JEE Main July 2022

Question Paper With Text Solution

28 July | Shift-1

MATHEMATICS

JEE MAIN JULY 2022 | 28TH JULY SHIFT-1

SECTION – A

Question ID : 100601

Differential Equation

1. Let the solution curve of the differential equation $xdy = \left(\sqrt{x^2+y^2}+y\right)dx$, x > 0, intersect the line x = 1 at y = 0 and the line x = 2 at y = α. Then the value of α is :

माना अवकल समीकरण $xdy = \left(\sqrt{x^2+y^2}+y\right)dx$, $x > 0$ का हल वक्र, रेखा $x = 1$ को $y = 0$ पर तथा रेखा $x = 2$ को $y = \alpha$ पर काटता है। तो α का मान है :

(1) $\frac{1}{2}$ (2) $\frac{3}{2}$ (3) $-\frac{3}{2}$ (4) $\frac{5}{2}$

Ans. Official Answer NTA (2)

Sol. $xdy = \left(\sqrt{x^2+y^2}+y\right)dx$

$xdy - ydx = \sqrt{x^2+y^2}\,dx$

$\frac{xdy-ydx}{x^2} = \sqrt{1+\frac{y^2}{x^2}}.\frac{dx}{x}$

$\frac{d(y/x)}{\sqrt{1+\left(\frac{y}{x}\right)^2}} = \frac{dx}{x}$

$\ln\left(\frac{y}{x}+\sqrt{\left(\frac{y}{x}\right)^2+1}\right) = \ln x + R$

$\frac{y+\sqrt{y^2+x^2}}{x} = cx$

$y+\sqrt{y^2+x^2} = cx^2$

$x = 1, y = 0 \Rightarrow 0 + 1 = C \Rightarrow C = 1$

Curve is $y+\sqrt{x^2+y^2} = x^2$

$x = 2, y = \alpha$

$2+\sqrt{4+\alpha^2} = 4$

$4+\alpha^2=16+\alpha^2=8\alpha$

$\alpha=\dfrac{3}{2}$

Question ID : 100602

ITF

2. Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x)=\cos^{-1}\left(\dfrac{x^2-4x+2}{x^2+3}\right)$ is :

प्रतिलोम त्रिकोणमितीय फलन के केवल मुख्य मान लेते हुए, फलन $f(x)=\cos^{-1}\left(\dfrac{x^2-4x+2}{x^2+3}\right)$ का प्रांत है:

(1) $\left(-\infty,\dfrac{1}{4}\right]$ (2) $\left[-\dfrac{1}{4},\infty\right)$ (3) $\left(-\dfrac{1}{3},\infty\right)$ (4) $\left(-\infty,\dfrac{1}{3}\right]$

Ans. Official Answer NTA (2)

Sol. $\left|\dfrac{x^2+4x+2}{x^2+3}\right|\le 1$

$\Leftrightarrow\left(x^2-4x+2\right)^2\le\left(x^2+3\right)^2$

$\Leftrightarrow\left(x^2-4x+2\right)^2-\left(x^2+3\right)^2\le 0$

$\Leftrightarrow\left(2x^2-4x+5\right)\left(-4x-1\right)\le 0$

$\Leftrightarrow -4x-1\le 0\rightarrow x\ge-\dfrac{1}{4}$

Question ID : 100603

Vectors

3. Let the vectors $\vec{a}=(1+t)\hat{i}+(1-t)\hat{j}+\hat{k}$, $\vec{b}=(1-t)\hat{i}+(1+t)\hat{j}+2\hat{k}$ and $\vec{c}=t\hat{i}-t\hat{j}+\hat{k}$, $t\in R$ be such that for $\alpha,\beta,\gamma\in R$, $\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}=\vec{0}\Rightarrow\alpha=\beta=\gamma=0$. Then, the set of all values of t is :

