JEE Main June 2022

Question Paper With Text Solution

28 June | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN JUNE 2022 | 28TH JUNE SHIFT-1

SECTION – A

Question ID : 171

Binomial Theorem

1. If $\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)}$, where $a \in R$, then the value of 16α is equal to :

   यदि $\sum_{k=1}^{31}\left({}^{31}C_k\right)\left({}^{31}C_{k-1}\right)-\sum_{k=1}^{30}\left({}^{30}C_k\right)\left({}^{30}C_{k-1}\right)=\frac{\alpha(60!)}{(30!)(31!)}$ है, जहाँ $a \in R$ है, तो 16α का मान बराबर है :

   (1) 1411 (2) 1320 (3) 1615 (4) 1855

Ans. Official Answer NTA (1)

Sol. $\sum_{k=1}^{31} {}^{31}C_k \cdot {}^{31}C_{k-1} = \sum_{k=1}^{31} {}^{31}C_k \cdot {}^{31}C_{32-k}$

$= \text{coefficient of } x^{32} \text{ in } (1+x)^{62}$

$= {}^{62}C_{32}$

Similary $\sum_{k=1}^{30} {}^{30}C_k \cdot {}^{30}C_{31-k} = {}^{60}C_{31}$

So ${}^{62}C_{32} - {}^{60}C_{31} = \frac{62}{32}\cdot\frac{61!}{30!31!}\left[\frac{62.61}{32}-30\right]$

$16\alpha = 31 \times 61 - 16 \times 30 = 1411$

Question ID : 172

Function

2. Let a function $f : N \to N$ be defined dy

   माना एक फलन $f : N \to N$

$$f(n)=\begin{cases} 2n, & n=2,4,6,8,..... \\ n-1, & n=3,7,11,15,..... \\ \frac{n+1}{2}, & n=1,5,9,13,..... \end{cases}$$

द्वारा परिभाशित है। तो f :

Then, f is :

(1) one-one but not onto

(2) onto but not one-one

(3) neither one-one nor onto

(4) one-one and onto

(1) एकैकी है परन्तु आच्छादक नहीं है

(2) आच्छादक है परन्तु एकैकी नहीं है

(3) न तो एकैकी है न ही आच्छादक है

(4) एकैकी तथा आच्छादक है

Ans. Official Answer NTA (4)

Sol. $f(x) = 1$ for $n = 1$

2 for $n = 3$

3 for $n = 5$

4 for $n = 2$

So onto and one-one

Question ID : 173

Determinant

3. If the system of linear equations

यदि रैखिक समीकरण निकाय

$2x + 3y - z = -2$

$x + y + z = 4$

$x - y + |\lambda|z = 4\lambda - 4$

where $\lambda \in R$, has no solution, then

जहाँ $\lambda \in R$ है, का कोई हल नहीं हैं, तो :

(1) $\lambda = 7$

(2) $\lambda = -7$

(3) $\lambda = 8$

(4) $\lambda^2 = 1$

Ans. Official Answer NTA (2)

Sol. $$D = \begin{vmatrix} 2 & 3 & -1 \\ 1 & 1 & 1 \\ 1 & -1 & |\lambda| \end{vmatrix} = 0 \Rightarrow |\lambda| \Rightarrow \lambda = \pm 7$$

for $\lambda = 7$, $D_1 = D_2 = D_3 = 0$

So $\lambda = -7$

Question ID : 174

Matrices

4. Let A be a matrix of order 3×3 and $\det(A) = 2$. Then $\det(\det(A)\ \text{adj}(5\ \text{adj}(A^3)))$ is equal to _____.

माना A एक क 3×3 कोटि का आव्यूह है। माना $\det(A) = 2$ है। तो $\det(\det(A)\ \text{adj}(5\ \text{adj}(A^3)))$ बराबर है _____.

(1) 512×10^6 (2) 256×10^6 (3) 1024×10^6 (4) 256×10^{11}

Ans. Official Answer NTA (1)

Sol. $|A| = 2$

$$\left|\,|A|.\text{adj}\left(5\ \text{adj}\left(A^3\right)\right)\right| = 2^3 \left|5^2.\text{adj}\left(\text{adj}\ A^3\right)\right|$$

$= 2^3 . 5^6 |\text{adj}(\text{adj}\ A^3)|$

$= 2^3.5^6 |A^3|^4$

$= 2^3. 5^6 .2^{12}$

$= 2^9 . 10^6$

$= 512 \times 10^6$

Question ID : 175

P & C

5. The total number of 5-digit numbers, formed by using the digits 1, 2, 3, 5, 6, 7 without repetition, which are multiple of 6, is :

