JEE Main July 2022

Question Paper With Text Solution

29 July | Shift-1

MATHEMATICS

JEE MAIN JULY 2022 | 29TH JULY SHIFT-1

SECTION – A

Question ID : 100201

Set & Relations

1. Let R be a relation from the set {1, 2, 3 ...., 60} to itself such that R = {(a, b) : b = pq, where p, q ≥ 3 are prime numbers.}. Then, the number of elements in R is :

माना समुच्चय S ={1, 2, 3 ...., 60} हैं। माना S से S में एक संबंध R, R = {(a, b) : b = pq, जहाँ p, q ≥ 3 अभाज्य संख्याएँ हैं} है। तो R में अवयवों की संख्या है :

(1) 600 (2) 660 (3) 540 (4) 720

Ans. Official Answer NTA (2)

Sol. Number of possible values of a = 60, for b = pq,
If p = 3, q = 3, 5, 7, 11, 13, 17, 19
If p = 5 q = 5, 7, 11
If p = 7 q = 7
Total cases = 60 × 11 = 660

Question ID : 100202

Complex number

2. If z = 2 + 3i, then $z^5 + (\bar{z})^5$ is equal to :

यदि $z = 2 + 3i$ है, तो $z^5 + (\bar{z})^5$ बराबर है :

(1) 244 (2) 224 (3) 245 (4) 265

Ans. Official Answer NTA (1)

Sol. $z^5 + (\bar{z})^5 = (2+3i)^5 + (2-3i)^5$

$= 2\left({}^5C_0 2^5 + {}^5C_2 2^3 (3i)^2 + {}^5C_4 2^1 (3i)^4\right)$

= 2 (32 + 10 × 8 (– 9) + 5 × 2 × 81) = 244

Question ID : 100203

Matrices

3. Let A and B be two 3 × 3 non-zero real matrices such that AB is a zero matrix. Then ;

(1) The system of linear equations AX = 0 has a unique solution

(2) The system of linear equations AX = 0 has infinitely many solution

(3) B is an invertible matrix

(4) adj(A) is an invertible matrix

माना A तथा B, 3×3 के दो अशून्य वास्तविक आव्यूह हैं जिसके लिए AB एक शून्य आव्यूह है। तो ;

(1) रैखिक समीकरण निकाय $AX = 0$ का अद्वितीय हल है

(2) रैखिक समीकरण निकाय $AX = 0$ के अनंत हल हैं

(3) B एक व्युत्क्रमणीय आव्यूह है

(4) adj(A) एक व्युत्क्रमणीय आव्यूह है

Ans. Official Answer NTA (2)

Sol. $AB = 0 \Rightarrow |AB| = 0$

$|A|\,|B| = 0$

$|A| = 0 \qquad |B| = 0$

If $|A| \neq 0, B = 0$ (not possible)

If $|B| \neq 0, A = 0$ (not possible)

Hence $|A| = |B| = 0$

$\Rightarrow AX = 0$ has infinitely many solutions

Question ID : 100204

Sequence & progression

4. If $\frac{1}{(20-a)(40-a)}+\frac{1}{(40-a)(60-a)}+....+\frac{1}{(180-a)(60-a)}=\frac{1}{256}$, then the maximum value of a is :

यदि $\frac{1}{(20-a)(40-a)}+\frac{1}{(40-a)(60-a)}+....+\frac{1}{(180-a)(60-a)}=\frac{1}{256}$ है, तो a का अधिकतम मान है :

(1) 198 (2) 202 (3) 212 (4) 218

Ans. Official Answer NTA (3)

Sol. $\frac{1}{20}\left[\left(\frac{1}{20-a}-\frac{1}{40-a}\right)+\left(\frac{1}{40-a}-\frac{1}{60-a}\right)+....+\left(\frac{1}{180-a}-\frac{1}{200-a}\right)\right]$

$\Rightarrow \frac{1}{20}\left(\frac{1}{20-a}-\frac{1}{200-a}\right)=\frac{1}{256}$

$(20 - a)(200 - a) = 256 \times 9$

$a^2 - 220a + 1696 = 0$

$a = 8, 212$

Hence maximum value of a is 212.

Question ID : 100205

Limit

5. If $\lim\limits_{x\to 0}\dfrac{\alpha e^{x}+\beta e^{-x}+\gamma\sin x}{x\sin^{2}x}=\dfrac{2}{3}$, where $\alpha,\gamma\in R$, then which of the following is NOT correct?

यदि $\lim\limits_{x\to 0}\dfrac{\alpha e^{x}+\beta e^{-x}+\gamma\sin x}{x\sin^{2}x}=\dfrac{2}{3}$ है, जहाँ $\alpha,\gamma\in R$ हैं, तो निम्न में से कौन सा सही नहीं है ?

(1) $\alpha^2+\beta^2+\gamma^2=6$

(2) $\alpha\beta+\beta\gamma+\gamma\alpha+1=0$

(3) $\alpha\beta^2+\beta\gamma^2+\gamma\alpha^2+3=6$

(4) $\alpha^2-\beta^2+\gamma^2=4$

Ans. Official Answer NTA (3)

Sol. $$\lim_{x\to 0}\frac{\alpha\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}\right)+\beta\left(1-x+\frac{x^2}{2!}+\frac{x^3}{3!}+...\right)+\gamma\left(x-\frac{x^3}{3!}+...\right)}{x^3}$$

constant terms should be zero

$\Rightarrow \alpha+\beta=0$

coeff of x should be zero

$\Rightarrow \alpha-\beta+\gamma=0$

coeff of x^2 should be zero

$$\lim_{x\to 0}\frac{x^3\left(\frac{\alpha}{3!}-\frac{\beta}{3!}-\frac{\gamma}{3!}\right)+x^4\left(\frac{\alpha}{3!}-\frac{\beta}{3!}-\frac{\gamma}{3!}\right)}{x^3}=\frac{2}{3}$$

$\Rightarrow \dfrac{\alpha}{2}+\dfrac{\beta}{2}=0$

$\dfrac{\alpha}{6}-\dfrac{\beta}{6}-\dfrac{\gamma}{6}=2/3$

$\Rightarrow \alpha=1,\beta=-1,\gamma=-2$

Question ID : 100206

Definite Integration

6. The integral $\displaystyle\int_0^{\frac{\pi}{2}}\frac{1}{3+2\sin x+\cos x}dx$ is equal to :

समाकलन $\displaystyle\int_0^{\frac{\pi}{2}}\frac{1}{3+2\sin x+\cos x}dx$ बराबर है :

