JEE Main June 2022

Question Paper With Text Solution

28 June | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JUNE 2022 | 28TH JUNE SHIFT-2

SECTION – A

Question ID : 101261

Set & Relations

1. Let $R_1 = \{(a, b) \in N \times N : |a - b| \leq 13\}$ and $R_2 = \{(a, b) \in N \times N : |a - b| \neq 13\}$. Then on N :

(1) Both R_1 and R_2 are equivalence relations

(2) Neither R_1 nor R_2 is an equivalence relation

(3) R_1 is an equivalence relation but R_2 is not

(4) R_2 is an equivalence relation but R_1 is not

माना $R_1 = \{(a, b) \in N \times N : |a - b| \leq 13\}$ तथा $R_2 = \{(a, b) \in N \times N : |a - b| \neq 13\}$ हैं। तो N पर :

(1) R_1 तथा R_2 दोनों तुल्यता संबंध हैं

(2) न तो R_1 ही R_2 एक तुल्यता संबंध है

(3) R_1 एक तुल्यता संबंध है परन्तु R_2 नहीं है

(4) R_2 एक तुल्यता संबंध है परन्तु R_1 नहीं है

Ans. Official Answer NTA (2)

Sol. $R_1 = \{(a, b) \in N \times N : |a - b| \leq 13\}$ and $R_2 = \{(a, b) \in N \times N : |a - b| \neq 13\}$ in R_1 for transitive let 3 numbers a = 1, b = 12, c = 19

$\because |a - b| \leq 13$

$\therefore |1 - 12| \leq 13 \Rightarrow (1, 12) \in R_1$,

similarly $(12, 19) \in R_1$ but $(1, 19) \notin R_1$. So R_1 is not transitive. Therefore R_1 is not equivalance.

In R_2 for transistive let 3 numbers.

a = 10, b = 15, c = 23

$\because |a - b| \neq 13$

$\therefore |10 - 15| \neq 13 \Rightarrow (10, 15) \in R_2$, Similarly $(15, 23) \in R_2$ but $(10, 23) \in R_2$

So R_2 is not transitive

Therefore R_2 is not equivalance

Question ID : 101262

Quadratic Equation

2. Let f(x) be a quadratic polynomial such that f(–2) + f(3) = 0. If one of the roots of f(x) = 0 is –1, then the sum of the roots of f(x) = 0 is equal to :

माना f(x) एक द्विघातीय बहुपद है जिसके लिए f(–2) + f(3) = 0 है। यदि f(x) = 0 का एक मूल –1 है, तो f(x) = 0 के मूलों का योगफल बराबर है :

(1) $\frac{11}{3}$ (2) $\frac{7}{3}$ (3) $\frac{13}{3}$ (4) $\frac{14}{3}$

Ans. Official Answer NTA (1)

Sol. Let $f(x) = a(x-\alpha)(x-\beta)$ ....(1)

for f(x) = 0; x = –1. [$\because$ one root is –1]

$\Rightarrow f(x) = 0 = a(x+1)(x-\beta)$ ....(2)

Now, f(–2) + f(3) = 0

$$a\left[(-2+1).(-2-\beta)\right]+a\left[(3+1)(3-\beta)\right]=0$$

$$a\left[2+\beta+12-4\beta\right]=0$$

$\Rightarrow 14-3\beta=0$ $\Rightarrow \beta=\frac{14}{3}$

$\alpha = -$ (given), $\beta=\frac{14}{3}$

sum of roots $=\frac{14}{3}-1=\frac{11}{3}$

Question ID : 101263

P & C

3. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to :

चार बच्चों C_1, C_2, C_3, C_4 में एक तरह की 30 कैंडी इस प्रकार बांटने के तरीकों की संख्या, जिनमें C_2 को कम से कम 4 तथा अधिक से अधिक 7 कैंडी, C_3 को कम से कम 2 तथा अधिक से अधिक 6 कैंडी मिलें, हैं :

(1) 205 (2) 615 (3) 510 (4) 430

Ans. Official Answer NTA (4)

Sol. By multinomial theorem, no. of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4. So we have to find coeff. of x^{30} in

$$\left(1+x+x^2+\text{..........upto }\infty\right)\times\left(x^4+x^5+x^6+x^7\right)\times\left(x^2+x^3+\text{.....}x^6\right)\times\left(1+x+x^2+\text{.....upto }\infty\right)$$

= coeff. of x^{30} in $(1 + x + ...)^2 (x^4 + ... x^7) (x^2 + ...x^6)$

= coeff. of x^{24} in $(1 + x + x^2 + ...)^2 \times x^4(x^1 + ... x^3) \times x^2(x + ... x^4)$ [x^4 and x^2 common from C_2, C_3]

Using formula for G.P. and ∞, G.P.

= coeff. of x^{24} in $\frac{1}{(1-x)^2}\times\frac{1-x^4}{1-x}\times\frac{1-x^5}{1-x}$

= coeff. of x^{24} in $(1-x^4)(1-x^5).(1-x)^{-4}$

= coeff. of x^{24} in $(1-x^4-x^5+x^9)(1-x)^{-4}$

$= {}^{27}C_{24} - {}^{23}C_{20} - {}^{22}C_{19} + {}^{18}C_{15} = 430$

Question ID : 101264

Binomial Theorem

4. The term independent of x in the expansion of $(1-x^2+3x^3)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$, $x \neq 0$ is :

$(1-x^2+3x^3)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$, $x \neq 0$ के प्रसार में से स्वतंत्र पद है :

(1) $\frac{7}{40}$ (2) $\frac{33}{200}$ (3) $\frac{39}{200}$ (4) $\frac{11}{50}$

Ans. Official Answer NTA (2)

Sol. $(1-x^2+3x^2)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$, $x \neq 0$

general term in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)$ is

$$T_{r+1} = {}^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(\frac{-1}{5x^2}\right)^r$$

$$= {}^{11}C_r\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^r x^{33-5r}$$

So term independent of x in given expression

$$= {}^{-11}C_7\left(\frac{5}{2}\right)^4\left(-\frac{1}{5}\right)^7 = \frac{11\times10\times9\times8}{24}\times\frac{1}{16\times125}$$

$$= \frac{33}{200}$$

Question ID : 101265

Sequence & progression

5. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1 : 7 and a + n = 33, then the value of n is :

