐसे $\alpha \in (3, 5)$ का अस्तित्व है, जहाँ f स्थानीय उच्चिष्ठ है

(2) $n_1 = 4, n_2 = 3$, के लिए ऐसे $\alpha \in (3, 5)$ का अस्तित्व है, जहाँ f स्थानीय निम्निष्ठ है

(3) $n_1 = 3, n_2 = 5$ के लिए ऐसे $\alpha \in (3, 5)$ का अस्तित्व है, जहाँ f स्थानीय उच्चिष्ठ है

(4) $n_1 = 4, n_2 = 6$ के लिए ऐसे $\alpha \in (3, 5)$ का अस्तित्व है, जहाँ f स्थानीय उच्चिष्ठ है

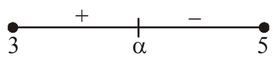
Sol. $f(3) = f(5) = 0$

$\to f'(\alpha) = 0$ $\quad \alpha \in (3, 5)$ (apply Rolles)

$$f'(x) = n_1(x-3)^{n_1-1}(x-5)^{x_2} + n_2(x-3)^{n_1}(x-5)^{x_2-1} = (x-3)^{n_1-1}(x-5)^{n_2-1}\left[n_1(x-5)+n_2(x-3)\right]$$

$n_2 \in$ even

$n_2 - 1 \in$ odd



For $n_2 \in$ even $\quad \alpha \to$ pt of maxima

$n_2 \in$ odd $\quad \alpha \to$ pt of minima

Correct option is (C)

Question ID : 50117

Definite Integration

7. Let f be a real valued continuous function on [0, 1] and $f(x) = x + \int_0^1 (x-t)f(t)\,dt$. Then, which of the following points (x, y) lies on the curve y = f(x)?

माना [0, 1] पर f एक वास्तविक मान संतत फलन है तथा $f(x) = x + \int_0^1 (x-t)f(t)\,dt$ है। तो निम्न में से कौनसा बिन्दु (x, y)वक्र y = f(x) पर स्थित है ?

(1) (2, 4) (2) (1, 2) (3) (4, 17) (4) (6, 8)

Ans. Official Answer NTA (4)

Sol. Given that $f(x) = x\int_0^1 (x-t)f(t)\,dt$

$f(x) = x + \int_0^1 f(t)dt - \int_0^1 tf(t)dt$

$f(x) = x\left[tf\int_0^1 f(t)dt\right] - \int_0^1 tf(t)dt$

Let $1 + \int_0^1 f(t)dt = a$ and $\int_0^1 tf(t)dt = 1$

$\therefore$ f(x) = a × –b

So, $a = 1 + \int_0^1 (at - b)dt = 1 + \frac{a}{2} - b$ $\Rightarrow \frac{a}{2} + b = 1$ ______(1)

$b = \int_0^t t(at - b)dt = \frac{a}{3} - \frac{b}{2}$ $\Rightarrow \frac{3b}{2} = \frac{a}{3}$ ______(2)

Solve (1) and (2)

$a = \frac{18}{13}, \; b = \frac{4}{13}$

$\therefore f(x) = = \frac{18\times(-4)}{13}$

So, (6, 8) lies on f(10).

Question ID : 50118

Definite Integration

8. If $\int_0^2 \left(\sqrt{2x} - \sqrt{2x - x^2}\right)dx = \int_0^1 \left(1 - \sqrt{1-y^2} - \frac{y^2}{2}\right)dy + \int_1^2 \left(2 - \frac{y^2}{2}\right)dy + I$ then I equals :

यदि $\int_0^2 \left(\sqrt{2x} - \sqrt{2x - x^2}\right)dx = \int_0^1 \left(1 - \sqrt{1-y^2} - \frac{y^2}{2}\right)dy + \int_1^2 \left(2 - \frac{y^2}{2}\right)dy + I$ है, तो I बराबर है :

(1) $\int_0^1 \left(1 + \sqrt{1-y^2}\right)dy$

(2) $\int_0^1 \left(\frac{y^2}{2} - \sqrt{1-y^2} + 1\right)dy$

(3) $\int_0^1 \left(1 - \sqrt{1-y^2}\right)dy$

(4) $\int_0^1 \left(\frac{y^2}{2} + \sqrt{1-y^2} + 1\right)dy$

Ans. Official Answer NTA (3)

Sol. Given $\int_0^2 \sqrt{2x}\,dx - \int_0^2 \sqrt{1-(x-1)^2}\,dx$

$= \int_0^2 \left(1-\frac{y^2}{2}\right)dy - \int_0^1 \sqrt{1-y^2}\,dy + 1 + I$

$\Rightarrow \frac{8}{3} - 2\int_0^1 \sqrt{1-y^2}\,dy = \frac{2}{3} + 1 - \int_0^1 \sqrt{1-y^2}\,dy + I$

$\Rightarrow I = 1 - \int_0^1 \sqrt{1-y^2}\,dy$

$\Rightarrow I = \int_0^1 \left(1-\sqrt{1-y^2}\,dy\right)$

Question ID : 50119

Differential Equation

9. If y = y(x) is the solution of the differential equation $\left(1+e^{2x}\right)\frac{dy}{dx} + 2\left(1+y^2\right)e^x = 0$ and y(0) = 0, then $6\left(y'(0)+\left(y\left(\log_e \sqrt{3}\right)\right)^2\right)$ is equal to :