(1) $\tan^{-1}(2)$

(2) $\tan^{-1}(2)-\dfrac{\pi}{4}$

(3) $\dfrac{1}{2}\tan^{-1}(2)-\dfrac{\pi}{8}$

(4) $\dfrac{1}{2}$

Ans. Official Answer NTA (2)

Sol. $I=\int_0^{\frac{\pi}{2}}\frac{dx}{3+2\sin x+\cos x}=\int_0^{\frac{\pi}{2}}\frac{\sec^2\frac{x}{2}.dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$

Put $\tan\frac{x}{2}=t,$ so

$I=\int_0^1\frac{dt}{(t+1)^2+1}=\tan^{-1}(x+1)\Big|_0^1=\tan^{-1}2-\frac{\pi}{4}$

Question ID : 100207

Differential Equation

7. Let the solution curve y = y(x) of the differential equation $\left(1+e^{2x}\right)\left(\frac{dy}{dx}+y\right)=1$ pass through the point $\left(0,\frac{\pi}{2}\right)$.

Then, $\lim_{x\to\infty}e^x y(x)$ is equal to :

माना अवकल समीकरण $\left(1+e^{2x}\right)\left(\frac{dy}{dx}+y\right)=1$ का हल वक्र y = y(x), $\left(0,\frac{\pi}{2}\right)$ से होकर जाता है। तो $\lim_{x\to\infty}e^x y(x)$ बराबर है :

(1) $\frac{\pi}{4}$ (2) $\frac{3\pi}{4}$ (3) $\frac{\pi}{2}$ (4) $\frac{3\pi}{2}$

Ans. Official Answer NTA (2)

Sol. $\frac{dy}{dx}+y=\frac{1}{1+e^{2x}}$

So integrating factor is $e^{\int 1.dx}=e^x$

So solution is $y.e^x=\tan^{-1}\left(e^x\right)+c$

Now as curve is passing through $\left(0,\frac{\pi}{2}\right)$ so

$\Rightarrow c=\frac{\pi}{4}$

$\Rightarrow \lim_{x\to\infty}\left(y.e^x\right)=\lim_{x\to\infty}\left(\tan^{-1}\left(e^x\right)+\frac{\pi}{4}\right)=\frac{3\pi}{4}$

Question ID : 100208

Ellipse

8. Let a line L pass through the point of intersection of the lines bx + 10y – 8 = 0 and 2x – 3y = 0,

$b \in R - \left\{\frac{4}{3}\right\}$. If the line L also passes through the point (1,1) and touches the circle $17(x^2 + y^2) = 16$, then the eccentricity of all ellipse $\frac{x^2}{5} + \frac{y^2}{b^2} = 1$ is :

माना एक रेखा L, रेखाओं $bx + 10y - 8 = 0$ तथा $2x - 3y = 0$, $b \in R - \left\{\frac{4}{3}\right\}$ के प्रतिच्छेदन बिन्दु से होकर जाती है। यदि रेखा L, बिन्दु (1,1) से भी होकर जाती है तथा वृत्त $17(x^2 + y^2) = 16$ को स्पर्श करती है, तो दीर्घवृत्त $\frac{x^2}{5} + \frac{y^2}{b^2} = 1$ की उत्केन्द्रता है :

(1) $\frac{2}{\sqrt{5}}$ (2) $\sqrt{\frac{3}{5}}$ (3) $\frac{1}{\sqrt{5}}$ (4) $\sqrt{\frac{2}{5}}$

Ans. Official Answer NTA (2)

Sol. Line is passing through intersection of
$bx + 10y - 8 = 0$ and $2x - 3y = 0$ is
$(bx + 10y - 8) + \lambda(2x - 3y) = 0$. As line is
passing through (1, 1) so $\lambda = b + 2$
Now line $(3b + 4)x - (3b - 4)y - 8 = 0$ is
tangent to circle $17(x^2 + y^2) = 16$

So $\frac{8}{\sqrt{(3b+4)^2 + (3b-4)^2}} = \frac{4}{\sqrt{17}}$

$\Rightarrow b^2 = 2 \Rightarrow e = \sqrt{\frac{3}{5}}$

Question ID : 100209

3D Geometry

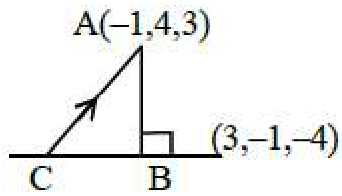
9. If the foot of the perpendicular from the point A(–1, 4, 3) on the plane P : $2x + my + nz = 4$, is $\left(-2, \frac{7}{2}, \frac{3}{2}\right)$, then the distance of the point A from the plane P, measured parallel to a line with direction ratios 3, –1, –4, is equal to :

यदि बिन्दु A(–1, 4, 3) से समतल P : $2x + my + nz = 4$, पर लंब का पाद $\left(-2, \frac{7}{2}, \frac{3}{2}\right)$ है, तो दिक् अनुपात 3, –1, –4 की एक रेखा के समांतर नापी गई बिन्दु A की समतल P से दूरी बराबर है :

(1) 1 (2) $\sqrt{26}$ (3) $2\sqrt{2}$ (4) $\sqrt{14}$

Ans. Official Answer NTA (2)

Sol.



Let B be foot of ⊥ coordinates of $B = \left(-2, \frac{7}{2}, \frac{3}{2}\right)$

Direction ratio of line AB is <2,1,3> so
m = 1, n = 3

So equation of AC is $\frac{x+1}{3} = \frac{y-4}{-1} = \frac{z-3}{-4} = \lambda$

So point C is $(3\lambda - 1, -\lambda + 4, -4\lambda + 3)$. But C lies on the plane, so

$6\lambda - 2 - \lambda + 4 - 12\lambda + 9 = 4$

$\Rightarrow \lambda = 1 \Rightarrow C(2,3,-1)$

$\Rightarrow AC = \sqrt{26}$

Question ID : 100210

Vectors

10. Let $\vec{a} = 3\hat{i} + \hat{j}$ and $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$. Let $\vec{c}$ be a vector satisfying $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda\vec{c}$. If $\vec{b}$ and $\vec{c}$ are non-parallel, then the value of λ is :