बिना पुरावृत्ति के अकों 1, 2, 3, 5, 6, 7 के उपयोग द्वारा बनाई गई -5 अकों की संख्याओं, जो कि 6 की गुणज हैं, कि कुल संख्या है :

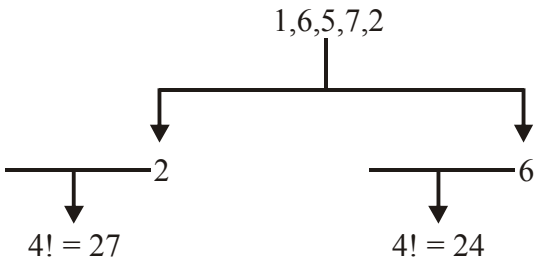
(1) 36 (2) 48 (3) 60 (4) 72

Ans. Official Answer NTA (4)

Sol. There are the numbers which are diviseble by 3 and even.

1, 2, 3, 5, 7.....(2)

$4! = 24$



total $= 3 \times 4! = 72$

Question ID : 176

Sequence & progression

6. Let $A_1, A_2, A_3, \ldots\ldots$ be an increasing geometric progression of positive real numbers. If $A_1A_3A_5A_7 = \frac{1}{1296}$ and $A_2 + A_4 = \frac{7}{36}$, then, the value of $A_6 + A_8 + A_{10}$ is equal to :

माना $A_1, A_2, A_3, \ldots\ldots$धनात्मक वास्तविक संख्याओं की वर्धमान गुणोत्तर श्रेढ़ी है। यदि $A_1A_3A_5A_7 = \frac{1}{1296}$ तथा $A_2 + A_4 = \frac{7}{36}$ हैं, तो $A_6 + A_8 + A_{10}$ का मान बराबर है :

(1) 33 (2) 37 (3) 43 (4) 47

Ans. Official Answer NTA (3)

Sol. Let $A_4 = a$ and common ratio = r

$$\frac{a}{r^3}.\frac{a}{r^2}.a^r.ar^3 = \frac{1}{1296} \Rightarrow a^4 = \frac{1}{64} \Rightarrow a = \frac{1}{6}$$

$$A_2 + A_4 = \frac{7}{36} \Rightarrow \frac{a}{r^2} + a = \frac{7}{36} \Rightarrow \frac{1}{r^2} + 1 = \frac{7}{6} \Rightarrow r^2 = 6$$

$$A_6 + A_8 + A_{10} = a(r^2 + r^4 + r^6) = 1/6\ (6 + 6^2 + 6^3) = 1 + 6 + 36 = 43$$

Question ID : 177

Definite Integration

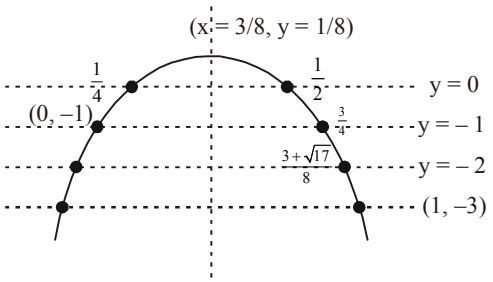
7. Let [t] denote the greatest integer less than or equal to t. Then, the value of the integral $\int_0^1 \left[-8x^2 + 6x - 1\right]dx$ is equal to :

माना [t] महत्तम पूर्णांक $\le t$ है। तो समाकलन $\int_0^1 \left[-8x^2 + 6x - 1\right]dx$ का मान बराबर है :

(1) -1 (2) $\frac{-5}{4}$ (3) $\frac{\sqrt{17} - 13}{8}$ (4) $\frac{\sqrt{17} - 16}{8}$

Ans. Official Answer NTA (3)

Sol.



Let $y = -8x^2 + 6x - 1$

for $y = -2$

$-8x^2 + 6x - 1 = 0 - 2$

$\Rightarrow x = \dfrac{3+\sqrt{17}}{8}$

for $y = -1$

$-8x^2 + 6x - 1 = -1$

$\Rightarrow x = \dfrac{3}{4}$

$$\text{so } \int_0^1 \left[-8x^2+6x-1\right]dx = \int_0^{1/4}(-1)\,dx + \int_{1/4}^{1/2} 0\,dx + \int_{1/2}^{3/4}(-1)\,dx + \int_{3/4}^{\frac{3+\sqrt{17}}{8}}(-2)\,dx + \int_{\frac{3+\sqrt{17}}{8}}^{1}(-3)\,dx$$