यदि अवकल समीकरण $\left(1+e^{2x}\right)\frac{dy}{dx} + 2\left(1+y^2\right)e^x = 0$, y(0) = 0 का हल y = y(x) है, तो $6\left(y'(0)+\left(y\left(\log_e \sqrt{3}\right)\right)^2\right)$ बराबर है :

(1) 2 (2) –2 (3) –4 (4) –1

Ans. Official Answer NTA (3)

Sol. Given $\left(1+e^{2x}\right)\frac{dy}{dx} + 2\left(1+y^2\right)e^x = 0$

$\int \frac{dy}{1+y^2} = -\int \frac{2e^x}{1+e^{2x}}dx = -\int \frac{2e^x}{1+\left(e^x\right)^2}dx$

let $e^x = t$ $\Rightarrow e^x\,dx = dt$

$\Rightarrow \int \frac{dy}{1+y^2} = -2\int \frac{1}{1+t^2}dt$

$\Rightarrow \tan^{-1} y = -2\tan^{-1} t + c$

$\Rightarrow \tan^{-1} y = -2\tan^{-1} e^x + c$

Given at x = 0, y = 0

$\Rightarrow c = \frac{\pi}{2}$

So, $\tan^{-1} y = -2\tan^{-1} e^x + \frac{\pi}{2}$

$\tan(\tan^{-1} y) = \tan\left(\frac{\pi}{2} - 2\tan^{-1} e^x\right)$

$$\boxed{y = \cot\left(2\tan^{-1} e^x\right)}$$

$\frac{dy}{dx} = -\operatorname{cosec}^2\left(2\tan^{-1} e^x\right)\left(\frac{2e^x}{1+e^{2n}}\right)$

$y'(0) = \left.\frac{dy}{dx}\right|_{x=0} = \frac{-2}{2} = -1$

$y\left(\ell n\sqrt{3}\right) = \cot\left(2\tan^{-1} e^{\log_e \sqrt{3}}\right)$

$y\left(\ell n\sqrt{3}\right) = \cot\left(2\tan^{-1}\sqrt{3}\right)$

$y\left(\ell n\sqrt{3}\right) = \cot 2\times\frac{\pi}{3} = \cot\frac{2\pi}{3} = \frac{-1}{\sqrt{3}}$

$\therefore 6\left(y'(0) + \left(y\left(\ell n\sqrt{3}\right)\right)^2\right) = 6\left[-1 + \left(\frac{-1}{\sqrt{3}}\right)^2\right]$

$= 6\left(-1 + \frac{1}{3}\right) = -4$

Question ID : 501110

Parabola

10. Let P : $y^2 = 4ax$, $a > 0$ be a parabola with focus S. Let the tangents to the parabola P make an angle of $\frac{\pi}{4}$ with the line y = 3x + 5 touch the parabola P at A and B. Then the value of a for which A, B and S are collinear is :

(1) 8 only (2) 2 only (3) $\frac{1}{4}$ only (4) any $a > 0$

माना P : $y^2 = 4ax$, $a > 0$ एक परवलय है, जिसकी नाभि S है। माना परवलय P की स्पर्श रेखाऐं, रेखा $y = 3x + 5$ से $\frac{\pi}{4}$ का कोण बनाती हैं तथा परवलय P को बिन्दुओं A तथा B पर स्पर्श करती हैं। तो बिन्दुओं A, B तथा a का मान है :

(1) केवल 8 (2) केवल 2 (3) केवल $\frac{1}{4}$ (4) कोई भी $a > 0$

Ans. Official Answer NTA (4)

Sol. Given P : $y^2 = 4ax$, $(a > 0)$ and focus $S \equiv (a, 0)$

equation of tangent on parabola $y = mn + \frac{a}{m}$

Given line, $y = 3x + 5$

Angle between given line and tangent is $\frac{\pi}{4}$

$\tan\frac{\pi}{4} = \left|\frac{m-3}{1+3m}\right| \Rightarrow m - 3 = \pm(1+3m)$

$\Rightarrow m - 3 = 1 + 3m$

$\boxed{m = -2}$

$m - 3 = -1 - 3m$

$\boxed{m = \frac{1}{2}}$

So, equation of one tangent : $y = -2 \times \frac{a}{2}$

equation of three tangent : $y = \frac{x}{z} + 29$

Point of contant are $\left(\frac{a}{(-2)^2}, \frac{-2a}{(-2)}\right)$ and $\left(\frac{a}{\left(\frac{1}{2}\right)^2}, \frac{-29}{\frac{1}{2}}\right)$

$\therefore A\left(\frac{a}{4}, a\right)$ and B(4a, –4a)

So, A, B and S are collineal

then $\boxed{\Delta ABS = 0}$

$$\frac{1}{2}\begin{vmatrix} \frac{a}{4} & a & 1 \\ 4a & -4a & 1 \\ a & 0 & 1 \end{vmatrix} = 0$$

$$\Rightarrow \frac{a}{4}(-4a-0)-a(4a-a)+\left(0-\left(-4a^2\right)\right)=0$$

$\Rightarrow -a^2-3a^2+4a^2=0$

$\Rightarrow 0=0$

It is not depend on a.