माना $\vec{a} = 3\hat{i} + \hat{j}$ तथा $\vec{b} = \hat{i} + 2\hat{j} + \hat{k}$ है। माना एक सदिश $\vec{c}$ के लिए $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda\vec{c}$ है। यदि $\vec{b}$ तथा $\vec{c}$ समांतर नहीं हैं, तो λ का मान है :

(1) –5 (2) 5 (3) 1 (4) –1

Ans. Official Answer NTA (1)

Sol. $\vec{a} = 3\hat{i} + \hat{j}, \vec{b} = \hat{i} + 2\hat{j} + \hat{k}$

As $\vec{a} \times (\vec{b} \times \vec{c}) = \vec{b} + \lambda\vec{c}$

$\Rightarrow \vec{a}.\vec{c}(\vec{b}) - (\vec{a}.\vec{b})\vec{c} = \vec{b} + \lambda\vec{c}$

$\Rightarrow \vec{a}.\vec{c} = 1, \vec{a}.\vec{b} = -\lambda$

$\Rightarrow (3\hat{i} + \hat{j}).(\hat{i} + 2\hat{j} + \hat{k}) = -\lambda$

$\Rightarrow \lambda = -5$

Question ID : 100211

Heights & Distances

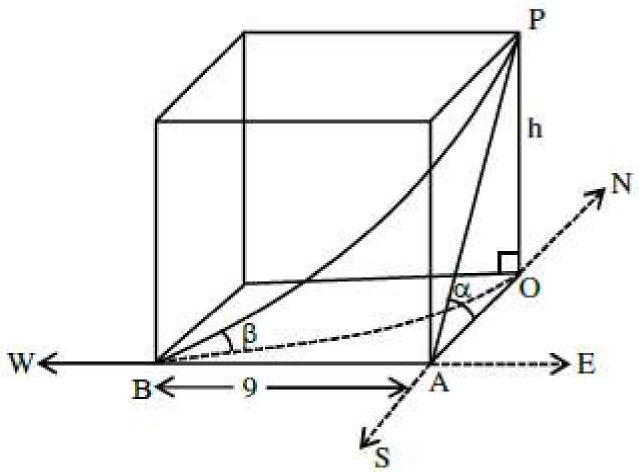
11. The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then cot α is equal to :

एक टावर के शिखर का, टावर के उत्तर की ओर एक बिन्दु A से उन्नयन कोण α है तथा A से पश्चिम की ओर 9 इकाई दूरी पर एक बिन्दु B से उन्नयन कोण $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$ है। यदि टावर से बिन्दु की दूरी 15 इकाई है, तो cot α बराबर है :

(1) $\frac{6}{5}$ (2) $\frac{9}{5}$ (3) $\frac{4}{3}$ (4) $\frac{7}{3}$

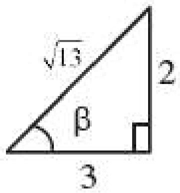
Ans. Official Answer NTA (1)

Sol.

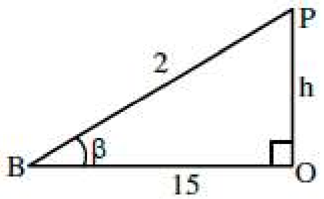


given OB = 15

$\cos\beta = \frac{3}{\sqrt{13}}$



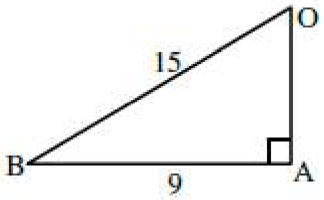
$\tan\beta = \frac{2}{3}$



$\tan\beta = \frac{h}{15}$

$\frac{2}{3} = \frac{h}{15}$

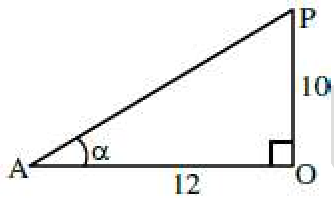
$10 = h$



$OA^2 + AB^2 = 225$

$OA^2 + 81 = 225$

$OA = 12$



$\tan\alpha = \frac{10}{12}$

$\cot\alpha = \frac{12}{10} = \frac{6}{5}$

Question ID : 100212

Mathematical Reasoning

12. The statement $(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to :

कथन $(p \wedge q) \Rightarrow (p \wedge r)$ किसके तुल्य है :

(1) $q \Rightarrow (p \wedge r)$ (2) $p \Rightarrow (p \wedge r)$ (3) $(p \wedge r) \Rightarrow (p \wedge q)$ (4) $(p \wedge q) \Rightarrow r$

Ans. Official Answer NTA (4)

Sol. $(p \wedge q) \Rightarrow (p \wedge r)$

$\sim(p \wedge q) \vee (p \wedge r)$

$(\sim p \vee \sim q) \vee (p \wedge r)$

$(\sim p \vee (p \wedge r)) \vee \sim q$

$(\sim p \vee p) \wedge (\sim p \vee r) \vee \sim q$

$(\sim p \vee r) \vee \sim q$

$(\sim p \vee \sim q) \vee r$

$\sim (p \wedge q) \vee r$

$(p \wedge q) \Rightarrow r$

Question ID : 100213

Straight Line

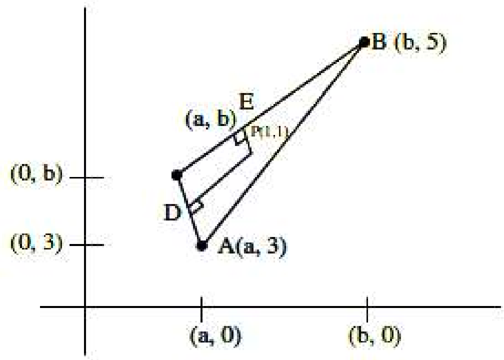
13. Let the circumcentre of a triangle with vertices A(a, 3), B(b, 5) and C(a, b), ab > 0 be P(1, 1). If the line AP intersects the line BC at the point $Q(k_1, k_2)$, then $k_1 + k_2$ is equal to :

माना एक त्रिभुज, जिसके शीर्ष A(a, 3), B(b, 5) तथा C(a, b), ab > 0 हैं, का परिकेन्द्र P(1, 1) है। यदि रेखा AP, रेखा BC को बिन्दु $Q(k_1, k_2)$ पर काटती है, तो $k_1 + k_2$ बराबर है :

(1) 2 (2) $\frac{4}{7}$ (3) $\frac{2}{7}$ (4) 4

Ans. Official Answer NTA (2)

Sol.