$$-\frac{1}{4}\times 1 + 0 + -1\left(\frac{3}{4}-\frac{1}{2}\right) - \left(\frac{3+\sqrt{17}}{8}-\frac{3}{4}\right)(2) - \left(1-\frac{3+\sqrt{17}}{8}\right)3$$

$$= \frac{\sqrt{17}-13}{8}$$

Question ID : 178

Continuity & Differentiability

8. Let $f : R \to R$ be defined as $f(x) = \begin{cases} \left[e^x\right], & x < 0 \\ ae^x + \left[x-1\right], & 0 \le x < 1 \\ b + \left[\sin(\pi x)\right], & 1 \le x < 2 \\ \left[e^{-x}\right] - c, & x \ge 2 \end{cases}$ where a, b, c $\in$ R and [t] denotes greatest integer less than or equal to t. Then, which of the following statements is true?

माना $f: R \to R$ $f(x) = \begin{cases} [e^x], & x < 0 \\ ae^x + [x-1], & 0 \le x < 1 \\ b + [\sin(\pi x)], & 1 \le x < 2 \\ [e^{-x}] - c, & x \ge 2 \end{cases}$ द्वारा परिभाषित है, $a, b, c \in R$ हैं तथा [t] महत्तम पूर्णांक t है।

तो निम्न कथनों में से कौनसा सत्य है ?

(1) There exists a, b, c $\in$ R such that f is continuous on R.

a, b, c $\in$ R का अस्तित्व है जिनके लिए f, R पर संतत है

(2) If f is discontinuos at exactly one point, then a + b + c = 1

यदि f मात्र एक बिन्दु पर असंतत है, तो a + b + c = 1

(3) If f is discontinuous at exactly one point, then a + b + c $\neq$ 1

यदि f मात्र एक बिन्दु पर असंतत है, तो a + b + c $\neq$ 1

(4) f is discontinuous at atleast two points, for any values of a, b and c

a, b, c के किन्हीं भी मानों के लिए f कम से कम बिन्दुओ पर असंतत है।

Ans. Official Answer NTA (3)

Sol.

Question ID : 179

Area Under Curve

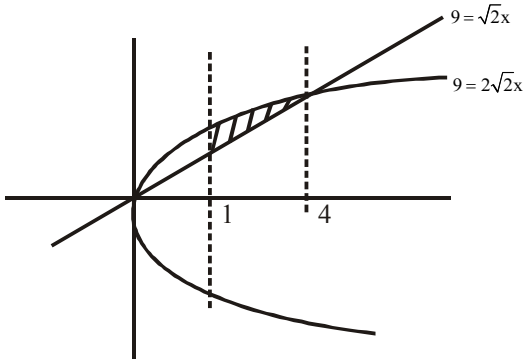
9. The area of the region $S = \{(x, y) : y^2 \le 8x, y \ge \sqrt{2}x, x \ge 1\}$ is :

क्षेत्र $S = \{(x, y) : y^2 \le 8x, y \ge \sqrt{2}x, x \ge 1\}$ का क्षेत्रफल है :

(1) $\frac{13\sqrt{2}}{6}$ (2) $\frac{11\sqrt{2}}{6}$ (3) $\frac{5\sqrt{2}}{6}$ (4) $\frac{19\sqrt{2}}{6}$

Ans. Official Answer NTA (2)

Sol. Required area



$$= \int_1^4 \left(2\sqrt{2x} - \sqrt{2}x\right)dx = \sqrt{2}\left[\frac{4.x^{3/2}}{3} - \frac{x^2}{2}\right]_1^4$$

$$= = \sqrt{2}\left[\frac{32}{3} - 8 - \frac{4}{3} + \frac{1}{2}\right]$$

$$= = \sqrt{2}\left[\frac{64 - 48 - 8 + 3}{6}\right] = \frac{11\sqrt{2}}{6}$$

Question ID : 1710

Differential Equation

10. Let the solution curve $y = y(x)$ of the differential equation $\left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right] x\frac{dy}{dx} = x + \left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right] y$ pass through the points $(1, 0)$ and $(2\alpha, \alpha)$, $\alpha > 0$. Then α is equal to :

माना अवकल समीकरण $\left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right] x\frac{dy}{dx} = x + \left[\frac{x}{\sqrt{x^2 - y^2}} + e^{\frac{y}{x}}\right] y$ का हल वक्र $y = y(x)$ बिन्दुओं $(1, 0)$ तथा $(2\alpha, \alpha)$, $\alpha > 0$ से होकर जाता है, तो α का मान बराबर है :

