JEE Main January 2023

Question Paper With Text Solution

29 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911

Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN JANUARY 2023 | 29TH JANUARY SHIFT-1

SECTION – A

Question ID : 3666942080

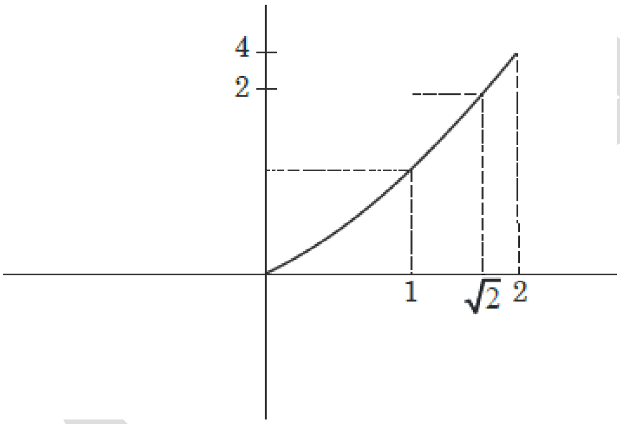
1. Let [x] denote the greatest integer $\le x$. Consider the function $f(x) = \max\{x^2, 1+[x]\}$. Then the value of the integral $\int_0^2 f(x)dx$ is

माना [x] महत्तम पूर्णांक $\le x$ है। माना फलन $f(x) = \max\{x^2, 1+[x]\}$ है। तब समाकलन $\int_0^2 f(x)dx$ का मान है :

(1) $\frac{8+4\sqrt{2}}{3}$ (2) $\frac{5+4\sqrt{2}}{3}$ (3) $\frac{1+5\sqrt{2}}{3}$ (4) $\frac{4+5\sqrt{2}}{3}$

Ans. Official Answer NTA (2)

Sol.



$$A = \int_0^1 1.dx + \int_1^{\sqrt{2}} 2dx + \int_{\sqrt{2}}^2 x^2 dx$$

$$= 1 + 2\sqrt{2} - 2 + \frac{8}{3} - \frac{2\sqrt{2}}{3}$$

$$\frac{5}{3} + \frac{4\sqrt{2}}{3}$$

Question ID : 3666942085

2. Three rotten apples are mixed accidently with seven good apples and four apples are drawn one by one without replacement. Let the random variable X denote the number of rotten apples. If μ and σ^2 represent

mean and variance of X, respectively, then $10(\mu^2+\sigma^2)$ is equal to

तीन गले हुए सेब सात अच्छे सेब के साथ संयोगवश मिल जाते है और बिना प्रतिस्थापना के एक–एक करके चार सेब निकाले जाते है। माना चादृच्छिक चर X सेबों की संख्या को दर्शाता है। यदि X के माध्य एवं प्रसरण क्रमशः μ एवं σ^2 है, तब $10(\mu^2+\sigma^2)$ बराबर है :

(1) 250 (2) 30 (3) 20 (4) 25

Ans. Official Answer NTA (3)

Sol.

X_i	0	1	2	3
p_i	$\frac{35}{210}=\frac{1}{6}$	$\frac{105}{210}=\frac{1}{2}$	$\frac{3\times21}{210}=\frac{3}{10}$	$\frac{7}{210}=\frac{1}{30}$

$$\mu=\Sigma\, p_i x_i=\frac{1}{2}\times1+\frac{3}{10}\times2+\frac{1}{30}\times3=\frac{1}{2}+\frac{3}{5}+\frac{1}{10}=\frac{5+6+1}{10}=\frac{6}{5}$$

$$\sigma^2=\Sigma p_i x_i^2-\mu^2=\frac{1}{2}+\frac{3}{10}\times4+\frac{1}{30}\times9-\frac{36}{25}=\frac{14}{25}$$

$$10\left(\mu^2+\sigma^2\right)=10\left(\frac{36}{25}+\frac{14}{25}\right)=20$$

Question ID : 3666942078

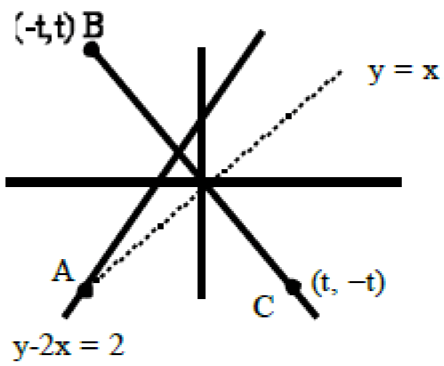
3. Let B and C be the two points on the line $y+x=0$ such that B and C are symmetric with respect to the origin. Suppose A is a point on $y-2x=2$ such that ΔABC is an equilateral triangle. Then, the area of the ΔABC is

माना रेखा $y+x=0$ पर दो बिंदु B व C मूल बिंदु के सममित है। माना $y-2x=2$ पर एक बिंदु A इस प्रकार है कि ΔABC एक समबाहु त्रिभुज है। तब ΔABC का क्षेत्रफल है :

(1) $3\sqrt{3}$ (2) $2\sqrt{3}$ (3) $\frac{8}{\sqrt{3}}$ (4) $\frac{10}{\sqrt{3}}$

Ans. Official Answer NTA (3)

Sol.



At A $x = y$

$Y - 2x = 2$

$(-2, -2)$

Height from line $x + y = 0$

$$h = \frac{4}{\sqrt{2}}$$

$$\text{Area of } \Delta = \frac{\sqrt{3}}{4}\frac{h^2}{\sin^2 60} = \frac{8}{\sqrt{3}}$$

Question ID : 3666942075

4. Let $A = \left\{(x, y) \in IR^2 : y \geq 0, 2x \leq y \leq \sqrt{4-(x-1)^2}\right.$ and

$B = \left\{(x, y) \in IR \times IR : 0 \leq y \leq \min\left\{2x, \sqrt{4-(x-1)^2}\right\}\right\}$.