So, any $a > 0$

Question ID :501111

Circle

11. Let a triangle ABC be inscribed in the circle $x^2-\sqrt{2}(x+y)+y^2=0$ such that $\angle BAC=\frac{\pi}{2}$. If the length of side AB is $\sqrt{2}$, then the area of the ΔABC is equal to :

माना वृत $x^2-\sqrt{2}(x+y)+y^2=0$ के अंतर्गत एक त्रिभुज ABC इस प्रकार है कि $\angle BAC=\frac{\pi}{2}$ है। यदि भुजा AB की लंबाई $\sqrt{2}$ है, तो ΔABC का क्षेत्रफल बराबर है :

(1) $\frac{\left(\sqrt{2}+\sqrt{6}\right)}{3}$ (2) $\frac{\left(\sqrt{6}+\sqrt{3}\right)}{2}$ (3) $\frac{\left(3+\sqrt{3}\right)}{4}$ (4) $\frac{\left(\sqrt{6}+2\sqrt{3}\right)}{4}$

Ans. Official Answer NTA (Dropped)

Sol. Given equation $x^2-\sqrt{2}x-\sqrt{2}y+y^2=0$

Centre $\equiv\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)\equiv(-9,-f)$

Radius $=\sqrt{9^2+f^2-c} = \sqrt{\frac{1}{2}+\frac{1}{2}-0}=1$

BC is a diameter

$a_1(\Delta ABC) = \frac{1}{2}\times\sqrt{2}\times\sqrt{2} = 1$

Question ID : 501112

3D Geometry

12. Let $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}$ lie on the plane $px - qy + z = 5$, for some $p, q \in R$. The shortest distance of the plane from the origin is :

माना $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}$ किसी $p, q \in R$ के लिए समतल $px - qy + z = 5$ पर स्थित है। मूल बिन्दु से इस समतल की न्यूनतम दूरी है :

(1) $\sqrt{\frac{3}{109}}$ (2) $\sqrt{\frac{5}{142}}$ (3) $\frac{5}{\sqrt{71}}$ (4) $\frac{1}{\sqrt{142}}$

Ans. Official Answer NTA (2)

Sol. Given equation of line : $\frac{x-2}{3}=\frac{y+1}{-2}=\frac{z+3}{-1}=\lambda$

$(3\lambda + 2, -2\lambda - 1, -\lambda - 3)$ lies on plane $px - qy + z = 5$

$p(3\lambda + 2) - q(-2\lambda - 1) + (-\lambda - 3) = 5$

$\lambda(3p + 2q - 1) + (2p + q - 8) = 0$

$\therefore 3p + 2q - 1 = 0$ _______(1)

$2p + q - 8 = 0$ _______(2)

Solve (1) and (2) we get

$p = 15,$ $q = -22$

equation of plane : $15x + 22y + z - 5 = 0$

$\therefore$ Shortest distance from origin $= \left|\frac{0+0+0-5}{\sqrt{15^2+22^2+1}}\right|$

$=\frac{5}{\sqrt{710}}=\sqrt{\frac{5}{142}}$

Question ID : 501113

3D Geometry

13. The distance of the origin from the centroid of the triangle whose two sides have the equations $x - 2y + 1 = 0$ and $2x - y - 1 = 0$ and whose orthocenter is $\left(\frac{7}{3},\frac{7}{3}\right)$ is :

एक त्रिभुज की दो भुजाओं के समीकरण $x - 2y + 1 = 0$ तथा $2x - y - 1 = 0$ इसका लंबकेन्द्र $\left(\frac{7}{3}, \frac{7}{3}\right)$ है। इस त्रिभुज के केन्द्र से मूल बिन्दु की दूरी है :

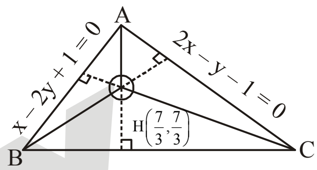
(1) $\sqrt{2}$ (2) 2 (3) $2\sqrt{2}$ (4) 4

Ans. Official Answer NTA (3)

Sol. Given equation of sides of ΔABC

AB : $x - 2y + 1 = 0$

AC : $2x - y - 1 = 0 \Rightarrow A(1,1)$



Altitude from B is BH = $x + 2y - 7 = 0 \Rightarrow B(3, 2)$

Altitude from C is CH = $2x + y - 7 = 0 \Rightarrow C(2, 3)$

Centroid of $\Delta ABC = C(2, 2)$

then, distance between origin and centroid of triangle = $\sqrt{(2-0)^2 + (2-0)^2} = 2\sqrt{2}$

Question ID : 501114

3D Geometry

14. Let Q be the mirror image of the point P(1, 2, 1) with respect to the plane $x + 2y + 2z = 16$. Let T be a plane passing through the point Q and contains the line $\vec{r} = -\hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k})$, $\lambda \in R$. Then, which of the following points lies on T?

माना समतल $x + 2y + 2z = 16$. के सापेक्ष बिन्दु P(1, 2, 1) का दर्पण प्रतिबिंब Q है। माना T एक समतल है जो बिन्दु Q से होकर जाता है तथा रेखा $\vec{r} = -\hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k})$, $\lambda \in R$ समतल T में स्थित है। तो निम्न में से कौनसा बिन्दु T पर स्थित है?