(1) $\frac{1}{2}\exp\left(\frac{\pi}{6} + \sqrt{e} - 1\right)$

(2) $\frac{1}{2}\exp\left(\frac{\pi}{3} + e - 1\right)$

(3) $\exp\left(\frac{\pi}{6} + \sqrt{e} + 1\right)$

(4) $2\exp\left(\frac{\pi}{3} + \sqrt{e} - 1\right)$

Ans. Official Answer NTA (1)

Sol. $\left(\frac{x}{\sqrt{x^2-y^2}}+e^{(y/x)}\right)\left(x\frac{dy}{dx}-y\right)=\alpha$

$$\Rightarrow\left(\frac{1}{\sqrt{1-\left(\frac{y}{x}\right)^2}}+e^{(y/x)}\right)d\left(\frac{y}{x}\right)=\frac{1}{x}dx$$

Inteqrating both sides

$$\sin^{-1}\left(\frac{y}{x}\right)+e^{(y/x)}=\ln x+c$$

as y (1) = 0 so c = 1

$$\sin^{-1}\left(\frac{y}{x}\right)+e^{(y/x)}=\ln x+1$$

$(2\alpha, \alpha)$ will satisfy it

$$\frac{\pi}{6}+\sqrt{e}-1=\ln(2\alpha)$$

$$\Rightarrow \alpha=\frac{1}{2}\exp\left(\frac{\pi}{6}+\sqrt{e}-1\right)$$

Question ID : 1711

Differential Equation

11. Let y = y(x) be the solution of the differential equation $x\left(1-x^2\right)\frac{dy}{dx}+\left(3x^2y-y-4x^3\right)=0, x>1$, with y(2) = – 2. Then y(3) is equal to :

माना अवकल समीकरण $x\left(1-x^2\right)\frac{dy}{dx}+\left(3x^2y-y-4x^3\right)=0, x>1$ का हल y = y(x) है तथा y(2) = – 2 तो y(3) बराबर है :

(1) –18 (2) –12 (3) –6 (4) –3

Ans. Official Answer NTA (1)

Sol. $\frac{dy}{dx}-\frac{y\left(3x^2-1\right)}{x\left(x^2-1\right)}=\frac{4x^3}{x\left(1-x^2\right)}$

If $= e^{-\int\left(\ln\left(x^2-1\right)+\ln x\right)} = \frac{1}{x\left(x^2-1\right)}$

$$\frac{y}{x\left(x^2-1\right)} = \int \frac{4x^3}{x\left(1-x^2\right)} \cdot \frac{1}{x\left(x^2-1\right)} dx = \int \frac{-4x}{\left(x^2-1\right)^2} dx = \frac{2}{x^2-1} + c$$

$\Rightarrow y = 2x + cx\,(x^2 - 1)$

Now $y\,(2) = -2 \Rightarrow c = -1$

So $y = 2x - x\,(x^2 - 1)$

$y\,(3) = 2 \times 3 - 3(8) = 6 - 24 = -18$

Question ID : 1712

Monotonocity

12. The number of real solutions of $x^7 + 5x^3 + 3x + 1 = 0$ is equal to _____.

$x^7 + 5x^3 + 3x + 1 = 0$ के वास्तविक हलों की संख्या है _____.

(1) 0 (2) 1 (3) 3 (4) 5

Ans. Official Answer NTA (2)

Sol. Let $f(x) = x^7 + 5x^3 + 3x + 1$

$f'(x) = 7x^6 + 15x^2 + 3 > 0$

So are real root as it is odd degree polynomial

Question ID : 1713

Hyperbola

13. Let the eccentricity of the hyperbola $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ be $\sqrt{\frac{5}{2}}$ and length of its latus retum be $6\sqrt{2}$, If $y = 2x + c$ is a tangent to the hyperbola H, then the value of c^2 is equal to :

माना अतिपरवलय $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ की उत्केन्द्रता $\sqrt{\frac{5}{2}}$ है तथा इसकी नाभिलंब जीवा की लंबाई $6\sqrt{2}$ है। यदि अतिपरलय H की एक स्पर्श रेखा $y = 2x + c$ है, तो c^2 का मान है :

(1) 18 (2) 20 (3) 24 (4) 32

Ans. Official Answer NTA (2)