(1) a non-empty finite set (2) equal to N

(3) equal to $R-\{0\}$ (4) equal to R

माना सदिश $\vec{a}=(1+t)\hat{i}+(1-t)\hat{j}+\hat{k}$, $\vec{b}=(1-t)\hat{i}+(1+t)\hat{j}+2\hat{k}$ तथा $\vec{c}=t\hat{i}-t\hat{j}+\hat{k}$, $t\in R$ इस प्रकार है कि $\alpha,\beta,\gamma\in R$, के लिए $\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c}=\vec{0}\Rightarrow\alpha=\beta=\gamma=0$ तो t के सभी मानों का समुच्चय है :

(1) एक अरिक्त परिमित समुच्चय है (2) N के बराबर है

(3) $R - \{0\}$ के बराबर है (4) R के बराबर है

Ans. Official Answer NTA (3)

Sol. By its given condition

$: \vec{a}, \vec{b}, \vec{c}$ are linearly independent vectors

$\Rightarrow \left[\bar{a}\,\bar{b}\,\bar{c}\right] \neq 0$ ....(i)

Now, $\left[\bar{a}\,\bar{b}\,\bar{c}\right]$

$$= \begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix}$$

$C_2 \to C_1 + C_2$

$$\begin{vmatrix} 1+t & 2 & 1 \\ 1-t & 2 & 2 \\ t & 0 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1+t & 2 & 1 \\ 1-t & 2 & 2 \\ t & 0 & 1 \end{vmatrix}$$

$= 2\left[(1+t) - (1-t) + t\right]$

$= 2[3t] = 6t$

$\left[\bar{a}\ \bar{b}\ \bar{c}\right] \neq 0 \Rightarrow t \neq 0$

Question ID : 100604

ITF

4. Considering the principal values of the inverse trigonometric functions, the sum of all the solutions of the equation $\cos^{-1}(x) - 2\sin^{-1}(x) = \cos^{-1}(2x)$ is equal to :

प्रतिलोम त्रिकोणमितीय फलन के मुख्य मान लेते हुए समीकरण $\cos^{-1}(x) - 2\sin^{-1}(x) = \cos^{-1}(2x)$ के सभी हलों का योग है :

(1) 0 (2) 1 (3) $\frac{1}{2}$ (4) $-\frac{1}{2}$

Ans. Official Answer NTA (1)

Sol. $\cos^{-1}x = 2\sin^{-1}x = \cos^{-1}2x$

$$\cos^{-1}x - 2\left(\frac{\pi}{2} - \cos^{-1}x\right) = \cos^{-1}2x$$

$$\cos^{-1}x - \pi + 2\cos^{-1}x = \cos^{-1}2x$$

$$3\cos^{2}x = \pi + \cos^{-1}2x \quad ....(1)$$

$$\cos\left(3\cos^{-1}x\right) = \cos\left(\pi + \cos^{-1}2x\right)$$

$$4x^{3} - 3x = -2x$$

$$4x^{3} = x \Rightarrow x = 0, \pm\frac{1}{2}$$

All satisfy the original equation

$$\text{sum} = -\frac{1}{2} \text{ to } + \frac{1}{2} = 0$$

Question ID : 100605

Mathematical Reasoning

5. Let the operations $*, \odot \in \{\wedge, \vee\}$. If $(p * q) \odot (p \odot \sim q)$ is a tautology, then the ordered pair $(*, \odot)$ is :

माना $*, \odot \in \{\wedge, \vee\}$ है। यदि $(p * q) \odot (p \odot \sim q)$ एक पुनरुक्ति है, तो क्रमित युग्म $(*, \odot)$ है :

(1) $(\vee, \wedge)$ (2) $(\vee, \vee)$ (3) $(\wedge, \wedge)$ (4) $(\wedge, \vee)$

Ans. Official Answer NTA (2)

