JEE Main July 2022

Question Paper With Text Solution

29 July | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JULY 2022 | 29TH JULY SHIFT-2

SECTION – A

Question ID : 15477154601

Complex number

1. If $z \neq 0$ be a complex number such that $\left|z-\frac{1}{z}\right|=2$, then the maximum value of $|z|$ is :

यदि सम्मिश्र संख्या $z \neq 0$ के लिए $\left|z-\frac{1}{z}\right|=2$ है, तो $|z|$ का अधिकतम मान है :

(1) $\sqrt{2}$ (2) 1 (3) $\sqrt{2}-1$ (4) $\sqrt{2}+1$

Ans. Official Answer NTA (4)

Sol. $z-1/z|=2$

$|z|-\frac{1}{|z|}|\leq|z-\frac{1}{z}|\leq|z|+\frac{1}{|z|}$ Let $|z|=r$

$\left|r-\frac{1}{r}\right|\leq 2\leq r+\frac{1}{r}$

$\left|r-\frac{1}{r}\right|\leq 2$ & $r+\frac{1}{r}\geq 2$ always true

$r-\frac{1}{r}\geq -2$ & $r-\frac{1}{r}\leq 2$

$r^2-1\leq 2r$

$r^2-2r\leq 1$

$(r-1)^2\leq 2$

$r-1\leq\sqrt{2}$

$\therefore |z|_{max}=1+\sqrt{2}$

Question ID : 15477154602

Matrices

2. Which of the following matrices can NOT be obtained from the matrix $\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$ by a single elementary row operation?

आव्यूह $\begin{bmatrix}-1 & 2\\ 1 & -1\end{bmatrix}$ पर केवल एक प्रारंभिक पंक्ति संक्रिया से निम्न में से कौनसा आव्यूह प्राप्त नहीं किया जा सकता है ?

(1) $\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ (2) $\begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ (3) $\begin{bmatrix} -1 & 2 \\ -2 & 7 \end{bmatrix}$ (4) $\begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$

Ans. Official Answer NTA (3)

Sol. $A = \begin{bmatrix} -1 & 2 \\ 1 & -1 \end{bmatrix}$

(1) $R_1 \to R_1 + R_2; \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$ possible

(2) $R_1 \leftrightarrow R_2; \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$ possible

(3) Option is not possible

(4) $R_2 \to R_2 + 2R_1; \begin{bmatrix} -1 & 2 \\ -1 & 3 \end{bmatrix}$ possible

Question ID : 15477154603

Determinant

3. If the system of equations

यदि समीकरण निकाय

$x + y + z = 6$

$2x + 5y + \alpha z = \beta$

$x + 2y + 3z = 14$

has infinitely many solutions, then $\alpha + \beta$ is equal to :

के अनंत हल है, तो $\alpha + \beta$ बराबर है :

(1) 8 (2) 36 (3) 44 (4) 48

Ans. Official Answer NTA (3)

Sol. $x + y + z = 6$ ___(1)

$2x + 5y + \alpha z = \beta$ ___(2)

$x + 2y + 3z = 14$ ____(3)

$x + y = 6 - z$

$x + 2y = 14 - 3z$

On solving

$x = z \alpha 2 \Rightarrow y = 8 - 2z$ in (2)

$2(z - 2) + 5(8 - 2z) + \alpha z = \beta$

$(\alpha - 8) z = \beta - 36$ For having infinite solutions

$\alpha - 8 = 0$ & $\beta - 36 = 0$

$\alpha = 8, \beta = 36 \qquad (\alpha + \beta = 44)$

Question ID : 15477154604

Continuity & Differentiability

4. Let the function $f(x)=\begin{cases}\dfrac{\log_e(1+5x)-\log_e(1+\alpha x)}{x} & ;\text{ if } x\neq 0\\ 10 & ;\text{ if } x=0\end{cases}$ be continuous at $x=0$. Then α is equal to :

माना फलन $f(x)=\begin{cases}\dfrac{\log_e(1+5x)-\log_e(1+\alpha x)}{x} & ;\text{ if } x\neq 0\\ 10 & ;\text{ if } x=0\end{cases}$, $x=0$ पर संतत है। तो α बराबर है :

(1) 10 (2) –10 (3) 5 (4) –5

Ans. Official Answer NTA (4)

Sol. $f(x)=\begin{cases}\dfrac{\ln(1+5x)-\ln(1+\alpha x)}{x} & ;x\neq 0\\ 10 & ;x=0\end{cases}$

$\lim\limits_{x\to 0}\dfrac{\ln(1+5x)-\ln(1+\alpha x)}{x}=10$

Using expension

$\lim\limits_{x\to 0}\dfrac{(5x+\ldots\ldots)-(\alpha x+\ldots\ldots)}{x}=10$

$5-\alpha=10\Rightarrow\alpha=-5$

Question ID : 15477154605

Definite Integration

5. If [t] denotes the greatest integer $\leq t$, then the value of $\int_0^1\left[2x-\left|3x^2-5x+2\right|+1\right]dx$ is :

यदि [t] महत्तम पूर्णांक $\leq t$ है, तो $\int_0^1\left[2x-\left|3x^2-5x+2\right|+1\right]dx$ का मान है :

(1) $\dfrac{\sqrt{37}+\sqrt{13}-4}{6}$ (2) $\dfrac{\sqrt{37}-\sqrt{13}-4}{6}$

(3) $\dfrac{-\sqrt{37}-\sqrt{13}+4}{6}$ (4) $\dfrac{-\sqrt{37}+\sqrt{13}+4}{6}$

Ans. Official Answer NTA (1)

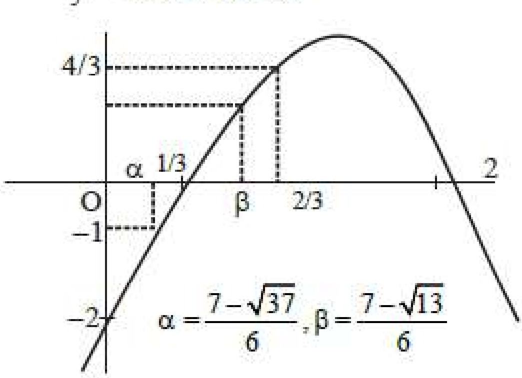
Sol. $I=\int_0^1\left[2x-|3x^2-3x-2x+2I+1\right]dx$