Sol. $1 + \frac{b^2}{a^2} = \frac{5}{2} \Rightarrow \frac{b^2}{a^2} = \frac{3}{2}$ .....(1)

and $\frac{2b^2}{a} = 6\sqrt{2}$ .....(2)

from (1) and (2) $a = 2\sqrt{2}$, $b = 2\sqrt{3}$

Now condition of tangency

$c^2 = a^2m^2 - b^2$

$\Rightarrow$ $c^2 = 8 \times 4 - 12 = 20$

Question ID : 1714

Circle

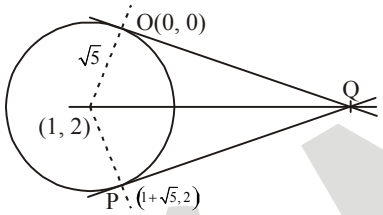
14. If the tangents drawn at the points O(0, 0) and $P\left(1+\sqrt{5}, 2\right)$ on the circle $x^2 + y^2 - 2x - 4y = 0$ intersect at the point Q, then the area of the triangle OPQ is euqal to :

यदि वृत्त $x^2 + y^2 - 2x - 4y = 0$ के बिन्दुओ O(0, 0) तथा $P\left(1+\sqrt{5}, 2\right)$ पर खींची गई स्पर्श रेखाऐं बिन्दु Q पर मिलती हैं, तो त्रिभुज OPQ का क्षेत्रफल बराबर है :

(1) $\frac{3+\sqrt{5}}{2}$ (2) $\frac{4+2\sqrt{5}}{2}$ (3) $\frac{5+3\sqrt{5}}{2}$ (4) $\frac{7+3\sqrt{5}}{2}$

Ans. Official Answer NTA (3)

Sol.



Tangent at (0, 0) : $x + 2y = 0$

$OP \perp PQ$ and Q is on ine $x + 2y = 0$

So $Q\left(\left(1+\sqrt{5}\right), -\frac{\left(1+\sqrt{5}\right)}{2}\right)$

So area of $\Delta = \begin{vmatrix} 0 & 0 & 1 \\ 1+\sqrt{5} & 2 & 1 \\ 1+\sqrt{5} & \frac{-\left(1+\sqrt{5}\right)}{2} & 1 \end{vmatrix} \times \frac{1}{2}$

$= \frac{5+3\sqrt{5}}{2}$

Question ID : 1715

3D Geometry

15. If two distinct points Q, R lie on the line of intersection of the planes $-x + 2y - z = 0$ and $3x - 5y + 2z = 0$ and $PQ = PR = \sqrt{18}$ where the point P is $(1, -2, 3)$, then the area of the triangle PQR is equal to :

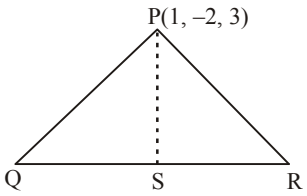
यदि समतलों $-x + 2y - z = 0$ तथा $3x - 5y + 2z = 0$ की प्रतिच्छेदन रेखा पर दो भिन्न बिन्दु Q, R हैं तथा $PQ = PR = \sqrt{18}$ है, जहां P बिन्दु $(1, -2, 3)$ है, तो त्रिभुज PQR का क्षेत्रफल बराबर है :

(1) $\frac{2}{3}\sqrt{38}$ (2) $\frac{4}{3}\sqrt{38}$ (3) $\frac{8}{3}\sqrt{38}$ (4) $\sqrt{\frac{152}{3}}$

Ans. Official Answer NTA (2)

Sol. Plane are $-x + 2y - z = 0$

$3x - 5y + 2z = 0$



Let R (k, k, k) Now $PQ = \sqrt{18}$

$\Rightarrow (k-2)^2 + (k+2)^2 + (k-3)^2 = 18$

$\Rightarrow 3k^2 - 4k - 4 = 0$

$\Rightarrow k = -\frac{2}{3}, 2$

So $Q\left(-\frac{2}{3}, -\frac{2}{3}, -\frac{2}{3}\right)$ and R(2, 2, 2)

mid point $S \equiv \left(\frac{2}{3}, \frac{2}{3}, \frac{2}{3}\right)$

base of $\Delta PQR = QR = \sqrt{3}.\frac{8}{3}$, height $= \frac{\sqrt{114}}{3} = PS$

area $= \frac{1}{2} \times \sqrt{3} \times \frac{8}{3} \times \frac{\sqrt{114}}{3} = \frac{4}{3}\sqrt{38}$

Question ID : 1716

3D Geometry

16. The acute angle between the planes P_1 and P_2, when P_1 and P_2 are the planes passing through the intersection of the planes $5x + 8y + 13z - 29 = 0$ and $8x - 7y + z - 20 = 0$ and the points (2, 1, 3) and (0, 1, 2), respectively, is :

मान समतलों $5x + 8y + 13z - 29 = 0$, $8x - 7y + z - 20 = 0$ के प्रतिच्छेदन और बिन्दुओं (2, 1, 3) तथा (0, 1, 2) से होकर जाने वाले दो समतल क्रमश: P_1 तथा P_2 हैं। तो समतलों P_1 तथा P_2 के बीच न्यून कोण है :