Then the ratio of the area of A to the area of B is :

माना $A = \left\{(x, y) \in IR^2 : y \geq 0, 2x \leq y \leq \sqrt{4-(x-1)^2}\right.$ और

$B = \left\{(x, y) \in IR \times IR : 0 \leq y \leq \min\left\{2x, \sqrt{4-(x-1)^2}\right\}\right\}$ है।

तब A के क्षेत्रफल तथा B के क्षेत्रफल का अनुपात है :

(1) $\frac{\pi+1}{\pi-1}$ (2) $\frac{\pi}{\pi+1}$ (3) $\frac{\pi}{\pi-1}$ (4) $\frac{\pi-1}{\pi+1}$

Ans. Official Answer NTA (4)

Sol. $2x \leq y \leq \sqrt{4-(x-1)^2} : y \geq 0$

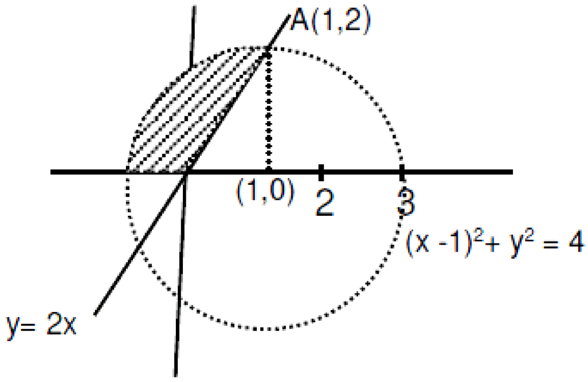


figure -1

area of A : area of shaded Region as show in figure–1

$\therefore$ area of A $= \frac{\pi \times 4}{2} - \left(\frac{1}{2} \times 1 \times 2\right)$

$= \pi - 1$

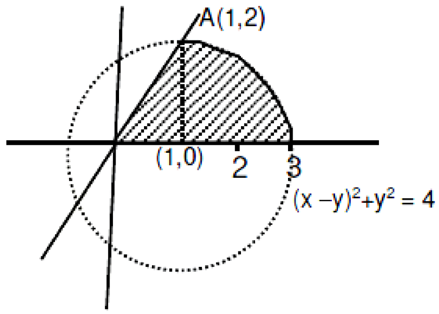


figure -2

area of B : area of shaded Region in as show in above figure–2

$\therefore$ area of B $= \frac{1}{2} \times 1 \times 2 + \frac{\pi \times 4}{4} = (1 + \pi)$

$\frac{A}{B} = \frac{\text{area of A}}{\text{area of B}} = \frac{\pi - 1}{\pi + 1}$

Question ID : 3666942074

5. Let $f(x) = x + \frac{a}{\pi^2 - 4}\sin x + \frac{b}{\pi^2 - 4}\cos x, x \in IR$ be a function which satisfies

$f(x) = x + \int_0^{\pi/2} \sin(x + y) f(y) dy$. Then $(a + b)$ is equal to

माना एक फलन $f(x) = x + \frac{a}{\pi^2 - 4}\sin x + \frac{b}{\pi^2 - 4}\cos x, x \in IR$,

$f(x) = x + \int_0^{\pi/2} \sin(x+y) f(y) dy$ को संतुष्ट करता है। तब $(a+b)$ का मान है :

(1) $-\pi(\pi+2)$ (2) $-2\pi(\pi+2)$ (3) $-2\pi(\pi-2)$ (4) $-\pi(\pi-2)$

Ans. Official Answer NTA (2)

Sol. $f(x) = x + \int_0^{\pi/2} (\sin x \cos y + \cos x \sin y) f(y) dy$

$$f(x) = x + \int_0^{\pi/2} \left((\cos y f(y) dy) \sin x + (\sin y f(y) dy) \cos x \right) ...(1)$$

On comparing with

$f(x) = x + \frac{a}{\pi^2 - 4} \sin x + \frac{b}{\pi^2 - 4} \cos x, x \in R$, then

$$\Rightarrow \frac{a}{\pi^2 - 4} = \int_0^{\pi/2} \cos y f(y) dy \quad ...(2)$$

$$\Rightarrow \frac{b}{\pi^2 - 4} = \int_0^{\pi/2} \sin y \ f(y) dy \quad ...(3)$$

Add (2) and (3)

$$\frac{a+b}{\pi^2 - 4} = \int_0^{\pi/2} (\sin y + \cos y) f(y) dy \ ....(4)$$

$$\frac{a+b}{\pi^2 - 4} = \int_0^{\pi/2} (\sin y + \cos y) f\left(\frac{\pi}{2} - y\right) dy \ ....(5)$$

Add (4) and (5)

$$\frac{2(a+b)}{\pi^2 - 4} = \int_0^{\pi/2} (\sin y + \cos y) \left(\frac{\pi}{2} + \frac{(a+b)}{\pi^2 - 4} (\sin y + \cos y) \right) dy$$

$$= \pi + \frac{a+b}{\pi^2 - 4} \left(\frac{\pi}{2} + 1 \right)$$

$(a + b) = -2\pi (\pi + 2)$

Question ID : 3666942070

6. Let $\lambda \neq 0$ be a real number. Let α, β be the roots of the equation $14x^2 - 31x + 3\lambda = 0$ and α, γ be the roots of the equation $35x^2 - 53x + 4\lambda = 0$. Then $\frac{3\alpha}{\beta}$ and $\frac{4\alpha}{\gamma}$ are the roots of the equation

माना $\lambda \neq 0$ एक वास्तविक संख्या है। माना α, β समीकरण $14x^2 - 31x + 3\lambda = 0$ के मूल हैं एवं α, γ समीकरण $35x^2 - 53x + 4\lambda = 0$ के मूल है। तब $\frac{3\alpha}{\beta}$ व $\frac{4\alpha}{\gamma}$ किस समीकरण के मूल है ?

(1) $49x^2 - 245x + 250 = 0$ (2) $7x^2 + 245x - 250 = 0$

(3) $49x^2 + 245x + 250 = 0$ (4) $7x^2 - 245x + 250 = 0$

Ans. Official Answer NTA (1)

Sol. $14\alpha^2 - 31\alpha + 3\lambda = 0$ ...(i)

$35\alpha^2 - 53\alpha + 4\lambda = 0$ ...(ii)

$\because$ α will satisfy both give equations

$\therefore$ $14\alpha^2 - 31\alpha + 3\lambda = 0 \Rightarrow \alpha^2 - \frac{31\alpha}{14} + \frac{3\lambda}{14} = 0$ .... (iii)

$35\alpha^2 - 53\alpha + 4\lambda = 0 \Rightarrow \alpha^2 - \frac{53\alpha}{35} + \frac{4\lambda}{35} = 0$ ....(iv)

from (iii) – (iv) we get

$\lambda = 7\alpha$ put in ... (iii)

$$\alpha^2 - \frac{31\alpha}{14} + \frac{3}{14}(7\alpha) = 0$$

$$\alpha^2 - \frac{31\alpha}{14} + \frac{3\alpha}{2} = 0$$

$\Rightarrow \alpha = 0$ or $\alpha = \frac{5}{7}$

($\alpha = 0$ not acceptable as $\lambda \neq 0$)