यदि a तथा 100 के बीच n समान्तर माध्य इस प्रकार डाले गए हैं कि पहले माध्य का अंतिम माध्य से अनुपात 1 : 7 है तथा a + n = 33 है, तो n का मान है :

(1) 21 (2) 22 (3) 23 (4) 24

Ans. Official Answer NTA (3)

Sol. $a, A_1, A_2, \ldots\ldots\ldots\ldots A_n, 100$

Let the common difference of above A.P. is d then $\frac{a+d}{100-d}=\frac{1}{7}$

$7a + 8d = 100$ ....(1)

and $a + n = 33$ ....(2)

also, $100 = a + (n + 1)d$

$\Rightarrow 100 = a+(34-a)\times\left(\frac{100-7a}{8}\right)$

$800 = 8a + 7a^2 - 338a + 3400$

$7a^2 - 330a + 2600 = 0$

$7a^2 - 260a - 70a + 2600 = 0$

$a = 10, \frac{260}{7}$ but $a \neq \frac{260}{7}$

$\therefore$ from eqn (2)

$10 + n = 33$

$\Rightarrow n = 23$

Question ID : 101266

Continuity & Differentiability

6. Let f, g : R → R be functions defined by $f(x)=\begin{cases}[x] & , \quad x<0 \\ |1-x| & , \quad x\geq 0\end{cases}$ and $g(x)=\begin{cases}e^x-x & , \quad x<0 \\ (x-1)^2-1 & , \quad x\geq 0\end{cases}$

where [x] denote the greatest integer less than or equal to x. Then, the function fog is discontinuous at exactly :

(1) one point (2) two points (3) three points (4) four points

माना फलन f, g : R → R $f(x)=\begin{cases}[x] & , \quad x<0 \\ |1-x| & , \quad x\geq 0\end{cases}$ तथा $g(x)=\begin{cases}e^x-x & , \quad x<0 \\ (x-1)^2-1 & , \quad x\geq 0\end{cases}$ द्वारा परिभाषित है,

जहाँ [x] महत्तम पूर्णांक ≤ x है। तो फलन fog कितने बिन्दुओं पर असंतत है :

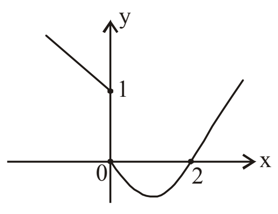
(1) एक बिन्दु (2) दो बिन्दु (3) तीन बिन्दु (4) चार बिन्दु

Ans. Official Answer NTA (2)

Sol. $f(x)=\begin{cases}[x] & ; \quad x<0 \\ |1-x| & ; \quad x\geq 0\end{cases}$ and $g(x)=\begin{cases}e^x-x & ; \quad x<0 \\ (x-1)^2-1 & ; \quad x\geq 0\end{cases}$

$$f(x)=\begin{cases}|1+x-e^x| & x<0 \\ 1 & x=0 \\ \left[(x-1)^2-1\right] & 0<x<2 \\ \left|2-(x-1)^2\right| & x\geq 2\end{cases}$$

graph of fog(x)



So x = 0 and 2 are the 2 points where fog(x) is discontinuous.

Question ID : 101267

Definite Integration

7. Let $f : R \rightarrow R$ be a differentiable function such that $f\left(\frac{\pi}{4}\right)=\sqrt{2}$, $f\left(\frac{\pi}{2}\right)=0$ and $f'\left(\frac{\pi}{2}\right)=1$ and let $g(x)=\int_{x}^{\frac{\pi}{4}}\left(f'(t)\sec t+\tan t\sec t\, f(t)\right)dt$ for $x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right)$. Then $\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-} g(x)$ is equal to :

माना $f : R \rightarrow R$ एक अवकलनीय फलन है जिसके लिए $f\left(\frac{\pi}{4}\right)=\sqrt{2}$, $f\left(\frac{\pi}{2}\right)=0$ तथा $f'\left(\frac{\pi}{2}\right)=1$ है। माना $g(x)=\int_{x}^{\frac{\pi}{4}}\left(f'(t)\sec t+\tan t\sec t\, f(t)\right)dt$, $x\in\left[\frac{\pi}{4},\frac{\pi}{2}\right)$ है। तो $\lim\limits_{x\to\left(\frac{\pi}{2}\right)^-} g(x)$ बराबर है :

(1) 2 (2) 3 (3) 4 (4) –3

Ans. Official Answer NTA (2)

Sol. Given $f\left(\frac{\pi}{4}\right)=\sqrt{2}$, $f\left(\frac{\pi}{2}\right)=0$ and $f'\left(\frac{\pi}{2}\right)=1$

Also

$$g(x)=\int_{x}^{\frac{\pi}{4}}\left[f'(t).\sec t+\tan t.\sec t.f(t)\right]dt$$

$$\because \int (u).v+v'.u).dx=\int (u.v).dx=u.v.$$

$$\therefore g(x)=\left[\sec t.f(t)\right]_{x}^{\frac{\pi}{4}}$$

g(x) = 2 – sec x.f(x)

Now,

$$\lim_{x\to\frac{\pi}{2}} g(x)=\lim_{h\to 0} g\left(\frac{\pi}{2}-h\right)=2-\lim_{h\to 0}\sec\left(\frac{\pi}{2}-h\right).f\left(\frac{\pi}{2}-h\right)$$

$$\lim_{x\to\frac{\pi}{2}} g(x)=2-\lim_{h\to 0}\operatorname{cosec} h\, f\left(\frac{\pi}{2}-h\right)$$

$$=2-\lim_{h\to 0}\frac{f\left(\frac{\pi}{2}-h\right)}{\cos h}$$

using Lopital rules

$$=2+\lim_{h\to 0}\frac{f'\left(\frac{\pi}{2}-h\right)}{\cos h}=2+1=3$$

Question ID : 101268

Definite Integration

8. Let $f : R \rightarrow R$ be a continuous function satisfying $f(x) + f(x + k) = n$, for all $x \in R$ where $k > 0$ and n is a positive integer. If $I_1 = \int_0^{4nk} f(x)dx$ and $I_2 = \int_{-k}^{3k} f(x)dx$, then :

माना $f : R \rightarrow R$ एक संतत फलन है जो सभी $x \in R$ के लिए $f(x) + f(x + k) = n$ को संतुष्ट करता है, जहाँ $x > 0$ है तथा n एक धनात्मक पूर्णांक है। यदि $I_1 = \int_0^{4nk} f(x)dx$ तथा $I_2 = \int_{-k}^{3k} f(x)dx$ हैं, तो :

(1) $I_1 + 2I_2 = 4nk$ (2) $I_1 + 2I_2 = 2nk$ (3) $I_1 + nI_2 = 4n^2k$ (4) $I_1 + nI_2 = 6n^2k$

Ans. Official Answer NTA (3)

Sol. Given $f : R \rightarrow R$ and $f(x) + f(x + k) = n \ \forall \ x \in R$ ——(1)

for $x \rightarrow x + k$

$f(x + k) + f(x + 2k) = n$ ——(2)

so $f(x + 2k) = f(x)$ [from (1) and (2)]

therefore period of f(x) is 2k.