(1) $\frac{\pi}{3}$ (2) $\frac{\pi}{4}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{12}$

Ans. Official Answer NTA (1)

Sol. The required plane $P_1 + \lambda P_2 = 0$

$\Rightarrow (5 + 8\lambda)x + (8 - 7\lambda)y + (13 + \lambda)z = 29 + 20\lambda$

(2, 1, 3) will satisfy if $\lambda = \frac{7}{2}$

and plane : $2x - y + z = 6$ (0, 1, 2) will satisfy if $\lambda = \frac{1}{5}$

So angle between then and plane : $x + y + 2z = 5$

$$\cos\theta = \left|\frac{2-1+2}{\sqrt{6}\sqrt{6}}\right| = \frac{1}{2}$$

$$\Rightarrow \theta = \frac{\pi}{3}$$

Question ID : 1717

Vectors

17. Let the plane P : $\vec{r}.\vec{a} = d$ contain the line of intersection of two planes $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right) = 6$ and $\vec{r}.\left(-6\hat{i}+5\hat{j}-\hat{k}\right) = 7$. If the plane P passes through the point $\left(2, 3, \frac{1}{2}\right)$, then the value of $\frac{|13\vec{a}|^2}{d^2}$ is equal to :

माना दो समतलतों $\vec{r}.\left(\hat{i}+3\hat{j}-\hat{k}\right) = 6$ तथा $\vec{r}.\left(-6\hat{i}+5\hat{j}-\hat{k}\right) = 7$ की प्रतिच्छेदन रेखा समतल P : $\vec{r}.\vec{a} = d$ में है। यदि समतल P, बिन्दु $\left(2, 3, \frac{1}{2}\right)$ से होकर जाता है, तो $\frac{|13\vec{a}|^2}{d^2}$ का मान है :

(1) 90 (2) 93 (3) 95 (4) 97

Ans. Official Answer NTA (2)

Sol. $P_1 : x + 3y - z = 6$

$P_2 : -6x + 5y - z = 7$

Let the required plane is $P_1 + \lambda P_2 = 0$

$\Rightarrow (1 - 6\lambda)x + (3 + 5\lambda)y - z(1 + \lambda) = 6 + 7\lambda$

$\left(2, 3, \frac{1}{2}\right)$ will satisfy it

$\lambda = 1$

So the plane : $-5x + 8y - 2z - 13 = 0$

$\Rightarrow \vec{r}.\left(-5\hat{i} + 8\hat{j} - 2\hat{k}\right) = 13$

$\frac{|13\vec{a}|^2}{d^2} = \frac{13^2}{13^2}.(25 + 64 + 4) = 93$

Question ID : 1718

Probability

18. The probability, that in a randomly selected 3-digit number at least two digits are odd, is :

3- अंकों की यादृच्छया चुनी गई संख्या में कम से कम दो अंक विषम होने की प्रायिकता है :

(1) $\frac{19}{36}$ (2) $\frac{15}{36}$ (3) $\frac{13}{36}$ (4) $\frac{23}{36}$

Ans. Official Answer NTA (1)

Sol. 000 + E00 + 0E0 + 00E

$P = \frac{5\times5\times5 + 4\times5\times5 + 5\times5\times5 + 5\times5\times5}{9\times10\times10}$

$= \frac{5\times5\times19}{9\times10\times10} = \frac{19}{36}$

Question ID : 1719

Heights & Distances

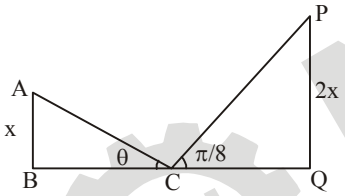
19. Let AB and PQ be two vertical poles, 160 m apart from each other. Let C be the middle point of B and Q, which are feet of these two poles. Let $\frac{\pi}{8}$ and θ be the angles of elevation from C to P and A, respectivley. If the height of pole PQ is twice the height of pole AB, then $\tan^2\theta$ is equal to :

माना दो ऊर्ध्वाधर पोल AB तथा PQ एक दूसरे से 160 मीटर की दूरी हैं। माना B तथा Q इन दो पोलों के पाद हैं, जिनके मध्य बिन्दु C से P तथा A के उन्नयन कोण क्रमशः $\frac{\pi}{8}$ तथा θ हैं। यदि पोल PQ की ऊँचाई, पोल AB की ऊँचाई की दो गुना है तो $\tan^2\theta$ का मान है :