$\therefore \alpha = \frac{5}{7}$

$\because \alpha + \beta = \frac{31}{14} \Rightarrow \beta = \frac{3}{2}$

$\because \alpha + \gamma = \frac{53}{35} \Rightarrow \gamma = \frac{4}{5}$

$\therefore \frac{3\alpha}{\beta} = \frac{10}{7}$ and $\frac{4\alpha}{\gamma} = \frac{25}{7}$

$\therefore$ Reqd. Q.E. is $\left(x - \frac{10}{7}\right)\left(x - \frac{25}{7}\right) = 0$

$(7x - 10)(7x - 25) = 0$

$49x^2 - 245x + 250 = 0$

Question ID : 3666942081

7. If the vectors $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ and $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ are coplanar and the projection of $\vec{a}$ on the vector $\vec{b}$ is $\sqrt{54}$ units, then the sum of all possible values of $\lambda + \mu$ is equal to

यदि सदिश $\vec{a} = \lambda\hat{i} + \mu\hat{j} + 4\hat{k}, \vec{b} = -2\hat{i} + 4\hat{j} - 2\hat{k}$ तथा $\vec{c} = 2\hat{i} + 3\hat{j} + \hat{k}$ समतलीय हैं एवं $\vec{a}$ का सदिश $\vec{b}$ पर प्रक्षेप $\sqrt{54}$ इकाई है, तब $\lambda + \mu$ के सभी संभव मानों का योग है :

(1) 6 (2) 18 (3) 24 (4) 0

Ans. Official Answer NTA (3)

Sol.
$$\begin{vmatrix} \lambda & \mu & 4 \\ -2 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

$\lambda(10) - \mu(2) + 4(-14) = 0$
$10\lambda - 2\mu = 56$
$5\lambda - \mu = 28$ ... (1)

$$\frac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = \sqrt{54}$$

$$\frac{-2\lambda + 4\mu - 8}{\sqrt{24}} = \sqrt{54}$$

$-2\lambda + 4\mu - 8 = \sqrt{54 \times 24}$ ... (2)

By solving equation (1) and (2)

$\Rightarrow \lambda + \mu = 24$

Question ID : 3666942079

8. Let the tangents at the point A(4,-11) and B(8,-5) on the circle $x^2 + y^2 - 3x + 10y - 15 = 0$, intersect at the point C. Then the radius of the circle,w hose centre is C and the line joining A and B is its tangent, is equal to

माना वृत्त $x^2 + y^2 - 3x + 10y - 15 = 0$ के बिंदु A(4,-11) व B(8,-5) पर खींची गई स्पर्श रेखाएँ बिंदु C पर मिलती है। उस वृत्त, जिसका केन्द्र C है एवं A व B को मिलाने वाली रेखा जिसकी स्पर्श रेखा है, की त्रिज्या है :

(1) $\frac{2\sqrt{13}}{3}$ (2) $2\sqrt{13}$ (3) $\frac{3\sqrt{3}}{4}$ (4) $\sqrt{13}$

Ans. Official Answer NTA (1)

Sol. Equation of line AB is $y+5=\frac{3}{2}(x-8)$

$2y + 10 = 3x - 24$

$\Rightarrow 3x - 2y - 34 = 0$ ....(1)

Let C be (h, k) then equation of AB

$$hx+ky-\frac{3}{2}(x+h)+5(y+k)-15=0$$

$$x\left(h-\frac{3}{2}\right)+y(k+5)-\frac{3}{2}h+5k-15=0 \quad ...(2)$$

comparing (1) and (2)

$$\frac{h-\frac{3}{2}}{3}=\frac{k+5}{-2}=\frac{\frac{-3}{2}h+5k-15}{-34}$$

$$(h,k)\equiv\left(8,\frac{-28}{3}\right)$$

$\therefore$ required radius = length of perpendicular drawn for (h, k) to line AB = $\frac{2\sqrt{13}}{3}$

Question ID : 3666942073

9. Let x = 2 be a root of the equation $x^2 + px + q = 0$ and

$$f(x)=\begin{cases}\dfrac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4} & , x\neq 2p\\ 0 & , x=2p\end{cases}$$

Then $\lim\limits_{x\to 2p^+}[f(x)]$,

where $[\cdot]$ denotes greatest integer function, is

माना x = 2 समीकरण $x^2 + px + q = 0$ का एक मूल है और

$$f(x)=\begin{cases}\dfrac{1-\cos(x^2-4px+q^2+8q+16)}{(x-2p)^4} & , x\neq 2p\\ 0 & , x=2p\end{cases}$$

है। तब $\lim_{x\to 2p^+}[f(x)]$, जहाँ $[\cdot]$ महत्तम पूर्णांक फलन है का मान है :

(1) -1 (2) 0 (3) 2 (4) 1

Ans. Official Answer NTA (2)

Sol. $\lim_{x\to 2p^+}\left(\frac{1-\cos\left(x^2-4px+q^2+8q+16\right)}{\left(x^2-4px+q^2+8q+16\right)^2}\right)\left(\frac{\left(x^2-4px+q^2+8q+16\right)^2}{(x-2p)^2}\right)$

$\lim_{h\to 0}\frac{1}{2}\left(\frac{(2p+h)^2-4p(2p+h)+q^2+82+16}{h^2}\right)^2=\frac{1}{2}$

Using L'Hospital's

$\lim_{x\to 2p^+}[f(x)]=0$

Question ID : 3666942076

10. Let Δ be the area of the region $\{(x,y)\in IR^2 : x^2+y^2\le 21, y^2\le 4x, x\ge 1\}$.

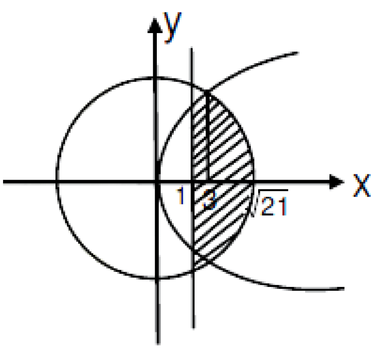
Then $\frac{1}{2}\left(\Delta-21\sin^{-1}\frac{2}{\sqrt{7}}\right)$ is equal to

माना क्षेत्र $\{(x,y)\in IR^2 : x^2+y^2\le 21, y^2\le 4x, x\ge 1\}$ का क्षेत्रफल Δ है, तब $\frac{1}{2}\left(\Delta-21\sin^{-1}\frac{2}{\sqrt{7}}\right)$ बराबर है :

(1) $2\sqrt{3}-\frac{2}{3}$ (2) $\sqrt{3}-\frac{2}{3}$ (3) $2\sqrt{3}-\frac{1}{3}$ (4) $\sqrt{3}-\frac{4}{3}$

Ans. Official Answer NTA (4)

Sol.