$I=\int_0^1\left[2x-\left|(3x-2)(x-1)\right|\right]dx+\int\limits_0^1 1dx$

$I=\int_0^{2/3}\left[\left(2x-\left(3x^2-5x+2\right)\right)\right]dx+\int_{2/3}^1\left(2x+\left(3x^2-5x+2\right)\right)dx+1$

$I=\int_0^{2/3}\left[-3x^2+7x-2\right]dx+\int_{2/3}^1\left(3x^2-3x+2\right)dx+1$

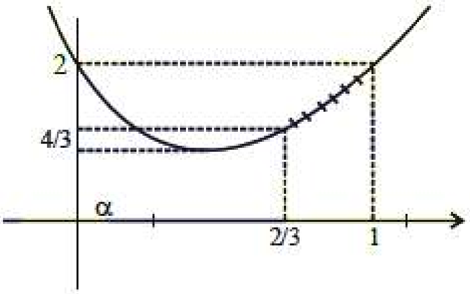
$y=-3x^2+7x-2$



$\int_0^{\alpha}(-2)dx+\int_{\alpha}^{1/3}(-1)dx+\int_{1/3}^{\beta}0dx+\int_{\beta}^{2/3}1\cdot dx$

$=-2\alpha-\left(\frac{1}{3}-\alpha\right)+\frac{2}{3}-\beta=-\alpha-\beta+\frac{1}{3}$

$y=3x^2-3x+2$



When $x\in\left(\frac{2}{3},1\right)$

$3x^2-3x+2\in\left(\frac{4}{3},2\right)$

$\left[3x^2-3x+2\right]=1$

$\therefore\int_{2/3}^1\left[3x^2-3x+2\right]dx=1\left(1-\frac{2}{3}\right)=\frac{1}{3}$

Hence $I=\left(\frac{1}{3}-(\alpha+\beta)\right)+\left(\frac{1}{3}\right)+1$

$=\frac{5}{3}-\left(\frac{7-\sqrt{37}}{6}+\frac{7-\sqrt{13}}{6}\right)$

$=\frac{-2}{3}+\frac{\sqrt{37}+\sqrt{13}}{6}$

$=\frac{\sqrt{37}+\sqrt{13}-4}{6}$

Question ID : 15477154606

Sequence & progression

6. Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 3a_{n+1} - 2a_n + 1,\ \forall\ n \geq 0$. Then $a_{25}a_{23}-2a_{25}a_{22}-2a_{23}a_{24}+4a_{22}a_{24}$ is equal to :

माना $\{a_n\}_{n=0}^{\infty}$ एक अनुक्रम है, जिसके लिए $a_0 = a_1 = 0$ तथा $a_{n+2} = 3a_{n+1} - 2a_n + 1,\ \forall\ n \geq 0$ हैं। तो $a_{25}a_{23}-2a_{25}a_{22}-2a_{23}a_{24}+4a_{22}a_{24}$ बराबर है :

(1) 483 (2) 528 (3) 575 (4) 624

Ans. Official Answer NTA (2)

Sol. $a_0=0, a_1=0$

$a_{n+2}=3a_{n+1}-2a_{n+1} : n\geq 0$

$a_{n+2}-a_{n+1}=2(a_{n+1}-a_n)+1$

$n=0 \quad a_2-a_1=2(a_1-a_0)+1$

$n=1 \quad a_3-a_2=2(a_2-a_1)+1$

$n=2 \quad a_4-a_3=2(a_3-a_2)+1$

$n=n \quad a_{n+2}-a_{n+1}=2(a_{n+1}-a_n)+1$

$(a_{n+2}-a_1)-2(a_{n+1}-a_0)-(n+1)=0$

$a_{n+2}=2a_{n+1}+(n+1)$

$n\rightarrow n-2$

$a_n-2a_{n-1}=n-1$

Now $a_{25}a_{23}-2a_{25}a_{22}-2a_{23}a_{24}+4a_{22}a_{24}$

$=(a_{25}-2a_{24})(a_{23}-2a_{22})=(24)(22)=528$

Question ID : 15477154607

P & C

7. $\sum_{r=1}^{20}\left(r^2+1\right)(r!)$ is equal to :

$\sum_{r=1}^{20}\left(r^2+1\right)(r!)$ बराबर है :

(1) 22! – 21!
(2) 22! – 2(21!)
(3) 21! – 2(20!)
(4) 21! – 20!

Ans. Official Answer NTA (2)

Sol. $\sum_{x=1}^{20}\left(r^2+1\right)r!$

$\sum_{x=1}^{20}\left((r+1)^2-2r\right)r!$

$\sum_{x=1}^{20}((r+1)(r+1)!-r\cdot r!)-\sum_{r=1}^{20}r\cdot r!$

$\sum_{x=1}^{20}((r+1)(r+1)!-r\cdot r!)-\sum_{r=1}^{20}((r+1)!-r!)$

$=(21\cdot\underline{|21}-1)-(\underline{|21}-1)$

$=20\cdot 21!=22!-2.21!$

Question ID : 15477154608

Indefinite Integration

8. For $I(x)=\int\frac{\sec^2 x-2022}{\sin^{2022}x}dx$, if $I\left(\frac{\pi}{4}\right)=2^{1011}$, then :

$I(x)=\int\frac{\sec^2 x-2022}{\sin^{2022}x}dx$ के लिए, यदि $I\left(\frac{\pi}{4}\right)=2^{1011}$ है, तो :

(1) $3^{1010}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$
(2) $3^{1010}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$
(3) $3^{1011}I\left(\frac{\pi}{3}\right)-I\left(\frac{\pi}{6}\right)=0$
(4) $3^{1011}I\left(\frac{\pi}{6}\right)-I\left(\frac{\pi}{3}\right)=0$

Ans. Official Answer NTA (1)