(1) (2, 1, 0) (2) (1, 2, 1) (3) (1, 2, 2) (4) (1, 3, 2)

Ans. Official Answer NTA (2)

Sol. Image of P(1, 2, 1) in plane $x + 2y + 2z = 16$

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-1}{2} = \frac{-2[1 + 2\times 2 + 2\times 1 - 16]}{1^2 + 2^2 + 2^2}$$

$$\frac{x-1}{1} = \frac{y-2}{2} = \frac{z-1}{2} = 2$$

Q(3, 6, 5) is image of point P(1, 2, 1) in the plane $x + 2y + 2z = 16$.

Given $\vec{r} = -\hat{k} + \lambda(\hat{i} + \hat{j} + 2\hat{k})$ plane passing through Q.

$\dot{a}(3,8,5)$

$\dot{A}(0,0,-1)$

$\overrightarrow{AQ} = 3\hat{i} + 6\hat{j} + 6\hat{k} = 3(\hat{i} + 2\hat{j} + 2\hat{k})$

$\vec{n} = (\hat{i} \times 2\hat{j} \times 2\hat{k}) \times (\hat{i} + \hat{j} + 2\hat{k})$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 2 \\ 1 & 1 & 2 \end{vmatrix} = 2\hat{i} - o\hat{j} - \hat{k}$$

Equation of plane = 2(x – 0)+ O(y – 0) – 1(z + 1) = 0
2x – 2 = 0
So, point lying on the plane is (1, 2, 1)

Question ID : 501115

Vectors

15. Let A, B, C be three points whose position vectors respectively are

$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in R$

$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$

If α is the smallest positive integer for which $\vec{a}, \vec{b}, \vec{c}$ are noncollinear, then the length of the median, in ΔABC, through A is :

माना तीन बिन्दुओं A, B, C के स्थिति सदिश क्रमशः

$\vec{a} = \hat{i} + 4\hat{j} + 3\hat{k}$

$\vec{b} = 2\hat{i} + \alpha\hat{j} + 4\hat{k}, \alpha \in R$

$\vec{c} = 3\hat{i} - 2\hat{j} + 5\hat{k}$

है। यदि α न्यूनतम धन पूर्णांक है जिसके लिए $\vec{a}, \vec{b}, \vec{c}$ असंरेख है, तो ΔABC की A से माध्यिका की लंबाई है :

(1) $\frac{\sqrt{82}}{2}$ (2) $\frac{\sqrt{62}}{2}$ (3) $\frac{\sqrt{69}}{2}$ (4) $\frac{\sqrt{66}}{2}$

Ans. Official Answer NTA (1)

Sol. Given $\overrightarrow{AB} \| \overrightarrow{AC}$ if $\frac{1}{2} = \frac{\alpha - 4}{-6} = \frac{1}{2}$

$\Rightarrow \alpha = 1$

$\therefore \vec{a}, \vec{b}, \vec{c}$ are non-collineal for $\alpha = 2$ (smallest positive integel)

Mid point of $BC = M\left(\frac{5}{2}, 0, \frac{9}{2}\right)$

$AM = \sqrt{\frac{9}{4} + 16 + \frac{9}{4}} = \frac{\sqrt{82}}{2}$

Question ID : 501116

Probability

16. The probability that a relation R from {x, y} to {x, y} is both symmetric and transitive, is equal to :

{x, y} से {x, y} में एक संबंध सममित तथा संक्रामक दोनों के होने की प्रायिकता है :

(1) $\frac{5}{16}$ (2) $\frac{9}{16}$ (3) $\frac{11}{16}$ (4) $\frac{13}{16}$

Ans. Official Answer NTA (1)

Sol. Total number of relation from {x, y} to {x,y}

$= 2^{2^2} = 2^4 = 16$

Relation which are symmetric as will as transitive are

$\phi, \{(x,x)\}, \{(y,y)\}, \{(x,x),(x,y),(y,y),(y,x)\} \{(x,x),(y,y)\}$

favourable cases = 5

$\text{Probability} = \frac{\text{favourable cases}}{\text{Total cases}} = \frac{5}{16}$

Question ID : 501117

Statistics

17. The number of values of $a \in N$ such that the variance of 3, 7, 12, a, 43 – a is a natural number is :

$a \in N$ के मानों की संख्या 3, 7, 12, a, 43 – a का प्रसरण एक धन पूर्णांक है :

(1) 0 (2) 2 (3) 5 (4) infinite (अनन्त)

Ans. Official Answer NTA (1)

Sol. Given numbers : 3, 7, 12, a, 43 – a

Mean $= \overline{x} = \dfrac{3+7+12+a+43-9}{5} = 13$

Variance $= \dfrac{9+49+144+a^2+(43-a)^2}{5} - 13^2$

So, variance $\in$ natural number

$\dfrac{2a^2-a+1}{5} \in N$

$2a^2 - a + 1 = 5N$

$2a^2 - a + 1 - 5N = 0$

It is quadratic in a

$a = \dfrac{1 \pm \sqrt{1-4(2)(1-5N)}}{4} = \dfrac{1 \pm \sqrt{40N-7}}{4}$

So, Δ cannot be perfect square & in the form of 4λ or 4λ +.