Required area $= 2\left(\int_1^2 2\sqrt{x}dx + \int_3^{\sqrt{21}} \sqrt{21-x^2}dx\right) = 2\left(2\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^3 + \left[\frac{x}{2}\sqrt{21-x^2} + \frac{21}{2}\sin^{-1}\left(\frac{x}{\sqrt{21}}\right)\right]_3^{\sqrt{21}}\right)$

$$2\left(\frac{4}{3}\left(3\sqrt{3}-1\right)+\frac{21\pi}{4}-\frac{3}{2}\times 2\sqrt{3}-\frac{21}{2}\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)\right) = 8\sqrt{3}-\frac{8}{3}+\frac{21\pi}{2}-6\sqrt{3}-21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$$

$$\therefore \Delta = 2\sqrt{3}+\frac{21\pi}{2}-\frac{8}{3}-21\sin^{-1}\left(\sqrt{\frac{3}{7}}\right)$$

$$\therefore \frac{1}{2}\left(\Delta - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right)$$

$$\frac{1}{2}\left[2\sqrt{3}-\frac{8}{3}+\frac{21\pi}{2}-21\left(\sin^{-1}\left(\frac{\sqrt{3}}{7}\right)+\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right)\right] \qquad \because \sin^{-1}\left(\frac{\sqrt{3}}{\sqrt{7}}\right)+\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)=\frac{\pi}{2}$$

$$\frac{1}{2}\left(\Delta - 21\sin^{-1}\left(\frac{2}{\sqrt{7}}\right)\right) = \frac{1}{2}\left[2\sqrt{3}-\frac{8}{3}+\frac{21\pi}{2}-21\left(\frac{\pi}{2}\right)\right] = \sqrt{3}-\frac{4}{3}$$

Question ID : 3666942086

11. Let $y = f(x)$ be the solution of the differential equation $y(x+1)dx - x^2dy = 0$, $y(1) = e$. Then $\lim_{x\to 0^+} f(x)$ is equal to

माना $y = f(x)$ अवकल समीकरण $y(x + 1)dx - x^2dy = 0$, $y(1) = e$ का हल है। तब $\lim_{x\to 0^+} f(x)$ बराबर है :

(1) e^2 (2) $\frac{1}{e}$ (3) 0 (4) $\frac{1}{e^2}$

Ans. Official Answer NTA (3)

Sol. $\frac{x+1}{x^2}dx = \frac{dy}{y}$

$\ln x - \frac{1}{x} = \ln y + c$

(1, e)

$c = -2$

$\ln x - \frac{1}{x} = \ln y - 2$

$y = e^{\ln x} - \frac{1}{x} + 2$

$\lim_{x \to 0^+} e^{\ln x - 1} - \frac{1}{x} + 2$

$= e^{-\infty}$

$= 0$

Question ID : 3666942077

12. A light ray emits from the origin making an angle 30° with the positive x-axis. After getting reflected by the line $x + y = 1$, if this ray intersects x-axis at Q, then the abscissa of Q is

X-अक्ष की धनात्मक दिशा से 30° का कोण बनाती हुई मूल बिंदु से एक प्रकाश किरण उत्सर्जित होती है। रेखा $x + y = 1$ से परावर्तित होने के बाद, यदि यह किरण x-अक्ष को Q पर मिलती है, तब Q का भुज है :

(1) $\frac{2}{3-\sqrt{3}}$ (2) $\frac{\sqrt{3}}{2(\sqrt{3}+1)}$ (3) $\frac{2}{(\sqrt{3}-1)}$ (4) $\frac{2}{3+\sqrt{3}}$

Ans. Official Answer NTA (4)

Sol. Image of O (0, 0) in line $x + y - 1 = 0$ lies on reflected ray.

$\frac{x-0}{1} = \frac{y-0}{1} = \frac{-2(0+0-1)}{2} \Rightarrow B\ (1, 1).$

Also, upon solving $y = \frac{x}{\sqrt{3}}$ and $x + y - 1 = 0$ we get $P \equiv \left(\frac{3-\sqrt{3}}{2}, \frac{\sqrt{3}-1}{2}\right)$

equation of reflected ray is same as line passing through BP.

$$\text{slope} = \frac{\frac{\sqrt{3}}{2} - \frac{1}{2} - 1}{\frac{3}{2} - \frac{\sqrt{3}}{2} - 1} = \frac{\sqrt{3}-3}{1-\sqrt{3}} = \sqrt{3}$$

Equation of line BP is

$y - 1 = \sqrt{3}(x-1)$

Put $y = 0 \Rightarrow -\frac{1}{\sqrt{3}} = x - 1$

Required point $\left(1 - \frac{1}{\sqrt{3}}, 0\right)$

Question ID : 3666942083

13. Let $f(\theta)=3\left(\sin^4\left(\frac{3\pi}{2}-\theta\right)+\sin^4(3\pi+\theta)\right)-2(1-\sin^2 2\theta)$ and

$S=\left\{\theta\in[0,\pi]:f'(\theta)=-\frac{\sqrt{3}}{2}\right\}$. If $4\beta=\sum_{\theta\in S}\theta$, then $f(\beta)$ is equal to

माना $f(\theta)=3\left(\sin^4\left(\frac{3\pi}{2}-\theta\right)+\sin^4(3\pi+\theta)\right)-2(1-\sin^2 2\theta)$ एवं

$S=\left\{\theta\in[0,\pi]:f'(\theta)=-\frac{\sqrt{3}}{2}\right\}$ है। यदि $4\beta=\sum_{\theta\in S}\theta$, है तब $f(\beta)$ बराबर है :

(1) $\frac{11}{8}$ (2) $\frac{3}{2}$ (3) $\frac{9}{8}$ (4) $\frac{5}{4}$

Ans. Official Answer NTA (4)

Sol. $f(\theta)=3\left(\sin^4\left(\frac{3\pi}{2}-\theta\right)+\sin^4(3x+\theta)\right)-2\left(1-\sin^2 2\theta\right)$

$S=\left\{\theta\in[0,\pi]:f'(\theta)=-\frac{\sqrt{3}}{2}\right\}$

$\Rightarrow f(\theta)=3(\cos^4\theta+\sin^4\theta)-2\cos^2 2\theta$

$\Rightarrow f(\theta)=3\left(1-\frac{1}{2}\sin^2 2\theta\right)-2\cos^2 2\theta$

$\Rightarrow f(\theta)=3-\frac{3}{2}\sin^2 2\theta-2\cos^2 2\theta$

$=\frac{3}{2}-\frac{1}{2}\cos^2 2\theta=\frac{3}{2}-\frac{1}{2}\left(\frac{1+\cos 4\theta}{2}\right)$