Sol. $I(x)=\int\sec^2 x\cdot\sin^{-2022}x\,dx-2022\int\sin^{-2022}x\,dx$

$=\tan x\cdot(\sin x)^{-2022}+\int(2022)\tan x\cdot(\sin x)^{-2023}\cos x\,dx$

$-2022\int(\sin x)^{-2022}dx$

$I(x)=(\tan x)(\sin x)^{-2022}+C$

At $X=\pi/4, 2^{1011}=\left(\frac{1}{\sqrt{2}}\right)^{-2022}+C \therefore C=0$

Hence $I(x)=\frac{\tan x}{(\sin x)^{2022}}$

$I(\pi/6)=\frac{1}{\sqrt{3}\left(\frac{1}{2}\right)^{2022}}=\frac{2^{2022}}{\sqrt{3}}$

$I(\pi/3)=\frac{\sqrt{3}}{\left(\frac{\sqrt{3}}{2}\right)^{2022}}=\frac{2^{2022}}{(\sqrt{3})^{2021}}=\frac{1}{3^{1010}}I\left(\frac{\pi}{6}\right)$

$3^{1010}I(\pi/3)=I(\pi/6)$

Question ID : 15477154609

Differential Equation

9. If the solution curve of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passes through the points (2, 1) and (k + 1, 2), k > 0, then :

यदि अवकल समीकरण $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ का हल वक्र बिन्दुओं (2, 1) तथा (k + 1, 2), k > 0, से होकर जाता है, तो :

(1) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$
(2) $\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(k^2+1\right)$
(3) $2\tan^{-1}\left(\frac{1}{k+1}\right)=\log_e\left(k^2+2k+2\right)$
(4) $2\tan^{-1}\left(\frac{1}{k}\right)=\log_e\left(\frac{k^2+1}{k^2}\right)$

Ans. Official Answer NTA (1)

Sol. $\frac{dy}{dx}=\frac{x+y-2}{x-y}=\frac{(x-1)+(y-1)}{(x-1)-(y-1)}$

$x-1=X, y-1=Y$

$\frac{dY}{dX}=\frac{X+Y}{X-Y}$

$Y = VX \quad \frac{dY}{dX} = V + X\frac{dV}{dX}$

$V + X\frac{dV}{dX} = \frac{1+V}{1-V} \qquad x\frac{dV}{dX} = \frac{v^2+1}{1-v}$

$\int \frac{1-V}{1+V^2}dV = \int \frac{dX}{X}$

$\int \frac{dV}{1+V^2} - \frac{1}{2}\int \frac{2VdV}{1+V^2} = \int \frac{dX}{XX}$

$\tan^{-1} V - \frac{1}{2}\ln\left(1+V^2\right) = \ln X + c$

$\tan^{-1}\left(\frac{Y}{X}\right) - \frac{1}{2}\ln\left(1+\frac{Y^2}{X^2}\right) = \ln(X) + c$

$\tan^{-1}\left(\frac{y-1}{x-1}\right) - \frac{1}{2}\ln\left(1+\frac{(y-1)^2}{(x-1)^2}\right) = \ln(x-1) + c$

Passes through (2, 1)

$\therefore \tan^{-1}\left(\frac{1}{k}\right) - \frac{1}{2}\text{In}\left(1+\frac{1}{k^2}\right) = \ln k$

$2\tan^{-1}\left(\frac{1}{k}\right) = \text{In}\left(\frac{1+k^2}{k^2}\right) + 2\ln k$

$2\tan^{-1}\left(\frac{1}{k}\right) = \text{In}\left(1+k^2\right)$

Question ID : 154771546010

Differential Equation

10. Let y = y(x) be the solution curve of the differential equation $\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right) y = \frac{(x+3)}{x+1}$, $x > -1$, which passes through the point (0, 1). Then y(1) is equal to :

माना $\frac{dy}{dx} + \left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right) y = \frac{(x+3)}{x+1}$, $x < -1$ का हल वक्र $y = y(x)$, बिन्दु (0, 1) से होकर जाता है। तो y(1) बराबर है :

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $\frac{5}{2}$ (4) $\frac{7}{2}$

Ans. Official Answer NTA (2)

Sol. $\frac{dy}{dx}+\left(\frac{2x^2+11x+13}{x^3+6x^2+11x+6}\right)y=\frac{x+3}{x+1}$

$\int p(x)dx \quad$ I.F. $=e^{\int p(x)dx}$

$\int p(x)dx=\int\frac{(2x^2+11x+13)dx}{(x+1)(x+2)(x+3)}$

Using partial fraction

$\frac{2x^2+11x+13}{(x+1)(x+2)(x+3)}=\frac{A}{x+1}+\frac{B}{x+2}+\frac{C}{x+3}$

$A=\frac{4}{2}=2$

$B=1$

$C=-1$

$\because \int p(x)dx=A\ln(x+1)+B\ln(x+2)+c\ln(x+3)$

$=\ln\left(\frac{(x+1)^2(x+2)}{x+3}\right)$

I.F. $=e^{\int p(x)dx}=\frac{(x+1)^2(x+2)}{(x+3)}$

Solution $y(IF)=\int Q\cdot(IF)dx$

$y\left(\frac{(x+1)^2(x+2)}{x+3}\right)=\int\left(\frac{x+3}{x+1}\right)\frac{(x+1)\,(x+2)}{(x+3)}dx$

$y\left(\frac{(x+1)^2(x+2)}{x+3}\right)=\frac{x^3}{3}+\frac{3x^2}{2}+2x+c$

Passes through (0, 1) $C=\frac{2}{3}$

$\Rightarrow y(1)=\frac{3}{2}$

Question ID : 154771546011

Straight Line

11. Let m_1, m_2 be the slopes of two adjacent sides of a square of side a such that $a^2+11a+3(m_1^2+m_2^2)=220$.

If one vertex of the square is $(10(\cos\alpha - \sin\alpha), 10(\sin\alpha + \cos\alpha))$, where $\alpha \in \left(0, \frac{\pi}{2}\right)$ and the equation of one diagonal is $(\cos\alpha - \sin\alpha)x + (\sin\alpha + \cos\alpha)y = 10$, then $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$ is equal to :