Now,

$$I_1 = \int_0^{4nk} f(x).dx = 2n\int_0^{2K} f(x).dx$$

$$= 2n\left[\int_0^{k} f(x).dx + \int_k^{2k} f(x).dx\right]$$

in $\int_k^{2k} f(x).dx$ put $x = t + k$ $\Rightarrow dx = dt$

$$= 2n\left[\int_0^{k} f(x).dx + \int_0^{k} f(t+k).dt\right] = 2n.(nk)$$

$=2n^2k$

Also

in $I_2 = \int_{-k}^{3k} f(x).dx = 2\int_0^{2k} f(x).dx = 2nk$

$\Rightarrow I_1 + I_2 \times n = 4n^2k$

Question ID : 101269

Area Under Curve

9. The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$ and the x-axis is :

वक्र $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$ तथा x-अक्ष से घिरे क्षेत्र का क्षेत्रफल है :

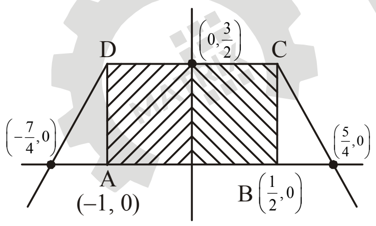
(1) $\frac{4}{9}$ (2) $\frac{45}{16}$ (3) $\frac{27}{8}$ (4) $\frac{63}{16}$

Ans. Official Answer NTA (3)

Sol. $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$

$$y = \begin{cases} 2x - \frac{7}{2} & x < -1 \\ \frac{3}{2} & -1 \le x \le \frac{1}{2} \\ \frac{5}{2} - 2x & x > \frac{1}{2} \end{cases}$$

graph of y.
required area is area of trapezium ABCD



$= \frac{1}{2}$ [sum of parallel sides] × distance between parallel sides

$= \frac{1}{2}\left(3 + \frac{3}{2}\right) . \frac{3}{2} = \frac{27}{8}$ sq. units

Question ID : 101270

Differential Equation

10. Let x = x(y) be the solution of the differential equation $2y\,e^{\frac{x}{y^2}}dx + \left(y^2 - 4xe^{\frac{x}{y^2}}\right)dy = 0$ such that x(1) = 0. Then, x(e) is equal to :

माना अवकल समीकरण $2y\,e^{\frac{x}{y^2}}dx + \left(y^2 - 4xe^{\frac{x}{y^2}}\right)dy = 0$ का हल x = x (y) इस प्रकार है कि x(1) = 0 है तो x(e) बराबर है :

(1) $e\log_e(2)$ (2) $-e\log_e(2)$ (3) $e^2\log_e(2)$ (4) $-e^2\log_e(2)$

Ans. Official Answer NTA (4)

Sol. $2.ye^{\left(\frac{x}{y^2}\right)}.dx + \left(y^2 - 4x.e^{\left(\frac{x}{y^2}\right)}\right).dy = 0$

$\Rightarrow 2e^{\left(\frac{x}{y^2}\right)}\left[y.dx - 2xdy\right] = -y^2.dy$

$\because d\left(\frac{x}{y^2}\right) = \frac{y^2.dx - x.2y.dy}{y^4}$

So multiple both sides by "$\frac{y}{y^4}$"

$\Rightarrow 2e^{\left(\frac{x}{y^2}\right)}\left[\frac{y^2.dx - 2xy.dy}{y^4}\right] = -\frac{dy}{y}$

$\Rightarrow \int 2e^{\left(\frac{x}{y^2}\right)}.d\left(\frac{x}{y^2}\right) = \int -\frac{dy}{y}$

$\Rightarrow 2e^{\left(\frac{x}{y^2}\right)} = -\ell n\,y + c$ ....(1)

given x(1) = 0 $\Rightarrow$ x = 0, y = 1 $\Rightarrow$ c = 2.

$\Rightarrow 2e^{\left(\frac{x}{y^2}\right)} = -\ell n\,y + 2$

for x(e)

$\Rightarrow 2e^{\left(\frac{x}{e^2}\right)} = -\ell n\,e + 2 = -1 + 2$

$\Rightarrow 2e^{\frac{x}{e^2}} = 1$ $\Rightarrow e^{\left(\frac{x}{e^2}\right)} = \frac{1}{2}$

$\frac{x}{e^2} = \ell n\left(\frac{1}{2}\right)$ $\quad\quad \Rightarrow x = e^2 \times -\ell n 2$

Question ID : 101271

Differential Equation

11. Let the slope of the tangent to a curve y = f(x) at (x, y) be given by 2 tan x(cos x – y). If the curve passes through the point $\left(\frac{\pi}{4}, 0\right)$, then the value of $\int_0^{\frac{\pi}{2}} y dx$ is equal to :

माना एक वक्र y = f(x) के बिन्दु (x, y) पर स्पर्श रेखा की प्रवणता 2 tan x(cos x – y) है। यदि यह वक्र बिन्दु $\left(\frac{\pi}{4}, 0\right)$ से होकर जाता है, तो $\int_0^{\frac{\pi}{2}} y dx$ का मान बराबर :

(1) $\left(2-\sqrt{2}\right)+\frac{\pi}{\sqrt{2}}$ (2) $2-\frac{\pi}{\sqrt{2}}$ (3) $\left(2+\sqrt{2}\right)+\frac{\pi}{\sqrt{2}}$ (4) $2+\frac{\pi}{\sqrt{2}}$

Ans. Official Answer NTA (2)

Sol. $\frac{dy}{dx} = 2\tan x\left(\cos x - y\right)$

$\frac{dy}{dx} + 2\tan x.y = 2.\sin x.$ [its a linear D.E.]