$f(\theta)=\frac{5}{4}-\frac{\cos 4\theta}{4}$

$f'(\theta)=\sin 4\theta$

$\Rightarrow f'(\theta)=\sin 4\theta=-\frac{\sqrt{3}}{2}$

$\Rightarrow 4\theta=n\pi+(-1)^n\frac{\pi}{3}$

$$\Rightarrow \theta = \frac{n\pi}{4} + (-1)^n \frac{\pi}{12}$$

$$\Rightarrow \theta = \frac{\pi}{12}, \left(\frac{\pi}{4} - \frac{\pi}{12}\right), \left(\frac{\pi}{2} + \frac{\pi}{12}\right), \left(\frac{3\pi}{4} - \frac{\pi}{12}\right)$$

$$\Rightarrow 4\beta = \frac{\pi}{4} + \frac{\pi}{2} + \frac{3\pi}{4} = \frac{3\pi}{2}$$

$$\Rightarrow \beta = \frac{3\pi}{8} \Rightarrow f(\beta) = \frac{5}{4} - \frac{\cos\frac{3\pi}{2}}{4} = \frac{5}{4}$$

Question ID : 3666942069

14. For two non-zero complex numbers z_1 and z_2, if $\text{Re}(z_1 z_2) = 0$ and $\text{Re}(z_1 + z_2) = 0$, then which of the following are possible?

A. $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) > 0$

B. $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) > 0$

C. $\text{Im}(z_1) > 0$ and $\text{Im}(z_2) < 0$

D. $\text{Im}(z_1) < 0$ and $\text{Im}(z_2) < 0$

Choose the correct answer from the options given below :

(1) B and C (2) B and D (3) A and B (4) A and C

z_1 व z_2 दो शून्येतर सम्मिश्र संख्याऐं है, यदि $\text{Re}(z_1 z_2) = 0$ और $\text{Re}(z_1 + z_2) = 0$ है, तब

A. $\text{Im}(z_1) > 0$ एवं $\text{Im}(z_2) > 0$

B. $\text{Im}(z_1) < 0$ एवं $\text{Im}(z_2) > 0$

C. $\text{Im}(z_1) > 0$ एवं $\text{Im}(z_2) < 0$

D. $\text{Im}(z_1) < 0$ एवं $\text{Im}(z_2) < 0$

नीचे दिये गये विकल्पों से सही उत्तर चुनिए :

(1) B तथा C (2) B तथा D (3) A तथा B (4) A तथा C

Ans. Official Answer NTA (1)

Sol. Let $z_1 = x_1 + iy_1$ and $z_2 = x_2 + iy_2$

$\because$ Re $(z_1 z_2) = 0$ and Re $(z_1 + z_2) = 0$,

$x_1 x_2 - y_1 y_2 = 0$ ...(i)

$x_1 + x_2 = 0$ ...(ii)

$x_1^2 + y_1y_2 = 0$

$y_1y_2 = -x_1^2$

$\Rightarrow$ Im (z_1) and Im (z_2) are of opposite sign.

Question ID : 3666942071

15. Consider the following system of questions

$\alpha x + 2y + z = 1$

$2\alpha x + 3y + z = 1$

$3x + \alpha y + 2z = \beta$

for some $\alpha, \beta \in R$. Then which of the following is NOT correct.

(1) It has no solution for $\alpha = -1$ and for all $\beta \in R$

(2) It has no solution for $\alpha = 3$ and for all $\beta \neq 2$

(3) It has no solution if $\alpha = -1$ and $\beta \neq 2$

(4) It has no solution for all $\alpha \neq -1$ and $\beta = 2$

किसी $\alpha, \beta \in R$ के लिए निम्न समीकरण निकाय का विचार कीजिए :

$\alpha x + 2y + z = 1$

$2\alpha x + 3y + z = 1$

$3x + \alpha y + 2z = \beta$

तब निम्न में से कौन सा सही नहीं है।

(1) $\alpha = -1$ एवं सभी $\beta \in R$ के लिए इसका कोई हल नहीं है

(2) $\alpha = 3$ एवं सभी $\beta \neq 2$ के लिए इसका कोई हल नहीं है

(3) इसका कोई हल नहीं है यदि $\alpha = -1$ एवं $\beta \neq 2$ हैं

(4) सभी $\alpha \neq -1$ एवं $\beta = 2$ के लिए इसका एक हल है

Ans. Official Answer NTA (1)

Sol. $D = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & 2 \end{vmatrix} = 0 \Rightarrow \alpha = -1, 3$

$$D_x = \begin{vmatrix} 2 & 1 & 1 \\ 3 & 1 & 1 \\ \alpha & 2 & \beta \end{vmatrix} = 0 \Rightarrow \beta = 2$$

$$D_y = \begin{vmatrix} \alpha & 1 & 1 \\ 2\alpha & 1 & 1 \\ 3 & 2 & \beta \end{vmatrix} = 0$$

$$D_z = \begin{vmatrix} \alpha & 2 & 1 \\ 2\alpha & 3 & 1 \\ 3 & \alpha & \beta \end{vmatrix} = 0$$

$\beta = 2, \alpha = -1$

$\alpha = -1, \beta = 2$ Infinite solution

Question ID : 3666942067

16. The domain of $f(x) = \dfrac{\log_{(x+1)}(x-2)}{e^{2\log_e x} - (2x+3)}, x \in R$ is

$f(x) = \dfrac{\log_{(x+1)}(x-2)}{e^{2\log_e x} - (2x+3)}, x \in R$ का प्रांत है :

(1) R - {-1,3}   (2) $(-1,\infty)$ - {3}   (3) R - {3}   (4) $(2,\infty)$ - {3}

Ans. Official Answer NTA (4)

Sol. $\therefore f(x) = \dfrac{\log_{(x+1)}(x-2)}{e^{2\log x} - (2x+3)}$

(i) $x - 2 > 0$

$x > 2$

$x \in (2, \infty)$

(ii) $x + 1 > 0$ & $x + 1 \neq 1$

$\Rightarrow x > -1$ $\Rightarrow x \neq 0$

$x \in (-1, 0) \cup (0, \infty)$

(iii) $x > 0$

$x \in (0, \infty)$

(iv) $e^{2\ell nx} - (2x+3) \Rightarrow x^2 - 2x - 3 \neq 0$

$\Rightarrow (x-3)(x+1) \neq 0$

$\Rightarrow x \neq 3, x \neq -1$

from (i) $\cap$ (ii) $\cap$ (iii) $\cap$ (iv)