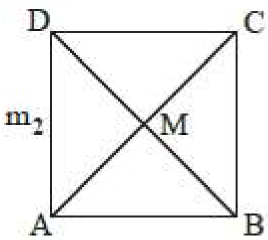
माना भुजा a के एक वर्ग की संलग्न भुजाओं की प्रवणताऐं m_1, m_2 हैं तथा $a^2 + 11a + 3\left(m_1^2 + m_2^2\right) = 220$ हैं। यदि वर्ग का एक शीर्ष $(10(\cos\alpha - \sin\alpha), 10(\sin\alpha + \cos\alpha))$, $\alpha \in \left(0, \frac{\pi}{2}\right)$ है तथा एक विकर्ण का समीकरण $(\cos\alpha - \sin\alpha)x + (\sin\alpha + \cos\alpha)y = 10$ है, तो $72(\sin^4\alpha + \cos^4\alpha) + a^2 - 3a + 13$ बराबर है :

(1) 119 (2) 128 (3) 145 (4) 155

Ans. Official Answer NTA (2)

Sol. $m_1 m_2 = -1$

$$a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1^2}\right) = 220$$



Eq. of AC

$AC = (\cos\alpha - \sin\alpha) + (\sin\alpha + \cos\alpha)y = 10$

$BD = (\sin\alpha - \cos\alpha)x + (\sin\alpha - \cos\alpha)y = 0$

$(10(\cos\alpha - \sin\alpha), 10(\sin\alpha - \cos\alpha))$

$$\text{Slope of AC} = \left(\frac{\sin\alpha - \cos\alpha}{\sin\alpha + \cos\alpha}\right) = \tan\theta = M$$

Eq. of line making an angle π_4 with AC

$$m_1, m_2 = \frac{m \pm \tan\frac{\pi}{4}}{1 \pm m\tan\frac{\pi}{4}}$$

$$= \frac{m+1}{1-m} \text{ or } \frac{m-1}{1+m}$$

$$\frac{\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}+1}{1-\left(\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}\right)}, \frac{\sin\alpha-\cos\alpha}{1+\frac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}}$$

$m_1, m_2 = \tan\alpha, \cot\alpha$

mid point of AC & BD

$= M(5(\cos\alpha-\sin\alpha), 5(\cos\alpha+\sin\alpha))$

$B(10(\cos\alpha-\sin\alpha), 10(\cos\alpha+\sin\alpha))$

$a = AB = \sqrt{2}BM = \sqrt{2}(5\sqrt{2}) = 10$

$a = 10$

$$\because a^2 + 11a + 3\left(m_1^2 + \frac{1}{m_1 2}\right) = 220$$

$$100 + 110 + 3\left(\tan^2\alpha + \cot^2\alpha\right) = 220$$

Hence $\tan^2\alpha = 3, \tan^2\alpha = \frac{1}{3} \Rightarrow \alpha = \frac{\pi}{3}$ or $\frac{\pi}{6}$

Now $72\left(\sin^4\alpha + \cos^4\alpha\right) + a^2 - 3a + 13$

$$= 72\left(\frac{9}{16} + \frac{1}{16}\right) + 100 - 30 + 13$$

$$= 72\left(\frac{5}{8}\right) + 83 = 45 + 83 = 128$$

Question ID : 154771546012

Trigonometric Equation

12. The number of elements in the set $S\left\{x \in R : 2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}\right\}$ is :

समुच्चय $S\left\{x \in R : 2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}\right\}$ में अवयवों की संख्या है :

(1) 1 (2) 3 (3) 0 (4) infinite (अनंत)

Ans. Official Answer NTA (1)

Sol. $2\cos\left(\frac{x^2+x}{6}\right) = 4^x + 4^{-x}$

L.H.S ≤ 2. & R.H.S. ≥ 2

Hence L.H.S = 2 & R.H.S = 2

$$2\cos\left(\frac{x^2+x}{6}\right)=24^x+4^{-x}=2$$

Check x = 0 Possible hence only one solution.

Question ID : 154771546013

Straight Line

13. Let $A(\alpha, -2)$, $B(\alpha, 6)$ and $C\left(\frac{\alpha}{4}, -2\right)$ be vertices of a ΔABC. If $\left(5, \frac{\alpha}{4}\right)$ is the circumcentre of ΔABC, then which of the following is NOT correct about ΔABC?

(1) area is 24 (2) perimeter is 25 (3) circumradius is 5 (4) inradius is 2

माना $A(\alpha, -2)$, $B(\alpha, 6)$ तथा $C\left(\frac{\alpha}{4}, -2\right)$ एक त्रिभुज ABC के शीर्ष हैं। यदि ΔABC का परिकेन्द्र $\left(5, \frac{\alpha}{4}\right)$ है, तो इस त्रिभुज के लिए निम्न में से कौनसा सही नहीं ?

(1) क्षेत्रफल 24 है (2) परिमाप 25 है (3) परिवृत्त की त्रिज्या 5 है (4) अंतवृत्त की त्रिज्या 2

Ans. Official Answer NTA (2)

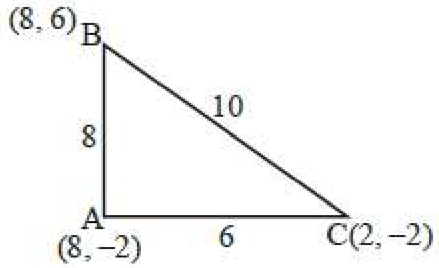
Sol. $A(\alpha,-2):B(\alpha,6):C\left(\frac{\alpha}{4},-2\right)$

since AC is perpendicular to AB.
So, ΔABC is right angled at A.

Circumcentre = mid point of BC. $=\left(\frac{5\alpha}{8},2\right)$

$\therefore \frac{5\alpha}{8}=5$ & $\frac{\alpha}{4}=2$

$\alpha = 8$



Area $=\frac{1}{2}(6)(8)=24$

Perimeter = 24
Circumradius = 5

Inradius $= \frac{\Delta}{s} = \frac{24}{12} = 2$

Question ID : 154771546014

3D Geometry

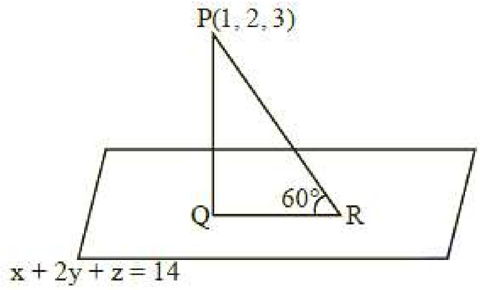
14. Let Q be the foot of perpendicular drawn from the point P(1, 2, 3) to the plane $x + 2y + z = 14$. If R is a point on the plane such that $\angle OPRQ = 60°$, then the area of ΔPQR is equal to :

माना बिन्दु P(1, 2, 3) से समतल $x + 2y + z = 14$ पर डाले गए लंब का पाद Q है। माना समतल पर बिन्दु R के लिए $\angle OPRQ = 60°$ है, तो ΔPQR का क्षेत्रफल बराबर :

(1) $\frac{\sqrt{3}}{2}$ (2) $\sqrt{3}$ (3) $2\sqrt{3}$ (4) 3

Ans. Official Answer NTA (2)

Sol.