Sol. Well check each option

For A π = v of 0 = A

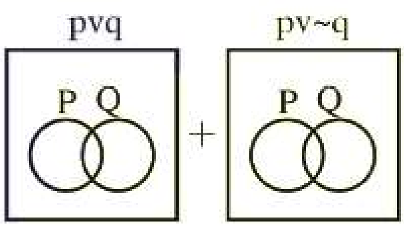
$$(pvq) \wedge (pv \sim q)$$

$$\equiv pv(q \wedge \sim q)$$

$$\equiv pv(c) \equiv p$$

For B : * = v, O = v

$(pvq) \vee (pv \sim q) \equiv t$ using Venn Diagrams



Question ID : 100606

Vectors

6. Let a vector $\vec{a}$ has magnitude 9. Let a vector $\vec{b}$ be such that for every $(x, y) \in R \times R - \{(0, 0)\}$, the vector $\left(x\vec{a} + y\vec{b}\right)$ is perpendicular to the vector $\left(6y\vec{a} - 18x\vec{b}\right)$. Then the value of $\left|\vec{a} \times \vec{b}\right|$ is equal to :

माना एक सदिश $\vec{a}$ का परिमाण 9 है। माना एक सदिश $\vec{b}$ इस प्रकार है कि प्रत्येक $(x, y) \in R \times R - \{(0, 0)\}$ के लिए, सदिश $\left(x\vec{a} + y\vec{b}\right)$, सदिश $\left(6y\vec{a} - 18x\vec{b}\right)$ के लंबवत है। तो $\left|\vec{a} \times \vec{b}\right|$ का मान बराबर है :

(1) $9\sqrt{3}$ (2) $27\sqrt{3}$ (3) 9 (4) 81

Ans. Official Answer NTA (2)

Sol. $|\vec{a}| = 9 \,\&\, \left(x\vec{a} + y\vec{b}\right).\left(6y\vec{a} - 18x\vec{b}\right) = 0$

$\Rightarrow 6xy|\bar{a}|^2 - 18x^2\left(\bar{a}.\bar{b}\right) + 6y^2\left(\bar{a}.\bar{b}\right) - 18xy|\bar{b}|^2 = 0$

$\Rightarrow 6xy\left(|\bar{a}|^2 - 3|\bar{b}|^2\right) + \left(\bar{a}.\bar{b}\right)\left(y^2 - 3x^2\right) = 0$

This should hold $\forall x, y \in R \times R$

$\therefore |\bar{a}|^2 = 3|\bar{b}|^2 \;\; \& \;\; \left(\bar{a}.\bar{b}\right) = 0$

Now $\left|\bar{a} \times \bar{b}\right|^2 = |\bar{a}|^2|\bar{b}|^2 - \left(\bar{a}.\bar{b}\right)^2$

$= \bar{a}.\dfrac{|\bar{a}|^2}{3}$

$\therefore |\bar{a} \times \bar{b}| = \dfrac{|\bar{a}|^2}{\sqrt{3}} = \dfrac{81}{\sqrt{3}} = 27\sqrt{3}$

Question ID : 100607

Circle

7. For $t \in (0, 2\pi)$, if ABC is an equilateral triangle with vertices $A(\sin t, -\cot t)$, $B(\cos t, \sin t)$ and $C(a, b)$ such that its orthocentre lies on a circle with centre $\left(1, \dfrac{1}{3}\right)$, then $(a^2 - b^2)$ is equal to :

$t \in (0, 2\pi)$ के लिए, यदि शीर्षों $A(\sin t, -\cot t)$, $B(\cos t, \sin t)$ तथा $C(a, b)$ के एक समबाहु त्रिभुज ABC का लंब केन्द्र, एक वृत्त जिसका केन्द्र $\left(1, \dfrac{1}{3}\right)$ है, पर स्थित है, तो $(a^2 - b^2)$ बराबर है :

(1) $\dfrac{3}{8}$ (2) 8 (3) $\dfrac{77}{9}$ (4) $\dfrac{80}{9}$

Ans. Official Answer NTA (2)

Sol. $s \equiv \sin t, c \equiv \cos t$

Let orthocentre be (h,k)

Since it if an equilateral triangle hence orthocentre coincides with centroid.