$m_{AC} \to \infty$

$m_{PD} = 0$

$D\left(\frac{a+a}{2}, \frac{b+3}{2}\right)$

$D\left(a, \frac{b+3}{2}\right)$

$m_{PD} = 0$

$\frac{b+3}{2} - 1 = 0$

$b + 3 - 2 = 0$

$b = -1$

$$E\left(\frac{b+a}{2},\frac{5+b}{2}\right)=\left(\frac{af}{2},2\right)$$

$m_{CB} . m_{EP} = -1$

$$\left(\frac{5-b}{b-a}\right)=\left(\frac{2}{a-3}\right)=-1$$

$$\left(\frac{6}{-1-a}\right)=\left(\frac{2}{a-3}\right)=-1$$

$12 = (1 + a) (a - 3)$
$12 = a^2 - 3a + a - 3$
$\Rightarrow a^2 - 2a - 15 = 0$
$(a - 5) (a + 3) = 0$
$a = 5$ or $a = -3$
Given $ab > 0$
$a(-1) > 0$
$-a > 0$
$a < 0$
$a = -3$ Accept
AP line A (–3, 3) P(1, 1)

$$y-1=\left(\frac{3-1}{-3-1}\right)(x-1)$$

$\Rightarrow x + 2y = 3$ Appling .....(1)
Line BC B(–1, 5)
C(–3, –1)

$$(y-5)=\frac{6}{2}(x+1)$$

$y - 5 = 3x + 3$
$y = 3x + 8$ .....(2)
Solving (1) & (2)
$x + 2 (3x + 8) = 3$
$\Rightarrow 7x + 16 = 3$
$7x = -13$

$$x=-\frac{13}{7}$$

$$y=3\left(-\frac{13}{7}\right)+8$$

$$=\frac{-39+56}{7}$$

$y = \frac{17}{7}$

$x + y = \frac{-13+17}{7} = \frac{4}{7}$

Question ID : 100214

Vectors

14. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$. If θ is the angle between the vectors $\left(\hat{a}+\hat{b}\right)$ and $\left(\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right)$, then the value of $164\cos^2\theta$ is equal to :

माना दो इकाई सदिशों $\hat{a}$ तथा $\hat{b}$ के बीच का कोण $\frac{\pi}{4}$ है। यदि सदिशों $\left(\hat{a}+\hat{b}\right)$ तथा $\left(\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right)$ के बीच का कोण θ है, तो $164\cos^2\theta$ का मान बराबर है :

(1) $90 + 27\sqrt{2}$ (2) $45 + 18\sqrt{2}$ (3) $90 + 3\sqrt{2}$ (4) $54 + 90\sqrt{2}$

Ans. Official Answer NTA (1)

Sol. $\hat{a}\,\hat{}\,\hat{b} = \frac{\pi}{4} = \phi$

$\hat{a}\cdot\hat{b} = |\hat{a}||\hat{b}|\cos\phi$

$\hat{a}\cdot\hat{b} = \cos\phi = \frac{1}{\sqrt{2}}$

$\cos\theta = \frac{(\hat{a}+\hat{b})\cdot(\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b}))}{|\hat{a}+\hat{b}||\hat{a}+2\hat{b}+2(\hat{a}\times\hat{b})|}$

$|\hat{a}+\hat{b}|^2 = (\hat{a}+\hat{b})\cdot(\hat{a}+\hat{b})$

$|\hat{a}+\hat{b}|^2 = 2 + 2\hat{a}\cdot\hat{b}$

$= 2+\sqrt{2}$

$\hat{a}\times\hat{b} = |\hat{a}||\hat{b}|\sin\phi\hat{n}$

$\hat{\mathbf{a}}\times\hat{\mathbf{b}} = \frac{\hat{n}}{\sqrt{2}}$ when $\hat{n}$ is vector $\perp\hat{a}$ and $\hat{b}$

let $\vec{c} = \hat{a}\times\hat{b}$

We know.

$\vec{c}\cdot\vec{a} = 0$

$\vec{c}\cdot\vec{b} = 0$

$|\hat{a}+2\hat{b}+2\vec{c}|^2$

$=1+4+\frac{(4)}{2}+4\hat{a}\cdot\hat{b}+8\hat{b}\cdot\vec{c}+4\vec{c}\cdot\hat{a}$

$=7+\frac{4}{\sqrt{2}}=7+2\sqrt{2}$

Now

$(\hat{a}+\hat{b})\cdot(\hat{a}+2\hat{b}+2\vec{c})$

$=|\hat{a}|^2+2\hat{a}\cdot\hat{b}+0+\hat{b}\cdot\hat{a}+2|\hat{b}|^2+0$

$=1+\frac{2}{\sqrt{2}}+\frac{1}{\sqrt{2}}+2$

$=3+\frac{3}{\sqrt{2}}$

$\cos\theta=\frac{3+\frac{3}{\sqrt{2}}}{\sqrt{2+\sqrt{2}}\sqrt{7+2\sqrt{2}}}$

$\cos^2\theta=\frac{9(\sqrt{2}+1)^2}{2(2+\sqrt{2})(7+2\sqrt{2})}$

$\cos^2\theta=\left(\frac{9}{2\sqrt{2}}\right)\frac{(\sqrt{2}+1)}{(7+2\sqrt{2})}$

$164\cos^2\theta=\frac{(82)(9)}{\sqrt{2}}\frac{(\sqrt{2}+1)}{(7+2\sqrt{2})}\frac{(7-2\sqrt{2})}{(7-2\sqrt{2})}$

$=\frac{(82)}{\sqrt{2}}\frac{(9)[7\sqrt{2}-4+7-2\sqrt{2}]}{(41)}$

$=(9\sqrt{2})[5\sqrt{2}+3]$

$=90+27\sqrt{2}$

Question ID : 100215

Definite Integration

15. If $f(\alpha)=\int_1^{\alpha}\frac{\log_{10}t}{1+t}dt$, $\alpha>0$, then $f(e^3)+f(e^{-3})$ is equal to :

यदि $f(\alpha)=\int_1^{\alpha}\frac{\log_{10}t}{1+t}dt$, $\alpha>0$ है, तो $f(e^3)+f(e^{-3})$ बराबर है :

(1) 9 (2) $\frac{9}{2}$ (3) $\frac{9}{\log_e(10)}$ (4) $\frac{9}{2\log_e(10)}$

Ans. Official Answer NTA (4)