Length of perpendicular

$$PQ = \left|\frac{1+4+3-14}{\sqrt{6}}\right| = \sqrt{6}$$

$$QR = (PQ)\cot 60° = \sqrt{2}$$

$$\therefore \text{ Area of } \Delta PQR = \frac{1}{2}(PQ)(QR) = \sqrt{3}$$

Question ID : 154771546015

Vectors

15. If (2, 3, 9), (5, 2, 1), (1, λ, 8) and (λ, 2, 3) are coplanar, then the product of all possible values of λ is :

यदि बिन्दु (2, 3, 9), (5, 2, 1), (1, λ, 8) तथा (λ, 2, 3) सह–तलीय है, तो λ के सभी संभव मानों का गुणनफल है :

(1) $\frac{21}{2}$ (2) $\frac{59}{8}$ (3) $\frac{57}{8}$ (4) $\frac{95}{8}$

Ans. Official Answer NTA (4)

Sol. $A(2,3,9); B(5,2,1); C(1,\lambda,8); D(\lambda,2,3)$

$$\left[\overrightarrow{AB} \quad \overrightarrow{AC} \quad \overrightarrow{AD}\right] = 0$$

$$\begin{vmatrix} 3 & -1 & -8 \\ -1 & \lambda-3 & -1 \\ \lambda-2 & -1 & -6 \end{vmatrix} = 0$$

$$\Rightarrow [-6(\lambda-3)-1]-8(1-(\lambda-3)(\lambda-2))+(6+(\lambda-2)=0$$

$$3(-6\lambda+17)-8\left(-\lambda^2+5\lambda-5\right)+(\lambda+4)=8$$

$$8\lambda^2-57\lambda+95=0$$

$$\lambda_1\lambda_2=\frac{95}{8}$$

Question ID : 154771546016

Probability

16. Bag I contains 3 red, 4 black and 3 white balls and Bag II contains 2 red, 5 black and 2 white balls. One ball is transferred from Bag I to Bag II and then a ball is drawn from Bag II. The ball so drawn is found to be black in colour. Then the probability, that the transferred ball is red, is :

थैले I में 3 लाल, 4 काली तथा 3 सफेद गेंद है तथा थैले II में 2 लाल, 5 काली तथा 2 सफेद गेंद में है। थैले I में से एक गेंद थैले II में स्थानांतरित की जाती है और तब थैले II से एक गेंद निकाली जाता है। इस प्रकार निकाली गई गेंद का रंग काला है। तो स्थानान्तरित गेंद का रंग लाल होने की प्रायिकता है :

(1) $\frac{4}{9}$ (2) $\frac{5}{18}$ (3) $\frac{1}{6}$ (4) $\frac{3}{10}$

Ans. Official Answer NTA (2)

Sol.

Bag I	Bag II
3R	2R
4B	5B
3W	2W

A : Drown ball from boy II is black
B : Red ball transferreed

$$P\left(\frac{B}{A}\right)=\frac{P(A\cap B)}{P(A)}$$

$$=\frac{\frac{3}{9}\times\frac{5}{10}}{\frac{3}{9}\times\frac{5}{10}+\frac{4}{9}\times\frac{6}{10}+\frac{3}{9}\times\frac{5}{10}}$$

$$=\frac{15}{15+24+15}=\frac{15}{54}=\frac{5}{18}$$

Question ID : 154771546017

Complex Number

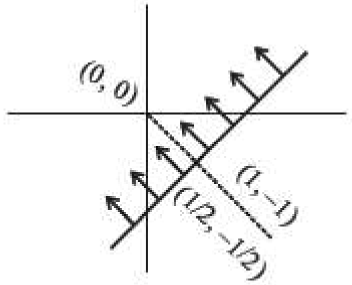
17. Let S = {z = x + *i*y : |z – 1 + *i*| ≥ |z|, |z| < 2, |z + *i*| = |z – 1|}. Then the set of all values of x, for which w = 2x + *i*y ∈ S for some y ∈ R, is :

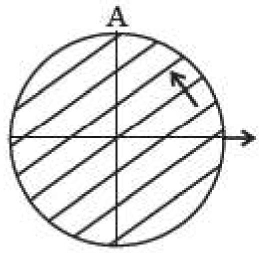
माना S = {z = x + *i*y : |z – 1 + *i*| ≥ |z|, |z| < 2, |z + *i*| = |z – 1|}है। तो x के उन सभी मानों, जिसके लिए किसी y ∈ R के लिए w = 2x + *i*y ∈ S है, का समुच्चय है :

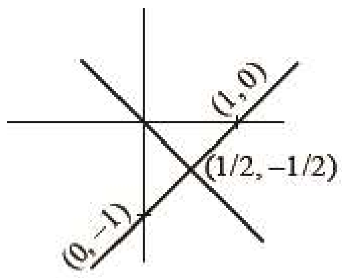
(1) $\left(-\sqrt{2}, \frac{1}{2\sqrt{2}}\right]$ (2) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{4}\right]$ (3) $\left(-\sqrt{2}, \frac{1}{2}\right]$ (4) $\left(-\frac{1}{\sqrt{2}}, \frac{1}{2\sqrt{2}}\right]$

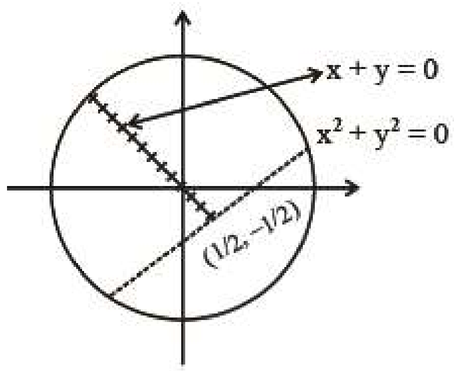
Ans. Official Answer NTA (2)