I.F. = $e^{\int 2\tan x.dx} = \sec^2 x$

∴ solution fo differential equation will be

$y.\left(I.f\right) = \int Q.\left(IF\right).dx$

$y.\sec^2 x = \int 2.\sin x.\sec^2 x.dx$

$y.\sec^2 x = 2\sec x + c$ ———(1)

Curve is passing through $\left(\frac{\pi}{4}, 0\right)$

$0 = 2\sqrt{2} + c$ $\quad\quad \Rightarrow c = -2\sqrt{2}$

from equation (1)

$y = 2.\cos x - 2\sqrt{2}\cos^2 x$

$\int_0^{\frac{\pi}{2}} y.dx = \int_0^{\frac{\pi}{2}} \left(2\cos x - 2\sqrt{2}\cos^2 x\right).dx$

$= 2-2\sqrt{2}\times\frac{\pi}{4} = 2-\frac{\pi}{\sqrt{2}}$

Question ID : 101272

Straight Line

12. Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3, which passes through the point P(2, 3), intersects L_2 at A and L_1 at B. If the point P divides the line-segmetn AB, internally in the ratio 1 : 3, then the area of the triangle is equal to :

माना एक त्रिभुज रेखाओं $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ तथा L_3 से घिरा है। रेखा L_3 बिन्दु P(2, 3) से होकर जाती है, L_2 को बिन्दु A पर काटती है तथा L_1 को बिन्दु B पर काटती है। यदि बिन्दु P, रेखा–खण्ड AB को अंतः 1 : 3 के अनुपात में विभाजित करता है, तो त्रिभुज का क्षेत्रफल बराबर है :

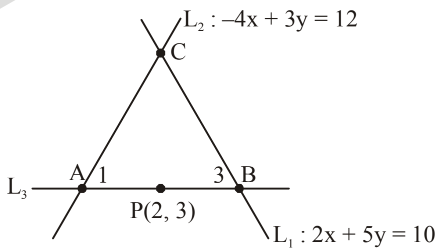
(1) $\frac{110}{13}$ (2) $\frac{132}{13}$ (3) $\frac{142}{13}$ (4) $\frac{151}{13}$

Ans. Official Answer NTA (2)

Sol. $L_1 : 2x + 5y = 10$ ———(1)

$L_2 : -4x + 3y = 12$ ———(2)

Solving eq. (1) and eq. (2) we get



$C = \left(\frac{-15}{13}, \frac{32}{13}\right)$

Now

let $B\left(x_1, \frac{1}{5}(10-2x_1)\right); A = \left(x_2, \frac{1}{3}12+4x_2\right)$

from section formula $2 = \frac{x_1+3x_2}{4}$

$\Rightarrow x_1 + 3x_2 = 8$ ....(3)

and

$$3 = \frac{12+4x_1+\frac{10-2x_1}{5}}{4}$$

$\Rightarrow 10x_2 - x_1 = -5$ ....(4)

from eq. (3) and (4) $x_1 = \frac{95}{13}, x_2 = \frac{3}{13}$

Point $A\left(\frac{3}{13},\frac{56}{13}\right)$, $B\left(\frac{95}{13},\frac{-12}{13}\right)$

Now area of ΔABC

Point area of $\Delta ABC = \left|\frac{1}{2}\left\{\frac{3}{13}\times\left(\frac{-44}{13}\right)+\frac{95}{13}\times\frac{-24}{13}+\frac{-15}{13}\times\frac{68}{13}\right\}\right|$

$=\frac{132}{13}$ sq. units

Question ID : 101273

Hyperbola

13. Let a > 0, b > 0. Let e and l repsectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. Let e′ and l' respectively be the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2=\frac{11}{14}l$ and $(e')^2=\frac{11}{8}l'$, then the value of 77a + 44b is equal to :

माना $a>0, b>0$ हैं। माना अतिपरवलय $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ की उत्केन्द्रता तथा नाभिलंब जीवा की लंबाई क्रमशः e तथा l है। माना इसके संयुग्मी अतिपरवलय की उत्केन्द्रता तथा नाभिलंब जीवा की लंबाई क्रमशः e′ तथा l' हैं। यदि $e^2=\frac{11}{14}l$ तथा $(e')^2=\frac{11}{8}l'$ हैं, तो $77a+44b$ का मान बराबर है :

(1) 100 (2) 110 (3) 120 (4) 130

Ans. Official Answer NTA (4)

Sol. Given $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ ——(1)

$e=\sqrt{1+\frac{b^2}{a^2}}$ and $\ell=\frac{2b^2}{a}$

$\Rightarrow e^2=\frac{11}{14}\ell \qquad \Rightarrow \frac{a^2+b^2}{a^2}=\frac{11}{4}\times\frac{2b^2}{a}$

$\Rightarrow a^2+b^2=\frac{11}{7}b^2a$ ——(2)

In conjugate hyperbola

$$-\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

$$e'=\sqrt{1+\frac{a^2}{b^2}}, \qquad \ell'=\frac{2a^2}{b}$$

given $(e')^2=\frac{11}{8}\ell'$ $\Rightarrow \frac{b^2+a^2}{b^2}=\frac{11}{8}\times\frac{2a^2}{b}$

$\Rightarrow a^2+b^2=\frac{11}{4}a^2b$ ———(3)

from eq. (2) and eq. (3)

$$7a=4b \Rightarrow a=\frac{4}{7}b$$

from eq. (2)

44b = 65 and 77a = 65

$\therefore$ 77a + 44b = 65 + 65 = 130

Question ID : 101274

Vectors

14. Let $\vec{a}=\alpha\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b}=-2\hat{i}+\alpha\hat{j}+\hat{k}$, where $\alpha \in R$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2+4)}$, then the value of $2|\vec{a}|^2+(\vec{a}.\vec{b})|\vec{b}|^2$ is equal to :