Sol. $f\left(e^3\right)=\int_1^{e^3}\frac{\ell nt}{\ell n10(1+t)}dt$ ....(1)

$f(\alpha)=\int_1^{\alpha}\frac{\ell nt}{(\ell n10)(1+t)}dt$

$t=\frac{1}{x}\Rightarrow x=\frac{1}{t}$

$dt=\frac{-1}{x^2}dx$

$=\int_1^{\frac{1}{\alpha}}\frac{-\ell nx}{(\ell n10)\left(1+\frac{1}{x}\right)}\left(-\frac{1}{x^2}\right)dx$

$f(\alpha)=\frac{1}{\ell n10}\int_1^{\frac{1}{\alpha}}\frac{\ell nx}{x(x+1)}dx$

$f\left(e^{-3}\right)=\frac{1}{\ell n10}\int_1^{e^3}\frac{\ell nt}{t(t+1)}dt$ ....(2)

Add (1) & (2)

$\left(e^3\right)+f\left(e^{-3}\right)$

$=\left(\frac{1}{\ell n10}\right)\int_1^{e^3}\frac{\ell nt}{(1+t)}\left[1+\frac{1}{t}\right]dt$

$=\left(\frac{1}{\ell n10}\right)\int_1^{3}\frac{\ln t}{t}dt$

$\ell nt=r$

$\frac{dt}{t}=dr$

$=\frac{1}{\ell n10}\int_0^3 rdr$

$=\left(\frac{1}{\ell n10}\right)\left(\frac{r^2}{2}\right)\Big|_0^3$

$=\left(\frac{1}{\log 10}\right)\left(\frac{9}{2}\right)$

$$=\frac{9}{2\log_e 10}$$

Question ID : 100216

Area Under Curve

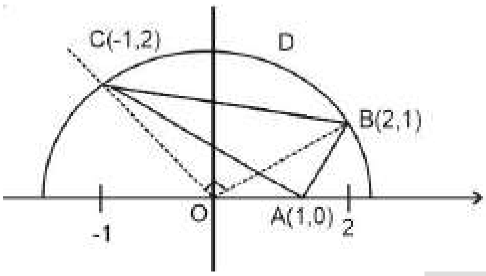
16. The area of the region $\left\{(x,y):|x-1|\le y\le\sqrt{5-x^2}\right\}$ is equal to :

क्षेत्र $\left\{(x,y):|x-1|\le y\le\sqrt{5-x^2}\right\}$ का क्षेत्रफल बराबर है :

(1) $\frac{5}{2}\sin^{-1}\left(\frac{3}{5}\right)-\frac{1}{2}$ (2) $\frac{5\pi}{4}-\frac{3}{2}$ (3) $\frac{3\pi}{4}+\frac{3}{2}$ (4) $\frac{5\pi}{4}-\frac{1}{2}$

Ans. Official Answer NTA (4)

Sol.



$|x-1|<y<\sqrt{5-x^2}$

When $|x-1|=\sqrt{5-x^2}$

$\Rightarrow (x-1)^2=5-x^2$

$\Rightarrow x^2-x-2=0$

$\Rightarrow x=2,-1$

Required Area = Area of ΔABC + Area of region BCD

$$=\frac{1}{2}\begin{vmatrix}1 & 0 & 1\\ 2 & 1 & 1\\ -1 & 2 & 1\end{vmatrix}+\frac{\pi}{4}(\sqrt{5})^2-\frac{1}{2}(\sqrt{5})^2$$

$$=\frac{5\pi}{4}-\frac{1}{2}$$

Question ID : 100217

Hyperbola

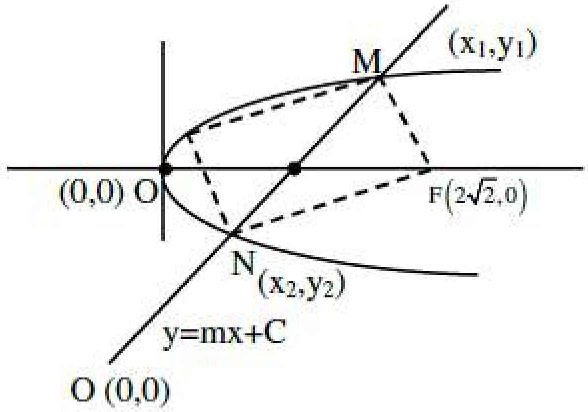
17. Let the focal chord of the parabola P : $y^2=4x$ along the line L : $y=mx+c$, $m>0$ meet the parabola at the points M and N. Let the line L be a tangent to the hyperbola H : $x^2-y^2=4$. If O is the vertex of P and F is the focus of H on the positive x-axis, then the area of the quadrilateral OMFN is :

माना रेखा $L : y = mx + c, m > 0$ के अनुदिश परवलय $P : y^2 = 4x$ की नाभिलंब जीवा परवलय को बिन्दुओं M तथा N पर मिलती है। माना रेखा L अतिपरवलय $H : x^2 - y^2 = 4$ की एक स्पर्श रेखा है। यदि P का शीर्ष O है तथा H की धनात्मक x-अक्ष पर नाभि F है, तो चतुर्भुज OMFN का क्षेत्रफल है :

(1) $2\sqrt{6}$ (2) $2\sqrt{14}$ (3) $4\sqrt{6}$ (4) $4\sqrt{14}$

Ans. Official Answer NTA (2)

Sol.



$H : \frac{x^2}{4} - \frac{y^2}{4} = 1$

Focus (ae, 0)

$F(2\sqrt{2}, 0)$

Line $L : y = mx + c$ pass (1, 0)

$o = m + C$ ....(1)

List L is tangent to Hyperbola. $\frac{x^2}{4} - \frac{y^2}{4} = 1$

$C = \pm\sqrt{a^2 m^2 - \ell^2}$

$C = \pm\sqrt{4m^2 - 4}$

From (1)

$-m = \pm\sqrt{4m^2 - 4}$

Squaring

$m^2 = 4m^2 - 4$

$4 = 3m^2$

$\frac{2}{\sqrt{3}} = m$ (as $m > 0$)