Sol. $|z-1+i| \geq |z|$; $|z| < 2$; $|z+i| = |z-1|$









Hence

$w = 2x + iy \in S$

$2x \le \frac{1}{2} \quad \therefore x \le \frac{1}{4}$

Now

$(2x)^2 + (2x)^2 < 4$

$x^2 < \frac{1}{2} \Rightarrow x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

$\therefore x \in \left(\frac{-1}{\sqrt{2}}, \frac{1}{4}\right]$

Question ID : 154771546018

Vectors

18. Let $\vec{a}, \vec{b}, \vec{c}$ be three coplanar concurrent vectors such that angles between any two of them is same. If the product of their magnitudes is 14 and $(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c}) + (\vec{b}\times\vec{c}).(\vec{c}\times\vec{a}) + (\vec{c}\times\vec{a}).(\vec{a}\times\vec{b}) = 168$, then $|\vec{a}| + |\vec{b}| + |\vec{c}|$ is equal to :

माना तीन सहतलीय संगामी सदिश $\vec{a}, \vec{b}, \vec{c}$ इस प्रकार हैं कि किन्हीं भी दो भी बीच के कोण बराबर हैं। यदि इन सदिशों के परिमाणों का गुणनफल 14 है तथा $(\vec{a}\times\vec{b}).(\vec{b}\times\vec{c}) + (\vec{b}\times\vec{c}).(\vec{c}\times\vec{a}) + (\vec{c}\times\vec{a}).(\vec{a}\times\vec{b}) = 168$ है, तो $|\vec{a}| + |\vec{b}| + |\vec{c}|$ बराबर है :

(1) 10 (2) 14 (3) 16 (4) 18

Ans. Official Answer NTA (3)

Sol. $\vec{a} \,||\, \vec{b} \,||\, \vec{c}| = 14$

$\vec{a} \,\hat{}\, \vec{b} = \vec{b} \wedge \vec{c} = \vec{c} \,\hat{}\, \vec{a} = \theta = \frac{2\pi}{3}$

So, $\vec{a}\cdot\vec{b} = -\frac{1}{2}ab, \vec{b}\cdot\vec{c} = -\frac{1}{2}bc, \vec{a}\cdot\vec{c}. = -\frac{1}{2}ac$

(let)

$(\vec{a}\times\vec{b})\cdot(\vec{b}\times\vec{c}) = (\vec{a}\cdot\vec{b})(\vec{b}\cdot\vec{c}) - (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{b})$

$= \frac{1}{4}ab^2c + \frac{1}{2}ab^2c = \frac{3}{4}ab^2c$

Similarly

$(\vec{b}\times\vec{c})\cdot(\vec{c}\times\vec{a}) = \frac{3}{4}abc^2$

$(\vec{c}\times\vec{a})\cdot(\vec{a}\times\vec{b})=\frac{3}{4}a^2bc$

$168=\frac{3}{4}abc(a+b+c)$

So, (a + b + c) = 16

Question ID : 154771546019

ITF

19. The domain of the function $f(x)=\sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ is :

फलन $f(x)=\sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ का प्रांत है :

(1) $[1,\infty)$ (2) $[-1,2]$ (3) $[-1,\infty)$ (4) $(-\infty,2]$

Ans. Official Answer NTA (3)

Sol. $f(x)=\sin^{-1}\left(\frac{x^2-3x+2}{x^2+2x+7}\right)$ Domain

$\frac{x^2-3x+2}{x^2+2x+7}\geq -1$ and $\frac{x^2-3x+2}{x^2+2x+7}\leq 1$

$2x^2-x+9\geq 0$ and $5x\geq -5\Rightarrow x\geq -1$

$x\in R$

Hence Domain $x\in[-1,\infty)$

Question ID : 154771546020

Mathematical Reasoning

20. The statement $(p\Rightarrow q)\vee(p\Rightarrow r)$ is NOT equivalent to :

कथन $(p\Rightarrow q)\vee(p\Rightarrow r)$ निम्न में से किस तुल्य नहीं है :

(1) $(p\wedge(\sim r))\Rightarrow q$ (2) $(\sim q)\Rightarrow((\sim r)\vee p)$

(3) $p\Rightarrow(q\vee r)$ (4) $(p\wedge(\sim q))\Rightarrow r$

Ans. Official Answer NTA (2)

Sol. $(p\to q)\vee(p\to r)$

$(\sim p\vee q)\vee(\sim p\vee r)$

$=\sim p\vee(q\vee r)$

$=p\to(q\vee r)\equiv(3)$ is true.

Now (1) $(p\wedge\sim r)\to q$

$\sim(p\wedge\sim r)\vee q=(\sim p\vee r)\vee q$

$=\sim p \vee (r \vee q) = p \rightarrow (q \vee r)$

(4) $(p \wedge \sim q) \rightarrow r = p \rightarrow (q \vee r)$

SECTION – B

Question ID : 154771546021

Statistics

21. The sum and product of the mean and variance of a binomial distribution are 82.5 and 1350 respectively. Then the number of trials in the binomial distributions is _________.

एक द्विपद बंटन के माध्य तथा प्रसरण का योग तथा गुणनफल क्रमश: 82.5 तथा 1350 हैं। तो द्विपद बंटन में परीक्षणों की संख्या है ________।

Ans. Official Answer NTA (96)

Sol. Let, mean = m = np

& variance = v = npq, p + q = 1

$\text{Sum } = m + v = \frac{165}{2}$

Product = mv = 1350

On solving,

$m = np = 60 \text{ \& } v = npq = \frac{45}{2} \therefore q = \frac{3}{8} \therefore P = \frac{5}{8}$

Hence n = 96

Question ID : 154771546022

Quadratic Equation

22. Let α, β $(\alpha > \beta)$ be the roots of the quadratic equation $x^2 - x - 4 = 0$. If $P_n = \alpha^n - \beta^n$, $n \in N$, then $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ is equal to _________.