$\therefore$ a + s + c = 3h, b + s – c = 3k

$\therefore (3h-a)^2+(3k-b)^2=(s+c)^2+(s-c)^2=2(s^2+c^2)=2$

$\therefore \left(h-\frac{a}{3}\right)^2+\left(K-\frac{b}{3}\right)^2=\frac{2}{9},$

circle centre at $\left(\frac{a}{3},\frac{b}{3}\right)$

Gioves, $\frac{a}{3}=1,\frac{b}{3}=\frac{1}{3}\Rightarrow a=3, b=1$

$\therefore a^2-b^2=8$

Question ID : 100608

Set & Relations

8. For $\alpha \in N$, consider a relation R on N given by R = {(x, y) : 3x + αy is a multiple of 7}. The relation R is an equivalence relation if and only if :

(1) $\alpha = 14$

(2) α is a multiple of 4

(3) 4 is the remainder when α is divided by 10

(4) 4 is the remainder wehn α is devided by 7

$\alpha \in N$ के लिए, N पर एक संबंध R,R = {(x, y) : 3x + αy, 7 का एक गुणज है} द्वारा दिया गया है। संबंध R एक तुल्यता संबंध है यदि और केवल यदि :

(1) $\alpha = 14$ है

(2) α, 4 का एक गुणज है

(3) α को 10 से विभाजित करने पर शेषफल 4 है

(4) α को 7 से विभाजित करने पर शेषफल है 4 है

Ans. Official Answer NTA (4)

Sol. For R to be reflexive $\Rightarrow$ x R x

$\Rightarrow 3x + \alpha x = 7x \Rightarrow (3 + \alpha) x = 7K$

$\Rightarrow 3 + \alpha = 7\lambda \Rightarrow \alpha = 7\lambda - 3 = 7N + 4$, K, λ,N $\in$ I

$\therefore$ when α divided by 7, remainder is 4.

R to be symmetric xRy $\Rightarrow$ yRx

$3x + \alpha y = 7N_1$, $3y + \alpha x = 7N_2$

$\Rightarrow (3 + \alpha)(x + y) = 7(N_1 + N_2) = 7N_3$

Which holds when 3 + α is multiple of 7

$\therefore \alpha = 7N + 4$ (as did earlier)

R to be transitive

$xRy \& yRz \Rightarrow xRz$.

$3x + \alpha y = 7N_1$ & $3y + \alpha z = 7N_2$ and

$3x + \alpha z = 7N_3$

$\therefore 3x + 7N_2 - 3y = 7N_3$

$\therefore 7N_1 - \alpha y + 7N_2 - 3y = 7N_3$

$\therefore 7(N_1 + N_2) - (3 + \alpha)y = 7N_3$

$\therefore (3 + \alpha)y = 7N$

Which is true again when $3 + \alpha$ divisible by 7, i.e.

when α divided by 7, remainder is 4.

Question ID : 100609

Probability

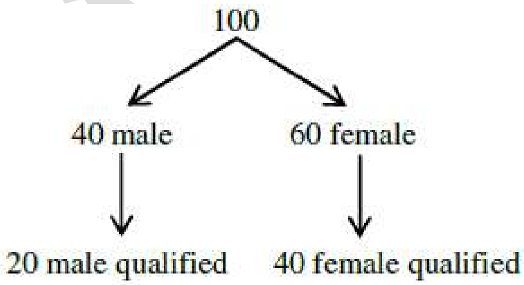
9. Out of 60% female and 40% male candidates appearing in an exam, 60% candidates qualify it. The number of females qualifying the exam is twice the number of males qualifying it. A candidate is randomly chosen from the qualified candidates. The probability, that the chosen candidate is a female, is :

60% महिला तथा 40% पुरुष अभ्यार्थियों द्वारा दी गई एक परीक्षा में 60% अभ्यर्थी सफल होते हैं। परीक्षा में सफल होने वाली महिलाओं की संख्या, परीक्षा में सफल होने वाले पुरूषों की संख्या की दो गुना है। सफल अभ्यार्थियों में एक अभ्यार्थी यादृच्छया चुना जाता है। चुने गए अभ्यार्थी के महिला होने की प्रायिकता है :

(1) $\frac{3}{4}$ (2) $\frac{11}{16}$ (3) $\frac{23}{32}$ (4) $\frac{13}{16}$

Ans. Official Answer NTA (1)

Sol.