$C = -m$

$C = \frac{-2}{\sqrt{3}}$

$y = \frac{2x}{\sqrt{3}} - \frac{2}{\sqrt{3}}$

$y^2 = 4x$

$\Rightarrow \left(\frac{2x-2}{\sqrt{3}}\right)^2 = 4x$

$\Rightarrow x^2 + 1 - 2x = 3x$

$\Rightarrow x^2 - 5x + 1 = 0$

$y^2 = 4\left(\frac{\sqrt{3}y + 2}{2}\right)$

$y^2 = 2\sqrt{3}y + 4$

$\Rightarrow y^2 - 2\sqrt{3}y - 4 = 0$

Area

$\left|\frac{1}{2}\right|\begin{matrix} 0 & x_1 & 2\sqrt{2} & x_2 & 0 \\ 0 & y_1 & 0 & y_2 & 0 \end{matrix}||$

$= \left|\frac{1}{2}\left[-2\sqrt{2}y_1 + 2\sqrt{2}y_2\right]\right|$

$= \sqrt{2}\left|y_2 - y_1\right| = \frac{(\sqrt{2})\sqrt{12+16}}{111}$

$= \sqrt{56}$

$= 2\sqrt{14}$

Question ID : 100218

Continuity & Differentiability

18. The number of points, where the function $f : R \to R$,

$f(x) = |x-1| \cos |x-2| \sin|x-1| + (x-3)|x^2 - 5x + 4|$, is NOT differentiable, is :

उन बिन्दुओं की संख्या, जहाँ पर फलन $f : R \to R$,

$f(x) = |x-1| \cos |x-2| \sin|x-1| + (x-3)|x^2 - 5x + 4|$, अवकलनीय नहीं है, है:

(1) 1 (2) 2 (3) 3 (4) 4

Ans. Official Answer NTA (2)

Sol. $f(x) = |x-1| \cos |x-2| \sin |x-1| + (x-3)| x^2 - 5x + 4|$

$= |x-1| \cos |x-2| \sin |x-1| + (x-3)| x-1||x-4|$

= |x – 1| [cos |x – 2| sin |x – 1| + (x – 3) |x – 4|]

Non differentiable at x = 1 and x = 4.

Question ID : 100219

Probability

19. Let S = {1, 2, 3, ..., 2022}. Then the probability, that a randomly chosen number n from the set S scuh that HCF (n, 2022) = 1, is :

माना S = {1, 2, 3, ..., 2022} है। तो समुच्चय S से यादृच्छया चुनी गई एक संख्या n के लिए HCF (n, 2022) = 1 होने की प्रायिकात है :

(1) $\frac{128}{1011}$ (2) $\frac{166}{1011}$ (3) $\frac{127}{337}$ (4) $\frac{112}{337}$

Ans. Official Answer NTA (4)

Sol. Total number of elements = 2022

2022 = 2×3×337

HCF (n, 2022) = 1

is feasible when the value of 'n' and 2022 has no common factor.

A = Number which are divisible by 2 from {1,2,3.....2022}

n(A) = 1011

B = Number which are divisible by 3 by 3

from {1,2,3......2022}

n(B) = 674

$A \cap B$ = Number which are divisible by 6

from {1,2,3........2022}

6,12,18........., 2022

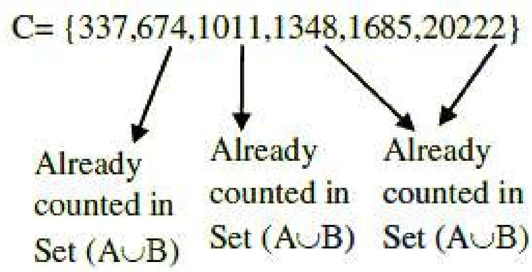
337 = n $(A \cap B)$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

= 1011+ 674 –337

= 1348

C= Number which divisible by 337 from {1,........1022}



Total elements which are divisible by 2 or 3 or 337

= 1348 +2 = 1350

Favourable cases = Element which are neither divisible by 2, 3 or 337

= 2022 – 1350

= 672

Required probability = $\frac{672}{2022} = \frac{112}{337}$

Question ID : 100220

Maxima & Minima

20. Let $f(x) = 3^{(x^2-2)^3+4}$, $x \in R$. Then which of the following statements are true?

P : x = 0 is a point of local minima of f

Q : $x = \sqrt{2}$ is a point of inflection of f

R : f ' is increasing for $x > \sqrt{2}$

(1) Only P and Q (2) Only P and R

(3) Only Q and R (4) All P, Q and R

माना $f(x) = 3^{(x^2-2)^3+4}$, $x \in R$ है। तो निम्न कथनों में से कौनसे सत्य है ?

P : x = 0, f का एक स्थानीय निम्ननिष्ठ बिन्दु है :

Q : $x = \sqrt{2}$,f का एक नति परिवर्तन बिन्दु है

R: $x > \sqrt{2}$ के लिए f' वर्धमान है

(1) केवल P तथा Q (2) केवल P तथा R

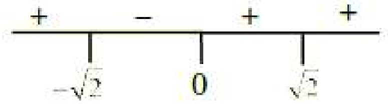
(3) केवल Q तथा R (4) सभी P, Q तथा R

Ans. Official Answer NTA (4)

Sol. $f(x) = 81.3^{(x^2-2)^3}$

$f'(x) = 81 \cdot 3^{(x^2-2)^3} \cdot \ln 3 \cdot 3(x^2-2)^2 \cdot 2x$

$= (81 \times 6)3^{(x^2-2)^3} x(x^2-2)^2 \ln 3$



x = 6 is point of local min

$f'(x) = \underbrace{(486 \cdot \ln 3)}_{k} \underbrace{3^{(x^2-2)^3} x(x^2-2)^2}_{g(x)}$

$g'(x) = 3^{(x^2-2)^3}(x^2-2)^2 + x \cdot 3^{(x^2-2)^3} \cdot 4x \cdot (x^2-2)$

$+x\cdot\left(x^2-2\right)^2\cdot 3^{\left(x^2-2\right)^3}\ln 3\cdot 3\left(x^2-2\right)^2\cdot 2x$

$=3^{\left(x^2-2\right)^3}\left(x^2-2\right)\left[x^2-2+4x^2+6x^2\ln 3\left(x^2-2\right)^3\right]$

$g'(x)=3^{\left(x^2-2\right)^3}\left(x^2-2\right)\left[5x^2-2+6x^2\ell\ln 3\left(x^2-2\right)^3\right]$

$f''(x)=k\cdot g'(x)$

$f''(\sqrt{2})=0, f''\left(\sqrt{2}^{+}\right)>0, f''\left(\sqrt{2}^{-}\right)<0$

$x=\sqrt{2}$ is point of inflection

$f''(x)>0$ for $x>\sqrt{2}$ so f' is increasing

SECTION – B

Question ID : 100221

Trigonometric Equation

21. Let $S = \{\theta \in (0, 2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\}$. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0, \theta \in S$, is _________.