माना $\vec{a}=\alpha\hat{i}+2\hat{j}-\hat{k}$ तथा $\vec{b}=-2\hat{i}+\alpha\hat{j}+\hat{k}$ हैं, जहाँ $\alpha \in R$ है। यदि समान्तर चतुर्भुज, जिसकी संलग्न भुजाएँ $\vec{a}$ तथा $\vec{b}$ हैं, क्षेत्रफल $\sqrt{15(\alpha^2+4)}$ हैं, तो $2|\vec{a}|^2+(\vec{a}.\vec{b})|\vec{b}|^2$ का मान बराबर है :

(1) 10 (2) 7 (3) 9 (4) 14

Ans. Official Answer NTA (4)

Sol. Given $\vec{a}=\alpha\hat{i}+2\hat{j}-\hat{k}$

$\vec{b}=-2\hat{i}+\alpha\hat{j}+\hat{k}$

area of parallelogram is $\sqrt{15(\alpha^2+4)}$

$\Rightarrow |\vec{a}\times\vec{b}|=\sqrt{15(\alpha^2+4)}$

$$\left\|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 2 & -1 \\ -2 & \alpha & 1 \end{vmatrix}\right\| = \sqrt{15(\alpha^2+4)}$$

$$\Rightarrow \sqrt{(2+\alpha)^2 + (\alpha-2)^2 + (\alpha^2+4)^2} = \sqrt{15\alpha^2+60}$$

Squaring both sides and solving it.

$\alpha^4 - 5\alpha^2 - 36 = 0$

$\alpha^2 = 9$ and $\alpha^2 = -4$ (not possible)

$\Rightarrow \alpha = \pm 3$

Therefore

$2|\vec{a}|^2 + (\vec{a}.\vec{b}).|\vec{b}|^2$

$= 2 \times 14 - 14 = 28 - 14 = 14$

Question ID : 101275

Parabola

15. If vertex of a parabola is (2, –1) and the equation of its directrix is $4x - 3y = 21$, then the length of its latus rectum is :

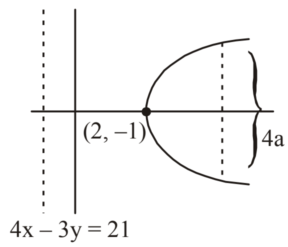
यदि एक परवलय का शीर्ष (2, –1) है इसकी नियता की समीकरण $4x - 3y = 21$ है, तो इसकी नाभिलंब जीवा की लंबाई है :

(1) 2 (2) 8 (3) 12 (4) 16

Ans. Official Answer NTA (2)

Sol. a = distance between vertex and direct x

$$a = \frac{|4 \times 2 - 3 \times 1 - 21|}{\sqrt{4^2 + 3^2}}$$



a = 2

Now, length of latus rectum = 4a

= 4 × 2 = 8 units

Question ID : 101276

3D Geometry

16. Let the plane $ax + by + cz = d$ pass through (2, 3, –5) and is perpendicular to the planes $2x + y - 5z = 10$

and $3x + 5y - 7z = 12$. If a, b, c, d are integers d > 0 and gcd (|a|, |b|, |c|, d) = 1, then the value of $a + 7b + c + 20d$ is equal to :

माना समतल $ax + by + cz = d$ बिन्दु (2, 3, –5) से होकर जाता है तथा समतलों $2x + y - 5z = 10$ और $3x + 5y - 7z = 12$ के लंबवत है। यदि a, b, c, d पूर्णांक हैं, d > 0 है तथा gcd (|a|, |b|, |c|, d) = 1 है, तो $a + 7b + c + 20d$ का मान बराबर है :

(1) 18 (2) 20 (3) 24 (4) 22

Ans. Official Answer NTA (4)

Sol. Equation of plane passing through (2, 3, –5) and perpendicular to the palnes $2x + y - 5z = 10$ and $3x + 5y - 7z = 12$ is

$$(\vec{r} - \vec{a}).\vec{N} = 0 \Rightarrow \begin{vmatrix} x-2 & y-3 & z+5 \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 0$$

$-18x + y - 7z = 2$

$a = -18, b = 1, c = -7, d = 2$

$\because$ gcd of (|a|, |b|, |c|, d) = 1

$\therefore a + 7b + c + 20d = 22$

Question ID : 101277

Probability

17. The probability that a randomly chosen one-one function from the set {a, b, c, d} to the set{1,2,3,4,5} satisfies $f(a) + 2f(b) - f(c) = f(d)$ is :

समुच्चय {a, b, c, d} से समुच्चय {1,2,3,4,5} में यादृच्छया चुने गए एक एकैकी फलन के $f(a) + 2f(b) - f(c) = f(d)$ का संतुष्ट करने की प्रायिकता है :

(1) $\frac{1}{24}$ (2) $\frac{1}{40}$ (3) $\frac{1}{30}$ (4) $\frac{1}{20}$

Ans. Official Answer NTA (4)

Sol. Given A = {a, b, c, d}

B = {1, 2, 3, 4, 5}

$f : A \rightarrow B$ is one – one

Total number of one – one functions from A to B = ${}^5P_4 = 5!$

also $f(a) + 2f(b) - f(c) = f(d)$

favourable case for this condition = 6

f(a)	f(b)	f(c)	f(d)
1	3	2	5
1	3	5	2
4	2	3	5
4	2	5	3
5	1	3	4
5	1	4	3

Probability of f(a) + 2f(b) – f(c) = f(d)

$$= \frac{6}{5!} = \frac{6}{120} = \frac{1}{20}$$

Question ID : 101278

Limit

18. The value of $\lim_{n\to\infty} 6\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$ is equal to :

$\lim_{n\to\infty} 6\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$ का मान बराबर है :

(1) 1 (2) 2 (3) 3 (4) 6

Ans. Official Answer NTA (3)