$x \in (2, \infty) - \{3\}$

Question ID : 3666942072

17. Let α and β be real numbers. Consider a 3×3 matrix A such that $A^2 = 3A + \alpha I$. If $A^4 = 21A + \beta I$, then

माना α व β वास्तविक संख्याएं है। एक 3×3 आव्यूह A है लिए $A^2 = 3A + \alpha I$. है। यदि $A^4 = 21A + \beta I$ है, तब

(1) $\beta = 8$ (2) $\beta = -8$ (3) $\alpha = 4$ (4) $\alpha = 1$

Ans. Official Answer NTA (2)

Sol. $A^2 = 3A + \alpha I$

$A^3 = 3A^2 + \alpha A$

$A^3 = 3(3A + \alpha I) + \alpha A$

$A^3 = 9A + \alpha A + 3\alpha I$

$A^4 = (9 + \alpha) A^2 + 3\alpha A$

$= (9 + \alpha)(3A + \alpha I) + 3\alpha A$

$= A(27 + 6\alpha) + \alpha(9 + \alpha)$

$\Rightarrow 27 + 6\alpha = 21 \Rightarrow \alpha = -1$

$\Rightarrow \beta = \alpha(9 + \alpha) = -8$

Question ID : 3666942082

18. Fifteen football players of a club-team are given 15 T-shirts with their names written on the backside. If the players pick up the T-shirts randomly, then the probability that at least 3 players pick the correct T-shirt is

एक क्लब–टीम के पन्द्रह फुटबाल खिलाड़ियों को 15 टी–शर्ट वितरित की गई जिनके पीछे की तरफ उनके नाम लिखे हुए थे। यदि खिलाड़ी टी–शर्टों का चयन यादृच्छया करते है, तब कम से कम तीन खिलाड़ियों द्वारा सही टी–शर्ट चयन करने की प्रायिकता है :

(1) $\frac{2}{15}$ (2) $\frac{5}{24}$ (3) $\frac{5}{36}$ (4) $\frac{1}{6}$

Ans. Official Answer NTA (Drop)

Sol. 15 players and 15 T–shirts

The answer of the question given by NTA is $\frac{1}{6}$

Which might be calculated by them like $\frac{{}^{15}C_3 \times 12!}{15!} = \frac{1}{6}$

But calculating favourable case by $({}^{15}C_3)$ 12! will be wrong because it will included repetitions also

The correct answer would be $\dfrac{15!-({}^{15}C_2D_{13}+{}^{15}C_1D_{14}+D_{15}!}{15!}$

Where D_n = De–arrangement of 'n' things

$\therefore$ P (Reqd) = $\dfrac{15!-(105D_{13}+15D_{14}+D_{15})}{15!}$

Question ID : 3666942084

19. If p, q and r are three propositions, then which of the following combination of truth values of p, q and r makes the logical epression $\{(p\vee q)\wedge((\sim p)\vee r)\}\rightarrow((\sim q)\vee r)$ false?

यदि p, q व तीन साध्य है, तब दिए गए विकल्पों में से p, q व r के कौन से सत्य मान $\{(p\vee q)\wedge((\sim p)\vee r)\}\rightarrow((\sim q)\vee r)$ को असत्य (F) बनाते है ?

(1) p = F, q = T, r = F (2) p = T, q = F, r = T

(3) p = T, q = T, r = F (4) p = T, q = F, r = F

Ans. Official Answer NTA (1)

Sol.

	p	q	r	$(p\vee q)\wedge((\sim p)\vee r)$	$\sim q\vee r$
(1)	T	F	T	T	T
(2)	T	T	F	F	F
(3)	F	T	F	T	F
(4)	T	F	F	F	T

Option (3) $(p\vee q)\wedge(\sim q\vee r)\rightarrow(\sim p\vee r)$ will be False.

Question ID : 3666942068

20. Let f: $R\rightarrow R$ be a function such that $f(x)=\dfrac{x^2+2x+1}{x^2+1}$. Then

(1) $f(x)$ is one-one in $[1,\infty)$ but not in $(-\infty,\infty)$

(2) $f(x)$ is one-one in $(-\infty,\infty)$

(3) $f(x)$ is many-one in $(1,\infty)$

(4) $f(x)$ is many-one in $(-\infty,-1)$

माना $f : R \to R$ एक फलन $f(x) = \frac{x^2+2x+1}{x^2+1}$ है। तब

(1) $[1,\infty)$ में $f(x)$ एकैकी है परन्तु $(-\infty,\infty)$ में एकैकी नहीं है।

(2) $(-\infty,\infty)$ में $f(x)$ एकैकी है।

(3) $(1,\infty)$ में $f(x)$ बहु–एक है।

(4) $(-\infty,-1)$ में $f(x)$ बहु–एक है।

Ans. Official Answer NTA (1)

Sol. $\because f(x) = \frac{(x+1)^2}{(x+1)}$

$$\because f'(x) = \frac{(x^2+1).2(x+1)-(x+1)^2(2x)}{(x^2+1)^2} = -\frac{2(x+1)(x-1)}{(x^2+1)^2}$$

clearly f(x) is one – one is $(-\infty,-1)$ and also in $(1,\infty)$ but f(x) is not one – one is $(-\infty,\infty)$

SECTION – B

Question ID : 3666942094

21. If the co-efficient of x^9 in $\left(\alpha x^3+\frac{1}{\beta x}\right)^{11}$ and the co-efficient of x^{-9} in $\left(\alpha x-\frac{1}{\beta x^3}\right)^{11}$ are equal, then $(\alpha\beta)^2$ is equal to ________ .

यदि $\left(\alpha x^3+\frac{1}{\beta x}\right)^{11}$ के प्रसार में x^9 का गुणांक एवं $\left(\alpha x-\frac{1}{\beta x^3}\right)^{11}$ के प्रसार में x^{-9} का गुणांक बराबर हैं, तब $(\alpha\beta)^2$ बराबर है।

Ans. Official Answer NTA (1)

Sol. Coefficient of x^9 in $\left(\alpha x^3+\frac{1}{\beta x}\right) = {}^{11}C_6 \cdot \frac{\alpha^5}{\beta^6}$

$\because$ Both are equal

$$\therefore \frac{11}{C_6}\cdot\frac{\alpha^5}{\beta^6} = -\frac{11}{C_5}\cdot\frac{\alpha^6}{\beta^5}$$

$\Rightarrow \frac{1}{\beta} = -\alpha$

$\Rightarrow \alpha\beta = -1$

$\Rightarrow (\alpha\beta)^2 = 1$

Question ID : 3666942088

22. Let the equation of the plane P containing the line $x+10 = \frac{8-y}{2} = z$ be ax + by + 3z = 2 (a + b) and the distance of the plane P from the point (1,27,7) be . Then $a^2 + b^2 + c^2$ is equal to ________ .