माना $S = \{\theta \in (0, 2\pi) : 7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2\}$ है। सभी समीकरणों $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0, \theta \in S$ के मूलों का योग है ________।

Ans. Official Answer NTA (16)

Sol. $7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$

$4\cos^2\theta + 3\cos^2\theta \,.\, 2\cos^2 2\theta = 2$

$2(1 + \cos 2\theta) + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2\cos^2 2\theta - 5\cos^2\theta = 0$

$\cos^2\theta(2\cos^2\theta - 5) = 0$

$\cos^2\theta = 0$

$2\theta=\left(2n+1\right)\frac{\pi}{2}$

$\theta=\left(2n+1\right)\frac{\pi}{4}$

$S=\left\{\frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}\right\}$

For all four values of θ

$x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

Sum of roots of all four equations = 4 × 4 = 16.

Question ID : 100222

Statistics

22. Let the mean and the variance of 20 observations $x_1, x_2, ...x_{20}$ be 15 and 9, respectively. For $\alpha \in R$, if the mean of $(x_1 + \alpha)^2, (x_2 + \alpha)^2, ..., (x_{20} + \alpha)^2$ is 178, then the square of the maximum value of α is equal to _________.

माना 20 प्रेक्षणों $x_1, x_2, ...x_{20}$ के माध्य तथा प्रसरण 15 तथा 9 हैं। $\alpha \in R$ के लिए, यदि $(x_1 + \alpha)^2, (x_2 + \alpha)^2, ..., (x_{20} + \alpha)^2$ का माध्य 178 है, तो α के अधिकतम मान का वर्ग बराबर है ________।

Ans. Official Answer NTA (4)

Sol. $\sum x_1 = 15 \times 20 = 300$ .....(i)

$\frac{\sum x_1^2}{20} - (15)^2 = 9$ .....(ii)

$\sum x_1^2 = 234 \times 20 = 4680$

$\frac{\sum (x_1 + \alpha)^2}{20} = 178 \Rightarrow \sum (x_1 + \alpha)^2 = 3560$

$\Rightarrow \sum x_1^2 + 2\alpha \sum x_1 + \sum \alpha^2 = 3560$

$4680 + 600\alpha + 20\alpha^2 = 3560$

$\Rightarrow \alpha^2 + 30\alpha + 56 = 0$

$\Rightarrow (\alpha + 28)(\alpha + 2) = 0$

$\alpha = -2, -28$

Square of maximum value of α is 4.

Question ID : 100223

3D Geometry

23. Let a line with direction ratio a, –4a, –7 be perpendicular to the lines with direction ratios 3, –1, 2b and b, a, –2. If the point of intersection of the line $\frac{x+1}{a^2+b^2} = \frac{y-2}{a^2-b^2} = \frac{z}{1}$ and the plane $x - y + z = 0$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to _________.

माना दिक् अनुपास a, –4a, –7 की एक रेखा, अनुपात 3, –1, 2b तथा b, a, –2 की रेखाओं के लंबवत है। यदि रेखा $\frac{x+1}{a^2+b^2} = \frac{y-2}{a^2-b^2} = \frac{z}{1}$ तथा समतल $x - y + z = 0$ का प्रतिच्छेदन बिन्दु (α, β, γ) है तो $\alpha + \beta + \gamma$ बराबर है _________।

Ans. Official Answer NTA (10)

Sol. (a, –4a, –7) ⊥ to (3, –1, 2b)

$a = 2b$ .....(i)

(a, –4a, –7) ⊥ to (b,a, –2)

$3a + 4a - 14b = 0$

$ab - 4a^2 + 14 = 0$ .....(ii)

From Equations (i) and (ii)

$2b^2 - 16b^2 + 14 = 0$

$b^2 = 1$

$a^2 = 4b^2 = 4$

$$\frac{x+1}{5} = \frac{y-2}{3} = \frac{z}{1} = k$$

$\alpha = 5k - 1, \beta = 3k + 2, \gamma = k$

As (α, β, γ) satisfies $x - y + z = 0$

$5k - 1 - (3k + 2) + k = 0$

$k = 1$

$\therefore \alpha + \beta + \gamma = 9k + 1 = 10$

Question ID : 100224

Sequence & progression

24. Let $a_1, a_2, a_3, ...$ be and A.P. If $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$, then $4a_2$ is equal to _________.

माना $a_1, a_2, a_3, ...$ एक A.P. है। यदि $\sum_{r=1}^{\infty} \frac{a_r}{2^r} = 4$ है, तो $4a_2$ बराबर है ________।

Ans. Official Answer NTA (16)

Sol. $$S = \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \dots$$

$$\frac{S}{2} = \frac{a_1}{2^2} + \frac{a_2}{2^3} + \dots$$

$$\frac{S}{2} = \frac{a_1}{2} + d\left(\frac{1}{2^2} + \frac{1}{2^3} + \dots\right)$$

$$\frac{S}{2} = \frac{a_1}{2} + d\left(\frac{\frac{1}{4}}{1-\frac{1}{2}}\right)$$

$\therefore S = a_1 + d = a_2 = 4$

Or $4a_2 = 16$

Question ID : 100225

Binomial Theorem

25. Let the ratio of the fifth term from the beginning to the fifth from the end in the binomial expansion of $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$, in the increasing powers of $\frac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6}$: 1. If the sixth term from the beginning is $\frac{\alpha}{\sqrt[4]{3}}$, then α is equal to _________.