माना $x^2 - x - 4 = 0$ के मूल $\alpha, \beta\ (\alpha > \beta)$ हैं। यदि $P_n = \alpha^n - \beta^n, n \in N$ है, तो $\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ बराबर है________।

Ans. Official Answer NTA (16)

Sol. $P_n = \alpha^n - \beta^n \qquad x^2 - x - 4 = 0$

$\frac{P_{15}P_{16} - P_{14}P_{16} - P_{15}^2 + P_{14}P_{15}}{P_{13}P_{14}}$ .....(1)

As $P_n - P_{n-1} = \left(\alpha^n - \beta^n\right) - \left(\alpha^{n-1} - \beta^{n-1}\right)$

$= \alpha^{n-2}\left(\alpha^2 - \alpha\right) - \beta^{n-2}\left(\beta^2 - \beta\right)$

$= 4\left(\alpha^{n-2} - \beta^{n-2}\right)$

$P_n - P_{n-1} = 4P_{n-2}$

Hence Expression (1)

$$\frac{P_{16}\left(P_{15} - P_{14}\right) - P_{15}\left(P_{15} - P_{14}\right)}{P_{13}P_{14}}$$

$$= \frac{\left(P_{15} - P_{14}\right)\left(P_{16} - P_{15}\right)}{P_{13}P_{14}} = \frac{\left(4P_{13}\right)\left(4P_{14}\right)}{P_{13}P_{14}} = 16$$

Question ID : 154771546023

Matrices

23. Let $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ and $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$. For $k \in N$, If $X' A^k X = 33$, then k is equal to _________.

माना $x = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ तथा $A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$ हैं। $k \in N$ के लिए, यदि $X' A^k X = 33$ है, तो k बराबर है_______।

Ans. Official Answer NTA (10)

Sol. $X = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}; A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}$

$X^T A^K X = 33$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix}^k \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

$$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$$

As $A^2 = \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^8 = \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{10} = \begin{bmatrix} 1 & 0 & 6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 24 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 30 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

for $K \to$ Even $A^K = \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$X^T A^K X = 33$ (This is not correct)

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3K \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & 1 & 3K+1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = [3K+3]$

$\therefore 3K + 3 = 33 \therefore K = 10$

But it should be dropeed as 33 is not matrix

If K is odd

$X^T A^K X = 33$

$X^T A A A^{K-1} X = 33$

$\begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} -1 & 2 & 3 \\ 0 & 1 & 6 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 3k-3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = 33$

$$\begin{bmatrix}-1 & 3 & 8\end{bmatrix}\begin{bmatrix}3k-2\\1\\1\end{bmatrix}=[33]$$

$[-3k+13]=[33]$

$k = 20/3$ (not possible)

Question ID : 154771546024

P & C

24. The number of natural numbers lying between 1012 and 23421 that can be formed using the digits 2, 3, 4, 5, 6 (repetition of digits is not allowed) and divisible by 55 is _________.

अंकों की पुनरावृत्ति के बिना, अंकों 2, 3, 4, 5, 6 के प्रयोग से 1012 तथा 23421 के बीच बनाई जा सकने वाली उन संख्याओं, जो 55 से विभाज्य हैं, की संख्या है ________।

Ans. Official Answer NTA (6)

Sol. 4 digit numbers

For divisibility by 55, no. should be

div. by 5 and 11 both

			5
a	b	c	d

Also, for divisibility by 11

$a + c = b + 5$

for $b = 1$ $a = 2, c = 4$

$a = 4, c = 2$

for $b = 2$ $a = 3, c = 4$

$a = 4, c = 3$

for $b = 3$ $a = 6, c = 2$

$a = 2, c = 6$

$\therefore$ 6 possible four digit no.s are div. by 55

(II) 5 digit number is not possible

(Not possible)

Question ID : 154771546025

Binomial Theorem

25. If $\sum_{k=1}^{10} K^2 \left(10_{C_k}\right)^2 = 22000L$, then L is equal to _________.

यदि $\sum_{k=1}^{10} K^2 \left(10_{C_k}\right)^2 = 22000L$ है, तो L बराबर _______ ।

Ans. Official Answer NTA (221)

Sol. $\sum_{K=1}^{10} K^2 \left({}^{10}C_K\right)^2$

$\sum_{K=1}^{10}\left(K \cdot {}^{10}C_K\right)^2 = \sum_{K=1}^{10}\left(10 \cdot {}^{9}C_{K-1}\right)^2$

$= 100\sum_{K=1}^{10} {}^{9}C_{K-1} \cdot {}^{9}C_{10-K}$

$= 100\left({}^{18}C_9\right) = 100\left(\frac{18!}{9!9!}\right)$

$\Rightarrow 4862000 = 22000L$

Hence $L = 221$

Question ID : 154771546026

Continuity & Differentiability

26. If [t] denotes the greatest integer ≤ t, then the number of points, at which the function $f(x) = 4|2x+3| + 9\left[x+\frac{1}{2}\right] - 12[x+20]$ is not differentiable in the open interval (–20, 20), is ________.

यदि [t] महत्तम पूर्णांक ≤ t है, तो उन बिन्दुओं, जिन पर फलन $f(x) = 4|2x+3| + 9\left[x+\frac{1}{2}\right] - 12[x+20]$ विवृत अंतराल (–20, 20) में अवकलनीय नहीं है, कि संख्या________.

Ans. Official Answer NTA (79)

Sol. $f(x) = 4|2x+3| + 9\left[x+\frac{1}{2}\right] - 12[x+20]$

$x \in (-20, 20)$

f(x) is not Diff. at $x = I \in \{-19, -18, \ldots\ldots 19\} = 39$

at $x = I + \frac{1}{2}$, $f(x)$ Not diff. at 39 points

Check at $x = \frac{-3}{2}$ Discount at $x = \frac{-3}{2} \therefore$ N.R(1)

No. of point of non-differentiabilty

= 39 + 39 + 1 = 79

Question ID : 154771546027

Tangent and normal

27. If the tangent to the curve $y = x^3 - x^2 + x$ at the point (a, b) is also tangent to the curve $y = 5x^2 + 2x - 25$ at the point (2, –1), then |2a + 9b| is equal to ________.