So, a cannot be perfect square

$\therefore$ Number of values = 0

Question ID : 501118

Heights & Distances

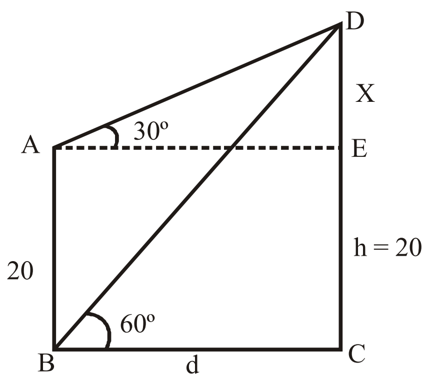
18. From the base of a pole of height 20 meter, the angle of elevation of the top of a tower is 60°. The pole subtends an angle 30° at the top of the tower. Then the height of the tower is :

20 मीटर ऊँचे एक पोल के आधार से एक टॉवर के शिखर का उन्नयन कोण 60° है। टॉवर के शिखर पर पोल 30° का कोण बनाता है। तो टॉवर की ऊँचाई है :

(1) $15\sqrt{3}$ (2) $20\sqrt{3}$ (3) $20+10\sqrt{3}$ (4) 30

Ans. Official Answer NTA (4)

Sol. Given



In ΔBDC

$\tan 60 = \frac{20+x}{d}$

$d = \frac{20+x}{\sqrt{3}}$

In ΔADE

$\tan 30 = x\sqrt{3}$

$\therefore x\sqrt{3} = \frac{20+x}{\sqrt{3}}$

$x = 10$

Height = 20 + 10

= 30

Question ID : 501119

Mathematical Reasoning

19. Negation of the boolean statement $(p \vee q) \Rightarrow ((\sim r) \vee p)$ is equivalent to :

बूलीय कथन $(p \vee q) \Rightarrow ((\sim r) \vee p)$ का निषेधन निम्न में से किसके तुल्य है :

(1) $p \wedge (\sim q) \wedge r$ (2) $(\sim p) \wedge (\sim q) \wedge r$ (3) $(\sim p) \wedge q \wedge r$ (4) $p \wedge q \wedge (\sim r)$

Ans. Official Answer NTA (3)

Sol. $p \vee q \Rightarrow ((\sim r) \vee p)$

$\equiv \sim(p \vee q) \vee (-r \vee p)$

$\equiv (\sim p \vee q) \vee (p \vee -r)$

$\equiv [(\sim p \vee p) \wedge (\sim q \vee p)] \vee \sim r$

$\equiv (-q \vee p) \vee \sim r$

then negative is $\sim((\sim q \vee p) \vee \sim r)$

Question ID : 501120

Binomial Theorem

20. Let $n \geq 5$ be an integer. If $9^n - 8n - 1 = 64\alpha$ and $6^n - 5n - 1 = 25\beta$, then $\alpha - \beta$ is equal to :

माना $n \geq 5$ एक पूर्णांक है। यदि $9^n - 8n - 1 = 64\alpha$ तथा $6^n - 5n - 1 = 25\beta$ हैं, तो $\alpha - \beta$ बराबर है :

(1) $1 + {}^nC_2 (8 - 5) + {}^nC_3 (8^2 - 5^2) + \dots\dots\dots + {}^nC_n (8^{n-1} - 5^{n-1})$

(2) $1 + {}^nC_3 (8 - 5) + {}^nC_4 (8^2 - 5^2) + \dots\dots\dots + {}^nC_n (8^{n-2} - 5^{n-2})$

(3) ${}^nC_3 (8 - 5) + {}^nC_4 (8^2 - 5^2) + \dots\dots\dots + {}^nC_n (8^{n-2} - 5^{n-2})$

(4) ${}^nC_4 (8 - 5) + {}^nC_5 (8^2 - 5^2) + \dots\dots\dots + {}^nC_n (8^{n-3} - 5^{n-3})$

Ans. Official Answer NTA (3)

Sol. Given $9^n - 8n - 1 = 64\alpha$

$\Rightarrow (1+8)^n - 8n - 1 = 64\alpha$

$= {}^nC_0 (1) + {}^nC_1 8 + {}^nC_2 8^2 + \dots\dots\dots + {}^nC_n 8^n - 8n - 1 = 8^2\alpha$

$= 1 + 8n + {}^nC_2 8^2 + \dots\dots\dots\dots + {}^nC_n 8^n - 8n - 1 = 8^2\alpha$

$= {}^nC_2 + {}^nC_2 8 + \dots\dots\dots + {}^nC_n 8^{n-2} = \alpha$ ....(1)

Similarly $6^n - 5n - 1 = 5^2\beta$

$(1 + 5)^n - 5n - 1 = 5^2 \beta$

$\Rightarrow 1 + 5n + {}^nC_1 5^2 + \dots\dots\ {}^nC_n 5^n - 5n - 1 = 5^2\beta$

$= {}^nC_2 + {}^nC_3 5 + \dots\dots + {}^nC_n 5^{n-2} = \beta$ ....(2)

$\therefore (\alpha - \beta) = {}^nC_6 (8-5) + {}^nC_4 (8^2 - 5^2) + \dots\dots + {}^nC_n (8^{n-2} - 5^{n-2})$

SECTION – B

Question ID : 501121

Vectors

21. Let $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{c}$ be a vector such that $\vec{a} + (\vec{b} \times \vec{c}) = \vec{0}$ and $\vec{b}.\vec{c} = 5$. Then the value of $3(\vec{c}.\vec{a})$ is equal to _________.