(1) $\frac{3-2\sqrt{2}}{2}$ (2) $\frac{3+\sqrt{2}}{2}$ (3) $\frac{3-2\sqrt{2}}{4}$ (4) $\frac{3-\sqrt{2}}{4}$

Ans. Official Answer NTA (3)

Sol.



$$\tan\theta = \frac{x}{BC} \quad \text{———(1)}$$

$$\tan\frac{\pi}{8} = \frac{2x}{BC} \quad \text{———(2)}$$

dividing $\frac{\tan\theta}{\tan\frac{\pi}{8}} = \frac{1}{2} \Rightarrow \tan\theta = \frac{\sqrt{2}-1}{2}$

$$\Rightarrow \tan^2\theta = \frac{3-2\sqrt{2}}{4}$$

Question ID : 1720

Mathematical Reasoning

20. Let p, q, r be three logical statements. Consider the compound statements

$S_1 : ((\sim p) \vee q) \vee ((\sim p) \vee r)$ and

$S_2 : p \rightarrow (q \vee r)$

Then, which of the following is NOT true ?

(1) If S_2 is True, then S_1 is True (2) If S_2 is False, then S_1 is False

(3) If S_2 is False, then S_1 is True (4) If S_1 is False, then S_2 is False

माना p, q, r तीन तर्कसंगत कथन हैं। निम्न कथनों पर विचार कीजिए :

$S_1 : ((\sim p) \vee q) \vee ((\sim p) \vee r)$ तथा

$S_2 : p \rightarrow (q \vee r)$

तो निम्न में से कौन–सा सत्य नहीं है ?

(1) यदि S_2 सत्य है, तो S_1 सत्य है (2) यदि S_2 असत्य है, तो S_1 असत्य है

(3) यदि S_2 असत्य है, तो S_1 सत्य है (4) यदि S_1 असत्य है, तो S_2 असत्य है

Ans. Official Answer NTA (3)

Sol. $S_1 : p \rightarrow (q \vee r) \equiv \sim p \vee (q \vee r)$

$= (\sim p \vee q) \vee (\sim p \vee r)$

$= S_1$

So S_1 and S_2 are same.

SECTION – B

Question ID : 1721

Set & Relations

21. Let R_1 and R_2 be relations on the set {1, 2, ..., 50} such that

$R_1 = \{(p, p^n) : p$ is a prime and $n \geq 0$ is an integer$\}$ and

$R_2 = \{(p, p^n) : p$ is a prime and $n = 0$ or $1\}$

Then, the number of elements in $R_1 - R_2$ is _____.

माना समुच्चय 1, 2, ..., 50} पर दो संबंध R_1 तथा R_2 निम्न प्रकार हैं :

$R_1 = \{(p, p^n) : p$ एक अभाज्य संख्या है तथा $n \geq 0$ एक पूर्णांक$\}$ तथा

$R_2 = \{(p, p^n) : p$ एक अभाज्य संख्या है तथा $n = 0$ या 1 है$\}$

तो $R_1 - R_2$ में अवयवों की संख्या है _____

Ans. Official Answer NTA (8)

Sol.

Question ID : 1722

Quadratic Equation

22. The number of real solutions of the equation $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ is :

समीकरण $e^{4x} + 4e^{3x} - 58e^{2x} + 4e^x + 1 = 0$ के वास्तविक हलों की संख्या है :

Ans. Official Answer NTA (2)

Sol.

Question ID : 1723

Statistics

23. The mean and standard deviation of 15 observations are found to be 8 and 3 respectively. On rechecking it was found that, in the observations, 20 was misread as 5. Then, the correct variance is equal to _____.

15 प्रेक्षणों के माध्य तथा मानक विचलन क्रमशः 8 तथा 3 ज्ञात किए गए। पुनः जाँच करने पर पाया गया कि प्रेक्षण 20 को गलती से 5 पढ़ा गया था। तो सही प्रसरण बराबर है _____.

Ans. Official Answer NTA (17)

Sol.

Question ID : 1724

Vectors

24. If $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ are coplanar vectors and $\vec{a}.\vec{c} = 5$, $\vec{b} \perp \vec{c}$ then $122(c_1 + c_2 + c_3)$ is equal to _____.

यदि $\vec{a} = 2\hat{i} + \hat{j} + 3\hat{k}$, $\vec{b} = 3\hat{i} + 3\hat{j} + \hat{k}$ तथा $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ के सह–तलीय है $\vec{a}.\vec{c} = 5$ है तथा $\vec{b} \perp \vec{c}$ के लंबवत है तब $122(c_1 + c_2 + c_3)$ का मान है _____.