माना समतल P, जिस पर रेखा $x+10 = \frac{8-y}{2} = z$ स्थित है, का समीकरण $ax + by + 3z = 2(a + b)$ है एवं बिंदु (1,27,7) से समतल P की दूरी c है। तब $a^2 + b^2 + c^2$ बराबर है।

Ans. Official Answer NTA (355)

Sol. $\frac{x+10}{1} = \frac{y-8}{-2} = \frac{z}{1}$ .....(1)

and ax + by + 3z = 2 (a + b) .....(2)

$\because$ (2) contains the line (1)

$\therefore$ (–10, 8, 0) will lie on (2)

$\Rightarrow$ –10a + 8b = 2a + 2b

b = 2a .....(3)

and a – 2b + 3 = 0 .....(4)

from (3) and (4), we get

a = 1 & b = 2

$\therefore$ eq^n of plane (2) is $x + 2y + 3z - 6 = 0$

$\therefore \left|\frac{1+54+21-6}{\sqrt{1+4+5}}\right| = 5\sqrt{14}$

$\therefore a_2 + b_2 + c_2 = 1 + 4 + 25 \times 14 = 5 + 350 = 355$

Question ID : 3666942087

23. Let the co-ordinates of one vertex of ΔABC be $A(0,2,\alpha)$ and the other two vertices lie on the line $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$. For $\alpha \in \mathbb{Z}$, if the area of ΔABC is 21 sq. units and the line segment BC has length $2\sqrt{21}$ units, then α^2 is equal to __________ .

माना ΔABC के एक शीर्ष के निर्देशांक $A(0,2,\alpha)$ है तथा अन्य दो शीर्ष रेखा $\frac{x+\alpha}{5} = \frac{y-1}{2} = \frac{z+4}{3}$ पर स्थित है।

$\alpha \in \mathbb{Z}$ के लिए, यदि ΔABC का क्षेत्रफल 21 वर्ग इकाई है एवं रेखा खण्ड BC की लम्बाई $2\sqrt{21}$ इकाई है, तब α^2 बराबर है।

Ans. Official Answer NTA (9)

Sol. A. $(O_1\ 2, \alpha)$

$$\left|\frac{1}{2}.2\sqrt{21}.\begin{vmatrix} i & j & k \\ \alpha & 1 & \alpha+4 \\ 5 & 2 & 3 \end{vmatrix}\frac{1}{\sqrt{25+4+9}}\right| = 21\sqrt{21}$$

$$\sqrt{(2\alpha+5)^2+(2\alpha+20)^2+(2\alpha-5)^2} = \sqrt{21}\sqrt{38}$$

$\Rightarrow 12\alpha^2 + 80\alpha + 450 = 798$

$\Rightarrow 12\alpha^2 + 80\alpha - 348 = 0$

$\Rightarrow \alpha = 3 \Rightarrow \alpha^2 = 9$

Question ID : 3666942096

24. Five digit numbers are formed using the digits 1,2,3,5,7 with repetitions and are written in descending order with serial numbers. For example, the number 77777 has serial number 1. Then the serial number of 35337 is ____________ .

अंकों 1,2,3,5,7 के प्रयोग से, पुनरावृति के साथ पाँच अंकों की संख्याऐं बनाई जाती है तथा इनको ह्यसमान क्रम में, क्रम संख्या के साथ लिखा जाता है। उदाहरण के लिए संख्या 77777 की क्रम संख्या 1 है। तब 35337 की क्रम संख्या है।

Ans. Official Answer NTA (1436)

Sol. Number of numbers starting with 7 $\rightarrow$ 625

Number of numbers starting with 5 $\rightarrow$ 625

Number of numbers starting with 37 $\rightarrow$ 125

Number of numbers starting with 357 $\rightarrow$ 25

Number of numbers starting with 3537 $\rightarrow$ 5

Number of numbers starting with 3535 $\rightarrow$ 5

Number of numbers starting with 35337 $\rightarrow$ 1

1436

The position of the number 35337 is 1436.

Question ID : 3666942089

25. Suppose f is a function satisfying $f(x+y)=f(x)+f(y)$ for all, $x, y \in \mathbb{N}$ and $f(1)=\frac{1}{5}$. If $\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)}=\frac{1}{12}$, then m is equal to _________ .

माना f एक फलन है जो सभी $x, y \in \mathbb{N}$ के लिए $f(x+y)=f(x)+f(y)$ को संतुष्ट करता है एवं $f(1)=\frac{1}{5}$ है। यदि $\sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)}=\frac{1}{12}$ है, तब m बराबर है।

Ans. Official Answer NTA (10)

Sol. $\because f(1)=\frac{1}{5} \therefore f(2)=f(1)+f(1)=\frac{2}{5}$

$f(2)=\frac{2}{5}\ f(3)=f(2)+f(1)=\frac{3}{5}$

$f(3)=\frac{3}{5}$

$\therefore \sum_{n=1}^{m} \frac{f(n)}{n(n+1)(n+2)}$

$=\frac{1}{5} \sum_{n=1}^{m}\left(\frac{1}{n+1}-\frac{1}{n+2}\right)$

$=\frac{1}{5}\left(\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\frac{1}{4}+\ldots .+\frac{1}{m+1}-\frac{1}{m+2}\right)$

$=\frac{1}{5}\left(\frac{1}{2}-\frac{1}{m+2}\right)=\frac{m}{10(m+2)}=\frac{1}{12}$

$\therefore m=10$

Question ID : 3666942092

26. If all the six digit numbers $x_1 x_2 x_3 x_4 x_5 x_6$ with $0<x_1<x_2<x_3<x_4<x_5<x_6$ are arranged in the increasing order, then the sum of the digits in the 72th number is _______ .

यदि सभी छः अंकों की संख्याओं $x_1 x_2 x_3 x_4 x_5 x_6$, जिनमें $0 < x_1 < x_2 < x_3 < x_4 < x_5 < x_6$ है, को वर्धमान क्रम में व्यवस्थित किया जाता है, तब 72वीं संख्या में अंकों का योग है।

Ans. Official Answer NTA (32)

Sol. Number of numbers starting with $1 = {}^8C_5 = 56$

Number of numbers starting with $23 = {}^6C_4 = 15$

Next number at 72nd option is 245678.