माना $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$, $\vec{b} = \hat{i} + \hat{j} + \hat{k}$ है तथा एक सदिश $\vec{c}$ इस प्रकार है कि $\vec{a} + (\vec{b} \times \vec{c}) = \vec{0}$ तथा $\vec{b}.\vec{c} = 5$ हैं। तो $3(\vec{c}.\vec{a})$ का मान बराबर है _________. (*Date error in question)

Ans. Official Answer NTA (10)

Sol. $\vec{a} \times (\vec{b} \times \vec{c}) = 0$

$\vec{c}.\vec{a} + \vec{c}.(\vec{a} \times \vec{c}) = 0$

$\vec{c}.\vec{a} + 0 = 0$

$\therefore 3(\vec{c}.\vec{a}) = 0$

Question ID : 501122

Differential Equation

22. Let y = y(x), x > 1, be the solution of the differential equation $(x-1)\frac{dy}{dx}+2xy=\frac{1}{x-1}$, with $y(2)=\frac{1+e^4}{2e^4}$. If $y(3)=\frac{e^\alpha+1}{\beta e^\alpha}$, then the value of $\alpha+\beta$ is equal to _______.

माना अवकल समीकरण $(x-1)\frac{dy}{dx}+2xy=\frac{1}{x-1}$, $y(2)=\frac{1+e^4}{2e^4}$ का हल y = y(x), x > 1 है। यदि $y(3)=\frac{e^\alpha+1}{\beta e^\alpha}$ है, तो $\alpha+\beta$ का मान बराबर है _______.

Ans. Official Answer NTA (14)

Sol. $\frac{dy}{dx}+\frac{2x}{(x-1)}.y=\frac{1}{(x-1)^2}$

I.F. = $e^{\int pdx}=e^{\int\frac{2x}{(x-1)}dx}=e^{\int\left(2+\frac{2}{x-1}\right)dx}$

$=e^{2x+2\log(x-1)}$

$=e^{2x}.e^{\log(x-1)^2}$

$=(x-1)^2.e^{2x}$

$\therefore y.(x-1)^2e^{2x}=\int\frac{1}{(x-1)^2}.(x-1)^2e^{2x}.dx+c$

$\therefore y.(x-1)^2e^{2x}=\frac{e^{2x}}{2}+c$

$\because y(2)=\frac{1+e^4}{2e^4}$

$\frac{(1+e^4)}{2e^4}.1.e^4=\frac{e^4}{2}+c$

$c=\frac{1}{2}$

$y(x-1)^2e^{2x}=\frac{e^{2x}}{2}+\frac{1}{2}$

If $y(3)=\frac{e^{\alpha}+1}{\beta.e^{\alpha}}$

$\frac{e^{\alpha}+1}{\beta.e^{\alpha}}.4.e^{6}=\frac{e^{6}}{2}+\frac{1}{2}$

$\frac{e^{\alpha}+1}{\beta.e^{\alpha}}=\frac{e^{6}+1}{8.e^{6}}$

$\alpha=6,\beta=8$

$\alpha+\beta=14$

Question ID : 501123

Sequence & progression

23. Let 3, 6, 9, 12, ... upto 78 terms and 5, 9, 13, 17, ... upto 59 terms be two series. Then, the sum of the terms common to both the series is equal to _______.

माना दो श्रेणियाँ 3, 6, 9, 12, ... (78 पदों तक) तथा 5, 9, 13, 17, ... (59 पदों तक) हैं। तो दोनों श्रेणियों के उभयनिष्ठ पदों का योगफल है____.

Ans. Official Answer NTA (2223)

Sol. S_1 = 3, 6, 9, 12 ....78 - terms

S_2 = 5, 9, 13, 17 ....59 - terms

Common terms are 9, 21, ...

Now T_{78} of S_1 =3 + (78 – 1) 3 = 234

T_{59} of S_1 =5 + (59 – 1) 4 = 237

So, common terms ≤ 234

9 + (n –1) 21 ≤ 234

$n<\frac{225}{12}+1$

n = 19

So, S_{19} of common terms

$5_{19}=\frac{19}{2}\left(2\times9+\left(19-1\right)\times12\right)=19\left(9+108\right)$

= 2223

Question ID : 501124

Trigonometric Equation

24. The number of solutions of the equation $\sin x = \cos^2 x$ in the interval (0, 10) is ________.

अंतराल (0, 10) में समीकरण $\sin x = \cos^2 x$ के हलों की संख्या है ________.