Sol. $6\lim_{n\to\infty}\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$

$$= 6\lim_{n\to\infty}\tan\left\{\sum_{r=1}^{n}\tan^{-1}\left[\frac{1}{1+\left(r^2+3r+2\right)}\right]\right\}$$

$$= 6\lim_{n\to\infty}\tan\left[\sum_{r=1}^{n}\tan^{-1}\left[\frac{1}{1+(r+2)(r+1)}\right]\right]$$

$$= 6\lim_{n\to\infty}\tan\left[\sum_{r=1}^{n}\tan^{-1}\left[\frac{(r+2)-(r+1)}{1+(r+2)(r+1)}\right]\right]$$

$$= 6\lim_{n\to\infty}\tan\left[\sum_{r=1}^{n}\left(\tan^{-1}(r+2)-\tan^{-1}(r+1)\right)\right]$$

$$= 6\lim_{n\to\infty}\tan\left[\left(\tan^{-1}(n+2)-\tan^{-1}2\right)\right]$$

$$= 6\tan\left[\lim_{n\to\infty}\tan^{-1}(n+2)-\lim_{n\to\infty}\tan^{-1}(2)\right]$$

$$= 6\tan\left[\tan^{-1}(\infty)-\tan^{-1}2\right] = 6\tan\left[\frac{\pi}{2}-\tan^{-1}2\right]$$

$$= \tan\left(\cot^{-2}2\right)$$

$$= 6\tan\left(\tan^{-1}\frac{1}{2}\right)$$

$$= 6\times\frac{1}{2} = 3$$

Question ID : 101279

Vectors

19. Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i}+\frac{1}{2}\hat{j}+2\hat{k}$. If $\vec{a}\times\left(2\hat{i}+\hat{k}\right)=2\hat{i}-13\hat{j}-4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i}+2\hat{j}+\hat{k}$ is :

माना सदिश $3\hat{i}+\frac{1}{2}\hat{j}+2\hat{k}$ के लंबवत एक सदिश $\vec{a}$ है। यदि $\vec{a}\times\left(2\hat{i}+\hat{k}\right)=2\hat{i}-13\hat{j}-4\hat{k}$ है, तो सदिश $\vec{a}$ का सदिश $2\hat{i}+2\hat{j}+\hat{k}$ पर प्रक्षेप है :

(1) $\frac{1}{3}$ (2) 1 (3) $\frac{5}{3}$ (4) $\frac{7}{3}$

Ans. Official Answer NTA (3)

Sol. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{a}.\left(3\hat{i}-\frac{1}{2}\hat{j}+2\hat{k}\right)=0$

$\Rightarrow 3a_1-\frac{a_2}{2}+2a_3=0$

$\Rightarrow 6a_1-a_2+4a_3=0$ ——(1)

Also

$$\vec{a}\times\left(2\hat{i}+\hat{k}\right)=2\hat{i}-13\hat{j}-4\hat{k}$$

$$-2a_2\hat{k}+2a_3\hat{j}-a_1\hat{j}+a_2\hat{i}=2\hat{i}-13\hat{j}-4\hat{k}$$

By comparison

$a_2=2$ and $a_1-2a_3=13$ ——(2)

from eq. (1) and (2)

$a_1 = 3$ and $a_3 = -5$

$\vec{a} = 3\hat{i} + 2\hat{j} - 5\hat{k}$

Projection of $\vec{a}$ on $2\hat{i} + 2\hat{j} + \hat{k} = \dfrac{6+4-5}{\sqrt{2^2+2^2+1^2}}$

$= \dfrac{5}{3}$ units

Question ID : 101280

Trigonometric Ratio and Identities

20. If cot α = 1 and sec β = $-\dfrac{5}{3}$, where $\pi < \alpha < \dfrac{3\pi}{2}$ and $\dfrac{\pi}{2} < \beta < \pi$, then the value of tan(α + β) and the quadrant in which α + β lies, respectively are :

(1) $-\dfrac{1}{7}$ and IVth quadrant
(2) 7 and Ist quadrant
(3) –7 and IVth quadrant
(4) $\dfrac{1}{7}$ and Ist quadrant

यदि $\cot\alpha = 1$ तथा $\sec\beta = -\dfrac{5}{3}$ हैं, जहाँ $\pi < \alpha < \dfrac{3\pi}{2}$ तथा $\dfrac{\pi}{2} < \beta < \pi$ हैं, तो $\tan(\alpha+\beta)$ का मान तथा वह चतुर्थांष जिसमें $\alpha + \beta$ स्थित हैं, क्रमशः है :

(1) $-\dfrac{1}{7}$ तथा चौथा IVth चतुर्थांश
(2) 7 तथा पहला चतुर्थांश
(3) –7 तथा चौथा चतुर्थांश
(4) $\dfrac{1}{7}$ तथा पहला चतुर्थांश

Ans. Official Answer NTA (1)

Sol. cot α = 1; $\alpha \in \left(\pi, \dfrac{3\pi}{2}\right)$

then tan α = 1

and $\sec\beta = \dfrac{-5}{3}$, $\beta \in \left(\dfrac{\pi}{2}, \pi\right)$

$\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\tan\beta = -\dfrac{4}{3}$

Also

$$\alpha+\beta=\left(\pi,\frac{3\pi}{2}\right)+\left(\frac{\pi}{2},\pi\right)$$

$$=\left(\frac{3\pi}{2},\frac{5\pi}{2}\right)$$

but tan $(\alpha + \beta)$ is –ve

$\therefore$ $\alpha + \beta \in$ iv quadrant

SECTION – B

Question ID : 101281

3D Geometry

21. Let the image of the point P(1, 2, 3) in the line L : $\frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}$ be Q. Let R(α, β, γ) be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22(\alpha + \beta + \gamma)$ is equal to __________.

माना रेखा L : $\frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}$ में बिन्दु P(1, 2, 3) का प्रतिबिंब Q है। माना बिन्दु R(α, β, γ), रेखा खण्ड PQ को अंत 1 : 3 के अनुपात में विभाजित करता है। तो $22(\alpha + \beta + \gamma)$ का मान बराबर है __________.

Ans. Official Answer NTA (125)

Sol. From point P(1, 2, 3), P′ is foot of perpendicular on line $\frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}=\lambda$

$\therefore$ P′ = $(3\lambda + 6, 2\lambda + 1, 3\lambda + 2)$

d.r's of PP′ $\equiv 3\lambda + 5, 2\lambda - 1, 3\lambda - 1$

as PP′ and line l is perpendicular

$\therefore$ $(3\lambda + 5)3 + 2(2\lambda - 1) + 3(3\lambda - 1) = 0$

$$\Rightarrow \lambda=\frac{-5}{11}$$

$$P'=\left(\frac{51}{11},\frac{1}{11},\frac{7}{11}\right)$$

The point dividing PQ in the ratio 1 : 3 will be mid point of P and foot of perpendicular from P on line

$\Rightarrow$ R will be mid point of PP′

$$\Rightarrow R=(\alpha,\beta,\gamma)=\left(\frac{\frac{51}{11}+1}{2},\frac{\frac{1}{11}+2}{2},\frac{\frac{7}{11}+3}{2}\right)$$

$$(\alpha,\beta,\gamma)=\left(\frac{62}{22},\frac{23}{22},\frac{40}{22}\right)$$

$$22(\alpha,\beta,\gamma)=22\left(\frac{62+23+40}{22}\right)=125$$

Question ID : 101282

Statistics

22. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62 and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is _________.