यदि वक्र $y = x^3 - x^2 + x$ के बिन्दु (a, b) पर स्पर्श रेखा, वक्र $y = 5x^2 + 2x - 25$ की भी बिन्दु (2, –1) पर स्पर्श रेखा है, तो |2a + 9b| बराबर ________ है।

Ans. Official Answer NTA (195)

Sol. $y = 5x^2 + 2x - 25$ P(2, –1)

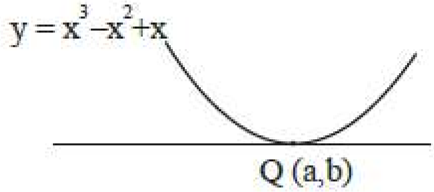
$y' = 10x + 2$

$y'_P = 22$

$\therefore$ tangent to curve at P

$y + 1 = 22(x - 2)$

$y = 22x - 45$



$$\left.\frac{dy}{dx}\right|_{C_2} = 3x^2 - 2x + 1$$

$$\left.\frac{dy}{dx}\right|_{Q} = 3a^2 - 2a + 1$$

Hence $3a^2 - 2a + 1 = 22$

$\therefore 3a^2 - 2a - 21 = 0$

$3a^2 - 9a + 7a - 21 = 0$

$(3a + 7)(a - 3) = 0$ < $a = 3$, $a = -7/3$

from curve $b = a^3 - a^2 + a$

$a = 3$

$b = 21$ $\qquad |2a + 9b| = 195$

at $a = -7/3$ tangent will be parallel

Hence it is rejected

Question ID : 154771546028

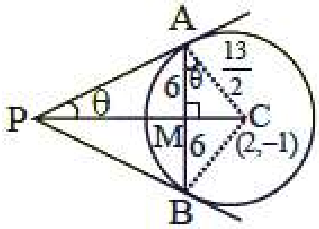
Circle

28. Let AB be a chord of length 12 of the circle $(x-2)^2 + (y+1)^2 = \frac{169}{4}$. If tangents drawn to the circle at points A and B intersect at the point P, then five times the distance of point P from chord AB is equal to ________.

माना वृत्त $(x-2)^2 + (y+1)^2 = \frac{169}{4}$ की एक जीवा AB की लंबाई 12 है। यदि A तथा B पर खींची गई वृत्त की स्पर्श रेखाएँ बिन्दु P पर मिलती हैं, तो बिन्दु P की जीवा AB से दूरी का पाँच गुना बराबर है ________।

Ans. Official Answer NTA (72)

Sol.



$\cos\theta = \dfrac{6}{\frac{13}{2}} = \dfrac{12}{13}$

$\sin\theta = \dfrac{5}{13}$

$PM = AM\cot\theta$

$PM = 6\left(\dfrac{12}{5}\right) \therefore 5(PM) = 72$

Question ID : 154771546029

Vectors

29. Let $\vec{a}$ and $\vec{b}$ be two vectors such that $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$, $\vec{a}.\vec{b} = 3$ and $|\vec{a}\times\vec{b}|^2 = 75$. Then $|\vec{a}|^2$ is equal to _________.

माना सदिशों $\vec{a}$ तथा $\vec{b}$ के लिए $|\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2$, $\vec{a}.\vec{b} = 3$ तथा $|\vec{a}\times\vec{b}|^2 = 75$ हैं, तो $|\vec{a}|^2$ बराबर है ____।

Ans. Official Answer NTA (14)

Sol. $\vec{a}+\vec{b}|^2 = |\vec{a}|^2 + 2|\vec{b}|^2; \vec{a}\cdot\vec{b} = 3$

As $|\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{a}|^2 + 2|\vec{b}|^2$

$|\vec{b}|^2 = 2\vec{a}\cdot\vec{b} = 6$

$|\vec{a}\times\vec{b}|^2 = 75$

$|\vec{a}|^2|\vec{b}|^2 - (\vec{a}\cdot\vec{b})^2 = 75$

$6|\vec{a}|^2 - 9 = 75 \Rightarrow |\vec{a}|^2 = 14$

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Set & Relations

30. Let $S = \{(x, y) \in N \times N : 9(x-3)^2 + 16(y-4)^2 \le 144\}$ and $T = \{(x, y) \in R \times R : (x-7)^2 + (y-4)^2 \le 36\}$. Then $n(S \cap T)$ is equal to _________.

माना $S = \{(x, y) \in N \times N : 9(x-3)^2 + 16(y-4)^2 \le 144\}$ तथा $T = \{(x, y) \in R \times R : (x-7)^2 + (y-4)^2 \le 36\}$ हैं। तो $n(S \cap T)$ बराबर ________ है।

Ans. Official Answer NTA (27)

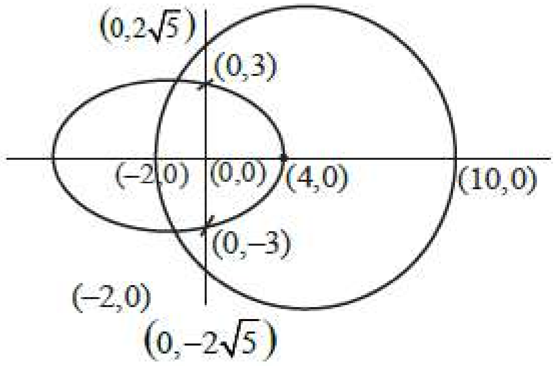
Sol. $S:\dfrac{(x-3)^2}{16}+\dfrac{(y-4)^2}{9}\le 1; x, y \in \{1, 2, 3, \dots\dots\}$

$T:(x-7)^2+(y-4)^2 \le 36x, y \in R$

Let $x-3=x : y-4=y$

$S:\dfrac{x^2}{16}+\dfrac{y^2}{9}\le 1; x \in \{-2, -1, 0, 1, \dots\dots\}$

$T:\left(x-4\right)^2+y^2 \le 36\ ;\ y \in \left\{-3, -2, -1, 0, \dots..\right\}$



$S \cap T$ = (–2, 0), (–1, 0), ...(4, 0) $\rightarrow$ (7)
(–1, 1), (0, 1), ...(3, 1) $\rightarrow$ (5)
(–1, –1), (0, –1), ...(3, –1) $\rightarrow$ (5)
(–1, 2), (0, 2), (1, 2), (2, 2) $\rightarrow$ (4)
(–1, –2), (0, –2), (1, –2), (2, –2) $\rightarrow$ (4)
(0, 3) (0, –3) $\rightarrow$ (2) ....