Ans. Official Answer NTA (150)

Sol.

Question ID : 1725

Straight Line

25. A ray of light passing through the point P(2, 3) reflects on the x-axis at point A and the reflected ray passes through the point Q(5, 4). Let R be the point that divides the line segment AQ internally into the ratio 2 : 1. Let the co-ordinates of the foot of the perpendicular M from R on the bisector of the angle PAQ be(α, β). Then, the value of $7\alpha + 3\beta$ is equal to _____.

बिन्दु P(2, 3) से होकर जाने वाली रोशनी की एक किरण x-अक्ष पर बिन्दु A से परावर्तित होती है तथा परावर्तित किरण बिन्दु Q(5, 4) से होकर जाती है। माना बिन्दु R रेखाखण्ड AQ को 2 : 1 के अनुपात में अंतः विभाजित करता है। माना कोण PAQ के समद्विभाजक पर से R लंब के पाद M के निर्देषांक (α, β) हैं। तो $7\alpha + 3\beta$ का मान बराबर है _____.

Ans. Official Answer NTA (31)

Sol.

Question ID : 1726

Tangent and normal

26. Let *l* be a line which is normal to the curve $y = 2x^2 + x + 2$ at a point P on the curve. If the point Q(6, 4) lies on the line *l* and O is origin, then the area of the triangle OPQ is equal to _____.

माना *l* एक रेखा है जो वक्र $y = 2x^2 + x + 2$ के बिन्दु P पर अभिलंब है। यदि बिन्दु Q(6, 4), रेखा *l* पर है तथा O मूल बिन्दु है, तो त्रिभुज OPQ का क्षेत्रफल बराबर है _____.

Ans. Official Answer NTA (13)

Sol.

Question ID : 1727

Sequence & progression

27. Let $A = \{1, a_1, a_2, \ldots\ldots a_{18}, 77\}$ be a set of integers with $1 < a_1 < a_2 < \ldots\ldots < a_{18} < 77$. Let the set $A + A = \{x + y : x, y \in A\}$ contain exactly 39 elements. Then, the value of $a_1 + a_2 + \ldots\ldots + a_{18}$ is equal to _____.

माना पूर्णांकों का एक समुच्चय $A = \{1, a_1, a_2, \ldots\ldots a_{18}, 77\}$ है तथा $1 < a_1 < a_2 < \ldots\ldots < a_{18} < 77$ हैं। माना समुच्चय $A + A = \{x + y : x, y \in A\}$ में मात्र 39 अवयव हैं। तो $a_1 + a_2 + \ldots\ldots + a_{18}$ का मान है _____.

Ans. Official Answer NTA (702)

Sol.

Question ID : 1728

Binomial Theorem

28. The number of positive integers k such that the constant term in the binomial expansion of $\left(2x^3+\frac{3}{x^k}\right)^{12}$, $x \neq 0$ is $2^8 . \ell$, where ℓ is an odd integer, is _____.

धनात्मक पूर्णांकों k जिनके लिए $\left(2x^3+\frac{3}{x^k}\right)^{12}$, $x \neq 0$ के द्विपद प्रसार में अचर पद $2^8 . \ell$ है, जहाँ ℓ एक विषम संख्या है, की संख्या है _____.

Ans. Official Answer NTA (2)

Sol.

uestion ID : 1729

omplex number

9. he number of elements in the set $\{z = a + ib \in C : a, b \in Z \text{ and } 1 < |z - 3 + 2i| < 4\}$ is _____.

मुच्चय $\{z = a + ib \in C : a, b \in Z$ तथा $1 < |z - 3 + 2i| < 4\}$हैं में अवयवों की संख्या है_____.

ns. fficial Answer NTA (40)

ol.

uestion ID : 1730

ircle

0. et the lines $y + 2x = \sqrt{11} + 7\sqrt{7}$ and $2y + x = 2\sqrt{11} + 6\sqrt{7}$ be normal to a circle . If the line $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ is tangent to the circle C, then the value f $(5h - 8k)^2 + 5r^2$ is equal to _____.

ाना रेखाऐं $y + 2x = \sqrt{11} + 7\sqrt{7}$ तथा $2y + x = 2\sqrt{11} + 6\sqrt{7}$ एक वृत्त $C : (x-h)^2 + (y-k)^2 = r^2$ के लंबवत है। दि $\sqrt{11}y - 3x = \frac{5\sqrt{77}}{3} + 11$ वृत्त C की स्पर्श रेखा है, तो $(5h - 8k)^2 + 5r^2$ का मान बराबर है _____.

ns. fficial Answer NTA (816)

ol.