Ans. Official Answer NTA (4)

Sol. $\Rightarrow \sin x = \cos^2 x$

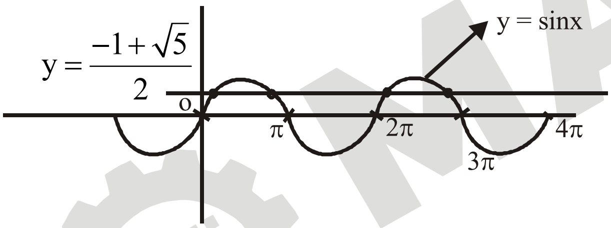
$\Rightarrow \sin x = 1 - \sin^2 x$

$\Rightarrow \sin^2 x + \sin x - 1 = 0$

$\Rightarrow \sin x = \frac{-1 \pm \sqrt{1-4(-1)(1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

$\sin x \neq \frac{-1-\sqrt{5}}{2}$ $\quad\quad \therefore \sin x \in [-1, 1]$

So, $\sin x = \frac{\sqrt{5}-1}{2} > 0$



So total number of solution =4

Question ID : 501125

Area Under Curve

25. For real numbers a, b (a > b > 0), let Area $\left\{(x, y) : x^2 + y^2 \le a^2 \ \& \ \frac{x^2}{a^2} + \frac{y^2}{b^2} \ge 1\right\} = 30\pi$ and

Area $\left\{(x, y) : x^2 + y^2 \ge b^2 \ \& \ \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\} = 18\pi$. Then the value of $(a - b)^2$ is equal to _______.

वास्तविक संख्याओं a, b (a > b > 0) के लिए यदि क्षेत्रफल $\left\{(x, y) : x^2 + y^2 \le a^2 \ \& \ \frac{x^2}{a^2} + \frac{y^2}{b^2} \ge 1\right\} = 30\pi$ तथा

क्षेत्रफल $\left\{(x, y) : x^2 + y^2 \ge b^2 \ \& \ \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\} = 18\pi$ हैं, तो $(a - b)^2$ का मान बराबर है _______.

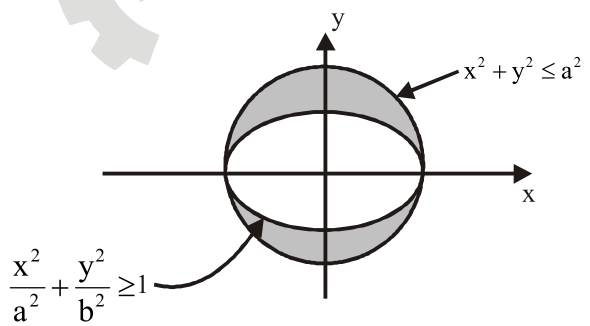
Ans. Official Answer NTA (12)

Sol. Given a,b $\in$ R or (a > b > 0)

$$\text{Area}\left\{(x, y) : x^2 + y^2 \le a^2 \text{and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\} = 30\pi$$

$$\text{Area}\left\{(x, y) : x^2 + y^2 \le b^2 \text{and } \frac{x^2}{a^2} + \frac{y^2}{b^2} \le 1\right\} = 18\pi$$

Now $x^2 + y^2 \le a^2 \ \& \ \frac{x^2}{a^2} + \frac{y^2}{b^2} \ge 1$



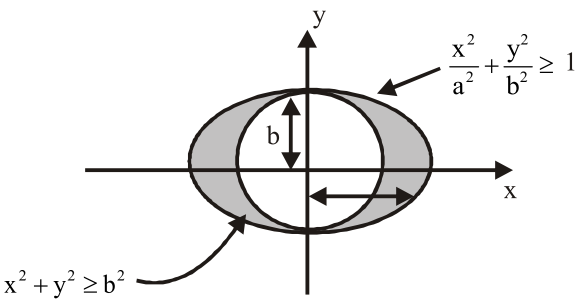
So, shaded region = area of circle - area of ellipse

$\Rightarrow \pi a^2 - \pi ab = 30\pi$

$\Rightarrow \pi a(a - b) = 30\pi$

$\Rightarrow a(a - b) = 30\pi$ ....(1)

Similarly $x^2 + y^2 \geq b^2$ & $\frac{x^2}{a^2} + \frac{y^2}{b^2} \leq 1$



So, shaded region = area of ellipse - area of circle

$\Rightarrow \pi ab - \pi b^2 = 18\pi$

$\Rightarrow \pi b(a - b) = 18\pi$

$\Rightarrow b(a - b) = 18\pi$ ....(2)

From (1) – (2)

(a – b) (a – b) = 30 – 18

$(a - b)^2 = 12$

Question ID : 501126

Monotonocity

26. Let f and g be twice differentiable even functions on (–2, 2) such that

$f\left(\frac{1}{4}\right) = 0, f\left(\frac{1}{2}\right) = 0, f(1) = 1$ and $g\left(\frac{3}{4}\right) = 0, g(1) = 2$

Then, the minimum number of solutions of $f(x)g''(x) + f'(x)g'(x) = 0$ in (–2, 2) is equal to ________.

माना (–2, 2) पर दो बार अवकलनीय सम फलनों f तथा g के लिए

$f\left(\frac{1}{4}\right) = 0, f\left(\frac{1}{2}\right) = 0, f(1) = 1$ तथा $g\left(\frac{3}{4}\right) = 0, g(1) = 2$

तो (–2,2) में $f(x)g''(x) + f'(x)g'(x) = 0$ के हलों की न्यूनतम संख्या है _______.

Ans. Official Answer NTA (4)

Sol.