माना एक कक्षा में 7 छात्र हैं। गणित की परीक्षा में इन छात्रों के अंकों का माध्य 62 तथा प्रसरण 20 हैं। कोई छात्र परीक्षा में फेल होता है। यदि वह 50 अंकों से कम प्राप्त करता है, तो फेल होने वाले छात्रों की अधिकतम संख्या है________.

Ans. Official Answer NTA (0)

Sol. According to given data

$$\bar{x}=62, \frac{\sum(x_i-\bar{x})^2}{7}=20$$

$\Rightarrow (x_1-62)^2+(x_2-62)^2+\ldots\ldots\ldots\ldots(x_7-62)^2=140$ for failure of any student $x_i<50$

Suppose $x_1=49$.

$(49-62)^2=169$ which is not possible. Therefore no student is going to score less. than 50.

= 0 students

Question ID : 101283

Circle

23. If one of the diameters of the circle $x^2+y^2-2\sqrt{2}x-6\sqrt{2}y+14=0$ is a chord of the circle $\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$, then the value of r^2 is equal to ___________.

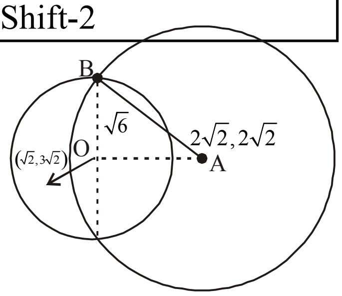
यदि वृत्त $x^2+y^2-2\sqrt{2}x-6\sqrt{2}y+14=0$ का एक व्यास, वृत्त $\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$ की एक जीवा है, तो r^2 का मान बराबर है ___________.

Ans. Official Answer NTA (10)

Sol. For $x^2+y^2-2\sqrt{2}x+6\sqrt{2}y+14=0$

$$r=\sqrt{\left(\sqrt{2}\right)^2+\left(3\sqrt{2}\right)^2-14}=\sqrt{6}$$

diameter = $2\sqrt{6}$



if diameter is chord to, $\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$, then

in ΔOAB,

OB = $\sqrt{6}$ units

OA = 2 units (distance between centres)

$r = AB = \sqrt{\left(\sqrt{6}\right)^2+(2)^2}$

$\Rightarrow r^2 = 10$ units

Question ID : 101284

Limit

24. If $\lim\limits_{x\to 1}\dfrac{\sin\left(3x^2-4x+1\right)-x^2+1}{2x^3-7x^2+ax+b}=-2$, then the value of (a – b) is equal to ________.

यदि $\lim\limits_{x\to 1}\dfrac{\sin\left(3x^2-4x+1\right)-x^2+1}{2x^3-7x^2+ax+b}=-2$ है, तो (a – b) का मान बराबर है ________।

Ans. Official Answer NTA (11)

Sol. $\lim\limits_{x\to 1}\dfrac{\sin\left(3x^2-4x+1\right)-x^2+1}{2x^3-7x^2+ax+b}=-2$

this will be $\frac{0}{0}$ form for finite answer denominator must be zero.

$\because\ 2-7+a+b=0$

$\therefore\ a+b=5$ ....(1)

As this will be $\frac{0}{0}$ form use De LHoptal Rule,

$\lim\limits_{x\to 1}\dfrac{\cos\left(3x^2-4x+1\right)(6x-4)-2x}{6x^2-14x+a}$

Again for finite answer denominator must be zero.

$\therefore\ a=8$

from eq. (1)

$b=-3$

required = a – b = 8 – (–3)

= 8 + 3 = 11

Question ID : 101285

Sequence & progression

25. Let for n = 1, 2, ...,50, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\frac{1}{(n+1)^2}$. Then the value of $\frac{1}{26}+\sum_{n=1}^{50}\left(S_n+\frac{2}{n+1}-n-1\right)$ is equal to ______.

माना n = 1, 2, ...,50 के लिए अनंत गुणोत्तर श्रेढ़ी जिसका पहला पद n^2 तथा सार्व अनुपात $\frac{1}{(n+1)^2}$ है का योग S_n है। तो $\frac{1}{26}+\sum_{n=1}^{50}\left(S_n+\frac{2}{n+1}-n-1\right)$ का मान बराबर है ______।

Ans. Official Answer NTA (41651)

Sol. Infinite series whose $a = n^2$.

$$r=\frac{1}{(n+1)^2}$$

$$S_n=S_\infty=\frac{n^2}{1-\frac{1}{(n+1)^2}}$$

$$S_n=\frac{(n+1)^2}{(n+2)}=\frac{n(n+1)^2}{(n+2)}$$

$$S_n=(n^2+1)-\frac{2}{n+2}\text{ [add and sub 2]}$$

Now,

$$\Rightarrow\frac{1}{26}+\sum_{n=1}^{50}\left[(n^2+1)-\frac{2}{n+2}+\frac{2}{n+1}-n-1\right]$$

$$\Rightarrow\frac{1}{26}+\sum_{n=1}^{50}\left[(n^2-n)+2\left[\frac{1}{n+1}-\frac{1}{n+2}\right]\right]$$

$$\Rightarrow\frac{1}{26}+\sum_{n=1}^{50}\left[(n^2-n)\right]+2.\sum_{n=1}^{50}\left[\frac{1}{n+1}-\frac{1}{n+2}\right]$$

$$\Rightarrow\frac{1}{26}+\frac{50\times51\times101}{6}-\frac{50\times51}{2}+2.\left[\frac{1}{2}-\frac{1}{52}\right]$$

$$= \frac{1}{26} + \frac{57550 - 7650}{6} + 1 - \frac{1}{26}$$
$$= 41650 + 1 = 41651$$

Question ID : 101286

Determinant

26. If the system of linear equations $2x - 3y = \gamma + 5$, $\alpha x + 5y = \beta + 1$, where $\alpha, \beta, \gamma \in R$ has infinitely many solutions, then the value of $|9\alpha + 3\beta + 5\gamma|$ is equal to ________.