माना $\frac{1}{\sqrt[4]{3}}$ की बढ़ती घातों में $\left(\sqrt[4]{2}+\frac{1}{\sqrt[4]{3}}\right)^n$ के द्विपद प्रसार में आरंभ पाँचवें पद का अंत से पाँचवें पद से अनुपात $\sqrt[4]{6}$: 1 है। यदि आरंभ से छठा पद $\frac{\alpha}{\sqrt[4]{3}}$ है, तो α बराबर है ________।

Ans. Official Answer NTA (84)

Sol. $$\frac{T_5}{T_{n-3}} = \frac{{}^nC_4\left(2^{1/4}\right)^{n-4}\left(3^{-1/4}\right)^4}{{}_nC_{n-4}\left(2^{1/4}\right)^4\left(3^{-1/4}\right)^{n-4}} = \frac{\sqrt[4]{6}}{1}$$

$$\Rightarrow 2^{\frac{n-8}{4}}3^{\frac{n-8}{4}} = 6^{1/4}$$

$$\Rightarrow 6^{n-8} = 6$$

$$\Rightarrow n-8=1 \Rightarrow n=9$$

$$T_6 = {}^9C_5\left(2^{1/4}\right)^4\left(3^{-1/4}\right)^5 = \frac{84}{\sqrt[4]{3}}$$

$$\therefore \alpha = 84$$

Question ID : 100226

Matrices

26. The number of matrices of order 3 × 3, whose entries are either 0 or 1 and sum of all the entries is a prime number, is _________.

कोटि 3 × 3 आव्यूहों, जिसके अवयव या तो 0 या 1 हैं तथा सभी अवयवों का योग एक अभाज्य संख्या है, की संख्या _________।

Ans. Official Answer NTA (282)

Sol. $$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} a_{ij} \in \{0,1\}$$

$\sum a_{ij} = 2,3,5,7$

Total matrix $= {}^9C_2 + {}^9C_3 + {}^9C_5 + {}^9C_7$

$= 282$

Question ID : 100227

Determinant

27. Let p and p + 2 be prime numbers and let $\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$

Then the sum of the maximum values of α and β, such that p^α and $(p+2)^\beta$ divide Δ, is _________.

माना p तथा p + 2 अभाज्य संख्याएँ हैं तथा $\Delta = \begin{vmatrix} p! & (p+1)! & (p+2)! \\ (p+1)! & (p+2)! & (p+3)! \\ (p+2)! & (p+3)! & (p+4)! \end{vmatrix}$

है। तो α तथा β के अधिकतम मानों, जिसके लिए p^α तथा $(p+2)^\beta$, Δ को विभाजित करते हैं, का योग है ____।

Ans. Official Answer NTA (4)

Sol. $\Delta = \begin{vmatrix} P! & (P+1)! & (P+2)! \\ (P+1)! & (P+2)! & (P+3)! \\ (P+2)! & (P+3)! & (P+4)! \end{vmatrix}$

$\Delta = P!(P+1)!(P+2)! \begin{vmatrix} 1 & 1 & 1 \\ P+1 & P+2 & P+3 \\ (P+2)(P+1) & (P+3)(P+2) & (P+4)(P+3) \end{vmatrix}$

$\Delta = 2P!(P+1)!(P+2)\,!$

Which is divisible by P^α & $(P+2)^\beta$

$\therefore \alpha = 3, \beta = 1$

Question ID : 100228

Sequence & progression

28. If $\frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \frac{1}{4\times5\times6} + ... + \frac{1}{100\times101\times102} = \frac{k}{101}$, then 34 k is equal to _________.

यदि $\frac{1}{2\times3\times4} + \frac{1}{3\times4\times5} + \frac{1}{4\times5\times6} + ... + \frac{1}{100\times101\times102} = \frac{k}{101}$, है, तो 34 k बराबर है ________।

Ans. Official Answer NTA (286)

Sol. $\frac{1}{2.3.4}+\frac{1}{3.4.5}+.....+\frac{1}{100.101.102}=\frac{k}{101}$

$\frac{4-2}{2.3.4}+\frac{5-3}{3.4.5}+.....+\frac{102-100}{100.101.102}=\frac{2k}{101}$

$\frac{1}{2.3}-\frac{1}{3.4}+\frac{1}{3.4}-\frac{1}{4.5}+.....+\frac{1}{100.101}-\frac{1}{101.102}=\frac{2k}{101}$

$\frac{1}{2.3}-\frac{1}{101.102}=\frac{2k}{101}$

$\therefore 2k=\frac{101}{6}-\frac{1}{102}$

$\therefore 34k=286$

Question ID : 100229

Set & Relations

29. Let S = {4, 6, 9} and T = {9, 10, 11, ..., 1000}. If $A = \{a_1 + a_2 + ...+a_k : k \in N, a_1, a_2, a_3, ..., a_k \in S\}$, then the sum of all the elements in the set T – A is equal to _________.

माना S = {4, 6, 9} तथा T = {9, 10, 11, ..., 1000}हैं। यदि $A = \{a_1 + a_2 + ...+a_k : k \in N, a_1, a_2, a_3, ..., a_k \in S\}$तो समुच्चय T – A में सभी अवयवों का योग है_________।

Ans. Official Answer NTA (11)

Sol. S = {4,6,9} T = {9,10,11..1000}

$A\{a_1 + a_2 +.....+ a_k :K\in N\}$ & $a_i \in S$

Here by the definition of set 'A'

A ={a : a = 4x + 6y + 9z}

Except the element 11, every element of set T is of of the form 4x + 6y + 9z for some $x, y, z \in W$

$\therefore$ T – A = {11}

Question ID : 100230

Circle

30. Let the mirror image of a circle $c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$ in line y = x + 1 be $c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$. If r is the radius of circle c_2, then $\alpha + 6r^2$ is equal to _________.

माना रेखा y = x + 1 में, वृत्त $c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$ का दर्पण प्रतिबिंब $c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$ है। यदि वृत्त c_2 की त्रिज्या r है, तो $\alpha + 6r^2$ बराबर है _________।

Ans. Official Answer NTA (12)

Sol. Image of centre $c_1 \equiv (1,3)$ in x – y + 1 = 0 is given by

$\frac{x_1-1}{1}=\frac{y_1-3}{-1}=\frac{-2(1-3+1)}{1^2+1^2}$

$\Rightarrow x_1 = 2, y_1 = 2$

$\therefore$ Centre of circle $c_2 \equiv (2,2)$

$\therefore$ Equation of c_2 be $x^2 + y^2 - 4x - 4y + \frac{38}{5} = 0$

Now radius of c_2 is $\sqrt{4+4-\frac{38}{5}} = \sqrt{\frac{2}{5}} = r$

$(\text{radius of } c_1)^2 = (\text{radius of } c_2)^2$

$\Rightarrow 10 - \alpha = \frac{2}{5} \Rightarrow \alpha = \frac{48}{5}$

$\therefore \alpha + 6r^2 = \frac{48}{5} + \frac{12}{5} = 12$