Question ID : 501127

Binomial Theorem

27. Let the coefficients of x^{-1} and x^{-3} in the expansion of $\left(2x^{\frac{1}{5}}-\frac{1}{x^{\frac{1}{5}}}\right)^{15}$, $x > 0$, be m and n respectively. If r is a positive integer such that $mn^2 = {}^{15}C_r \cdot 2^r$, then the value of r is equal to _______.

माना $\left(2x^{\frac{1}{5}}-\frac{1}{x^{\frac{1}{5}}}\right)^{15}$, $x > 0$ के प्रसार में x^{-1} तथा x^{-3} के गुणांक क्रमशः m तथा n है। यदि $\left(2x^{\frac{1}{5}}-\frac{1}{x^{\frac{1}{5}}}\right)^{15}$, $x > 0$, धनपूर्णांक r के लिए $mn^2 = {}^{15}C_r \cdot 2^r$ है, तो r का मान बराबर है _______.

Ans. Official Answer NTA (5)

Sol. $\left(2x^{\frac{1}{5}}-\frac{1}{x^{\frac{1}{5}}}\right)^{15}$, $x > 0$

$$T_{r+1} = {}^{15}C_r\left(2x^{\frac{1}{5}}\right)^{15-r}\left(\frac{-1}{x^{\frac{1}{5}}}\right)^r$$

for coefficient of x^{-1}

$\frac{15-r}{5}-\frac{r}{5}=-1 \Rightarrow 15-2r=-5 \Rightarrow r=10$

$\therefore m = {}^{15}C_{10}.2^5$

& coefficient of x^{-3}, $\frac{15-2r}{5}=-3$

$r = 15$

$n = -{}^{15}C_{15}$

Given, $mn^2 = {}^{15}C_r.2^r$

${}^{15}C_{10}.2^5 = {}^{15}C_r.2^r$

$\Rightarrow r = 5$

So, $(a-b)^2 = \left(5\sqrt{5}-3\sqrt{5}\right)^2 = 3.4 = 12$

Question ID : 501128

P & C

28. The total number of four digit numbers such that of first three digits is divisible by the last digit, is equal to ________.

चार अंकों की संख्याओं, जिनके लिए प्रथम तीन अंकों में से प्रत्येक, अंतिम अंक से विभाज्य है, कि कुल संख्या है ________.

Ans. Official Answer NTA (1086)

Sol. Let the number is abcd, where a, b, c are divisible by

When (d)	No. of such	numbers
d = 1	$9 \times 10 \times 10$	= 900
d = 2	$4 \times 5 \times 5$	= 100
d = 3	$3 \times 4 \times 4$	= 48
d = 4	$2 \times 3 \times 3$	= 18
d = 5	$1 \times 2 \times 2$	= 4
d = 6, 7, 8, 9	4×4	= 16
	sum	1086

Question ID : 501129

Matrices

29. Let $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$, where α is a non-zero real number an $N = \sum_{k=1}^{49} M^{2k}$. If $(I - M^2)N = -2I$, then the positive integral value of α is ________.

माना $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}$ है, जहाँ α एक शून्येतर वास्तविक संख्या है तथा $N = \sum_{k=1}^{49} M^{2k}$ यदि $(I - M^2)N = -2I$ है, तो α का धन पूर्णांक मान है ________.

Ans. Official Answer NTA (1)

Sol. Given $M = \begin{bmatrix} 0 & -\alpha \\ \alpha & 0 \end{bmatrix}, M^2 = \begin{bmatrix} -\alpha^2 & 0 \\ 0 & -\alpha^2 \end{bmatrix} = -\alpha^2 I$

$N = M^2 + M^4 + \dots\dots\dots + M^{98}$

$N = [-\alpha^2 + \alpha^4 - \alpha^6 + \dots\dots\dots]I$

$I - M^2 = (1 + \alpha^2)I$

$(I - M^2)N = 1 + -\alpha^2(\alpha^{98} + 1) = -2$

$\alpha = 1$

Question ID : 501130

Function

30. Let f(x) and g(x) be two polynomials of degree 2 and 1 respectively. If $f(g(x)) = 8x^2 - 2x$, and $g(f(x)) = 4x^2 + 6x + 1$, then the value of $f(2) + g(2)$ is ________.

माना दो वास्तविक बहुपद f(x) तथा g(x) क्रमशः घात 2 तथा 1 के हैं। यदि $f(g(x)) = 8x^2 - 2x$ तथा $g(f(x)) = 4x^2 + 6x + 1$ हैं, तो $f(2) + g(2)$ का मान है ________.

Ans. Official Answer NTA (18)

Sol. $f(x) = ax^2 + bx + c$

$g(x) = dx + e$

$fog(x) = a(g(x))^2 + bg(x) + c$

$= a[d^2x^2 + e^2 + 2dex] + b[dx + e] + c$

$= ad^2x^2 + x(2ade + bd) + e^2 + be + c$

gof (x)

$= df(x) + e$

$= adx^2 + bdx + cd + e$

on comparing

$ad = 4$, $bd = 6$, $cd + e = 1$

$ad^2 = 8$, $2ade + bd = -2$

$e^2 + be + c = 0$

$ae = -2$

On solving

$a = 2$, $b = 3$, $c = 1$, $d = 2$, $e = -1$

$f(x) = 2x^2 + 3x + 1$

$g(x) = 2x - 1$