यदि रैखिक समीकरण निकाय $2x - 3y = \gamma + 5$, $\alpha x + 5y = \beta + 1$ जहाँ $\alpha, \beta, \gamma \in R$ है, के अनंत हल है, तो $|9\alpha + 3\beta + 5\gamma|$ का मान बराबर है ________।

Ans. Official Answer NTA (58)

Sol. Given $2x - 3y = \gamma + 5$

$\alpha x + 5y = \beta + 1$. have infinite sol^n

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2} = \frac{2}{\alpha} = \frac{-3}{5} = \frac{\gamma + 5}{\beta + 1}$$

$\Rightarrow \alpha = \frac{-10}{3}$ and $3\beta + 5\gamma = -28$

So $|9.\alpha + 3\beta + 5\gamma| = \left|9 \times \frac{-10}{3} - 28\right| = 58$

Question ID : 101287

Matrices

27. Let $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ where $i = \sqrt{-1}$. Then, the number of elements in the set $\{n \in \{1, 2, ..., 100\} : A^n = A\}$ is ________.

माना $A = \begin{pmatrix} 1+i & 1 \\ -i & 0 \end{pmatrix}$ है, जहाँ$i = \sqrt{-1}$ है। तो समुच्चय $\{n \in \{1, 2, ..., 100\} : A^n = A\}$ में अवयवों की संख्या है ________।

Ans. Official Answer NTA (25)

Sol. $A^2 = \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} \begin{bmatrix} 1+i & 1 \\ -i & 0 \end{bmatrix} = \begin{bmatrix} i & 1+i \\ 1-i & -i \end{bmatrix}$

$A^4 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

So $A^5 = A$, $A^9 = A$ and So on.

$\therefore$ n = 1, 5, 9 ...., 97

Number of values = 25

Question ID : 101288

Complex number

28. Sum of squares of modulus of all the complex numbers z satisfying $\overline{z} = iz^2 + z^2 - z$ is equal to ________.

$\overline{z} = iz^2 + z^2 - z$ का संतुष्ट करने वाली सभी सम्मिश्र संख्याओं z के मापाकों के वर्गों का योगफल बराबर है___।

Ans. Official Answer NTA (2)

Sol. Let z = x + iy given $\overline{z} = i(z^2) + z^2 - z$

So $2x = (1+i)(x^2 - y^2 + 2ixy)$

Separating real and imaginary parts

$\Rightarrow 2x = x^2 - y^2 + 2xy$ ....(1)

$x^2 - y^2 - 2xy = 0$ ....(2)

from eqn. (1) and (2)

$x = 0$ or $y = -\frac{1}{2}$

When x = 0 $\Rightarrow y = 0$

When $y = -\frac{1}{2}$ $\Rightarrow x^2 - x - \frac{1}{4} = 0$

$x = \frac{-1 \pm \sqrt{2}}{2}$

So there will be 3 complex numbers.

$(0+0i)$; $\frac{-1-\sqrt{2}}{2} + i\left(-\frac{1}{2}\right)$; $\frac{-1+\sqrt{2}}{2} + \left(-\frac{i}{2}\right)$

Sum of squares of modulus of these

$$= 0 + \left(\frac{\sqrt{2}-1}{2}\right)^2 + \frac{1}{4} + \left(\frac{\sqrt{2}+1}{2}\right)^2 + \frac{1}{4}$$

$$= \frac{3}{2} + \frac{1}{2} = 2$$

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Function

29. Let S = {1, 2, 3, 4}. Then the number of elements in the set $\{f : S \times S \to S : f$ is onto and $f(a,b) = f(b,a) \geq a \forall (a,b) \in S\times S\}$ is _________.

माना $S = \{1, 2, 3, 4\}$ है। तो समुच्चय $\{f : S \times S \to S : f$ आच्छादक है तथा $f(a,b) = f(b,a) \geq a \forall (a,b) \in S\times S\}$ में अवयवों की संख्या है __________।

Ans. Official Answer NTA (37)

Sol. There are 16 ordered pass in 5 × 5. We write all these ordered pairs in 4 sets as follows :

A = {(1, 1)}

B = {(1, 4), (2, 4), (3, 4), (4, 4), (4, 3), (4, 2), (4, 1)}

C = {(1, 3), (2, 3), (3, 3), (3, 2), (3, 1)}

D = {(1, 2), (2, 2), (2, 1)}

All elements of set B have image 4 and element of A has only image 1.

All elements of set C have image 3 or 4 and all elements of set D have image 2 or 3 or 4.

Case - I : When no element of set C has image 3 number of onto functions = 2 (when elements of set D have images 2 or 3)

Case - II : When at least one element of set C has image 3. Number of onto functions = $(2^3 - 1)(1 + 2 + 2)$

$= 35$

Total number of functions = 37.

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Mathematical Reasoning

30. The maximum number of compound propositions, out of $p\vee r\vee s$, $p\vee r\vee \sim s$, $p\vee \sim q\vee s$, $\sim p\vee \sim r\vee s$, $\sim p\vee \sim r\vee \sim s$,

$\sim p \vee q \vee \sim s$, $q \vee r \vee \sim s$, $q \vee \sim r \vee \sim s$, $\sim p \vee \sim q \vee \sim s$ that can be made simultaneously true by an assignement of the truth values to p, q, r and s, is equal to _______.

$p \vee r \vee s$, $p \vee r \vee \sim s$, $p \vee \sim q \vee s$, $\sim p \vee \sim r \vee s$, $\sim p \vee \sim r \vee \sim s$, $\sim p \vee q \vee \sim s$, $q \vee r \vee \sim s$, $q \vee \sim r \vee \sim s$, $\sim p \vee \sim q \vee \sim s$ में से उन मिश्र साध्यों, जिन्हें p, q, r तथा s के सत्यमान देने से एक साथ सत्य बनाया जा सकता है, की अधिकतम संख्या है___.

Ans. Official Answer NTA (9)

Sol. There are total 9 compound propositions, out of which 6 contain ~s(negation of s) so if we assign s as false; 6 propositions will be true for remaining 3 are can opt

$$P \to T$$
$$r \to F$$

Hence maximum number of propositions that can be true are 9.