$\therefore$ sum of digits $= 2 + 4 + 5 + 6 + 8 = 32$

Question ID : 3666942095

27. Let the coefficients of three consecutive terms in the binomial expansion of $(1 + 2x)^n$ be in the ratio 2 : 5 : 8. Then the coefficient of the term, which is in the middle of these three terms, is ________ .

माना $(1 + 2x)^n$ द्विपद प्रसार में तीन क्रमागत पदों के गुणंकों का अनुपात 2 : 5 : 8 है। इन तीन पदों में मध्य पद का गुणांक है।

Ans. Official Answer NTA (1120)

Sol. $t_{r+1} = {}^nC_r (2x)^r$

$$\Rightarrow \frac{{}^nC_r(2)^{r-1}}{{}^nC_r(2)^r} = \frac{2}{5}$$

$$\Rightarrow \frac{\frac{n!}{(r-1)!(n-r+1)!}}{\frac{n!(2)}{r!(n-r)!}} = \frac{2}{5}$$

$$\Rightarrow \frac{r}{n-r+1} = \frac{4}{5} \Rightarrow 5r = 4n - 4r + 4$$

$\Rightarrow 9r = 4(n + 1)$ ....(1)

$$\Rightarrow \frac{{}^nC_r(2)^r}{{}^nC_{r+1}(2)^{r+1}} = \frac{5}{8}$$

$$\Rightarrow \frac{\frac{n!}{r!(n-r)!}}{\frac{n!}{(r+1)!(n-r-1)!}} = \frac{5}{4} \Rightarrow \frac{r+1}{n-r} = \frac{5}{4}$$

$\Rightarrow 4r + 4 = 5n - 5r \Rightarrow 5n - 4 = 9r$ ....(2)

From (1) and (2)

$\Rightarrow 4n + 4 = 5n - 4 \Rightarrow n = 8$

(1) $\Rightarrow r = 4$

so, coefficient of middle term is

$${}^{8}C_{4}2^{4} = 16 \times \frac{8\times7\times6\times5}{4\times3\times2\times1} = 16\times70 = 1120$$

Question ID : 3666942093

28. Let f: $\mathbb{R} \to \mathbb{R}$ be a differentiable function that satisfies the relation $f(x+y) = f(x) + f(y) - 1$, $\forall x, y \in \mathbb{R}$. If $f'(0) = 2$, then $|f(-2)|$ is equal to ________ .

माना f: $\mathbb{R} \to \mathbb{R}$ एक अवकलनीय फलन है जो संबंध $f(x+y) = f(x) + f(y) - 1$, $\forall x, y \in \mathbb{R}$ को संतुष्ट करता है। यदि $f'(0) = 2$ है, तब $|f(-2)|$ बराबर है।

Ans. Official Answer NTA (3)

Sol. $f(x+y) = f(x) + f(y) - 1$ ....(1)

Partial differential w.r.t. x

$f'(x+y) = f'(x)$

put $x = 0$

$f'(y) = f'(0) = 2$

$\Rightarrow f(y) = 2y + c$

$f(x) = 2x + c$

Now put $x = y = 0$ in (1) we get $f(0) = 1$

$\Rightarrow f(0) = 0 + c$

$\Rightarrow c = 1$

$\Rightarrow f(x) = 2x + 1$

$\Rightarrow |f(-2)| = 3$

Question ID : 3666942091

29. Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non-zero non-coplanar vectors. Let the position vectors of four points A, B, C and D be $\vec{a}-\vec{b}+\vec{c}$, $\lambda\vec{a}-3\vec{b}+4\vec{c}$, $-\vec{a}+2\vec{b}-3\vec{c}$ and $2\vec{a}-4\vec{b}+6\vec{c}$ respectively. If $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{AD}$ are coplanar, then λ is equal to ___________ .

माना $\vec{a}$, $\vec{b}$ तथा $\vec{c}$ तीन शून्येत्तर असहतलीय सदिश है। माना चार बिंदुओं A, B, C व D के स्थिति सदिश क्रमशः $\vec{a}-\vec{b}+\vec{c}$, $\lambda\vec{a}-3\vec{b}+4\vec{c}$, $-\vec{a}+2\vec{b}-3\vec{c}$ व $2\vec{a}-4\vec{b}+6\vec{c}$ है, यदि $\overrightarrow{AB}$, $\overrightarrow{AC}$ तथा $\overrightarrow{AD}$ समतलीय है। तब λ बराबर है।

Ans. Official Answer NTA (2)

Sol. $\overline{AB} = (\lambda-1)\bar{a} - 2\bar{b} + 3\bar{c}$

$\overline{AC} = 2\bar{a} + 3\bar{b} - 4\bar{c}$

$\overline{AD} = \bar{a} - 3\bar{b} + 5\bar{c}$

$$\begin{vmatrix} \lambda-1 & -2 & 3 \\ -2 & 3 & -4 \\ 1 & -3 & 5 \end{vmatrix} = 0$$

$\Rightarrow (\lambda - 1)(15 - 12) + 2(-10 + 4) + 3(6 - 3) = 0$

$\Rightarrow (\lambda - 1) = 1 \Rightarrow \lambda = 2$

Question ID : 3666942090

30. Let $a_1, a_2, a_3,$ .... be a GP of increasing positive numbers. If the product of fourth and sixth terms is 9 and the sum of fifth and seventh terms is 24, then $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ is equal to ________ .

माना $a_1, a_2, a_3,$ .... वर्धमान धनात्मक संख्याओं की एक GP है। यदि चौथे व छटवें पदों का गुणनफल 9 है और पाँचवे व सातवें पदों का योग 24 है, तब $a_1a_9 + a_2a_4a_9 + a_5 + a_7$ बराबर है।

Ans. Official Answer NTA (60)

Sol. $\because a_1, a_2, a_3$ .... increasing G.P. of positive nos.

$\because a_4a_6 = 9$ and $a_5 + a_7 = 24$

$(ar^3)(ar^5) = 9$

$a^2r^8 = 9$ and $ar^4 + ar^6 = 24$

$\Rightarrow ar^4 = 3$ ....(1) $ar^4(1 + r^2) = 24$ ....(2)

$\therefore$ from (1) and (2) we get $1 + r^2 = 8$

$r^2 = 7 \Rightarrow r = \sqrt{7}$

$\therefore a = \frac{3}{49}$

$\therefore a_1.a_9 + a_2 a_4a_9 + a_5 + a_7$

$= a(ar^8) + (ar)(ar^3)(ar^8) + ar^4 + ar^6$

$= a^2r^8 + a^3r^{12} + ar^4 + ar^6$

$= 9 + 3^3 + 3 + \frac{3}{49} \times 7^3$

$= 9 + 27 + 3 + 21$

$= 60$