Probability that chosen candidate is female $= \frac{40}{60} = \frac{2}{3}$

Question ID : 100610

Differential Equation

10. If $y = y(x)$, $x \in (0, \pi/2)$ be the solution curve of the differential equation

$\left(\sin^2 2x\right)\frac{dy}{dx} + \left(8\sin^2 2x + 2\sin 4x\right)y = 2e^{-4x}\left(2\sin 2x + \cos 2x\right)$, with $y\left(\frac{\pi}{4}\right) = e^{-\pi}$, then $y\left(\frac{\pi}{6}\right)$ is equal to :

यदि अवकल समीकरण $\left(\sin^2 2x\right)\frac{dy}{dx} + \left(8\sin^2 2x + 2\sin 4x\right)y = 2e^{-4x}\left(2\sin 2x + \cos 2x\right)$, $x \in (0, \pi/2)$ $y\left(\frac{\pi}{4}\right) = e^{-\pi}$ का हल वक्र $y = y(x)$ है, तो $y\left(\frac{\pi}{6}\right)$ बराबर है :

(1) $\frac{2}{\sqrt{3}}e^{-2\pi/3}$ (2) $\frac{2}{\sqrt{3}}e^{2\pi/3}$ (3) $\frac{1}{\sqrt{3}}e^{-2\pi/3}$ (4) $\frac{1}{\sqrt{3}}e^{2\pi/3}$

Ans. Official Answer NTA (1)

Sol. Given differential equation can be re-written as

$$\frac{dy}{dx} + \left(8 + 4\cot 2x\right)y = \frac{2e^{-4x}}{\sin^2 2x}\left(2\sin x + \cos 2x\right)$$

which is a linear diff. equation.

$$\text{I.f.} = e^{\int(8+4\cot 2x)dx} = e^{8x + 2Cu(\sin 2x)}$$

$$= e^{8x}.\sin^2 2x$$

$\therefore$ solution is

$$y\left(e^{8x}.\sin^2 2x\right) = \int 2e^{4x}\left(2\sin 2x + \cos 2x\right)dx + C$$

$$= e^{4x}.\sin 2x + C$$

Given $y\left(\frac{\pi}{4}\right) = e^{-\pi} \Rightarrow C = 0$

$$\therefore y = \frac{e^{-4x}}{\sin 2x}$$

$$\therefore y\left(\frac{\pi}{6}\right) = \frac{e^{-4.\frac{\pi}{6}}}{\sin\left(2.\frac{\pi}{6}\right)} = \frac{2}{\sqrt{3}}e^{-\frac{2\pi}{3}}$$

Question ID : 100611

Parabola

11. If the tangents drawn at the points P and Q on the parabola $y^2 = 2x - 3$ intersect at the point R(0, 1), then the orthocentre of the triangle PQR is :

यदि परवलय $y^2 = 2x - 3$ के बिन्दुओं P तथा Q पर खींची गई स्पर्श रेखाऐं बिन्दु R(0, 1), पर मिलती हैं। तो त्रिभुज PQR का लंब केन्द्र है :

(1) (0, 1) (2) (2, –1) (3) (6, 3) (4) (2, 1)

Ans. Official Answer NTA (2)

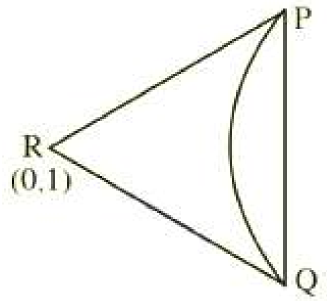
Sol. $y^2 = 2x - 3$ …(1)

Equation of chord of contact

PQ : r = 0

$y \times 1 = (x + 0) - 3$

$y = x - 3$ ....(2)



from (1) and (2)

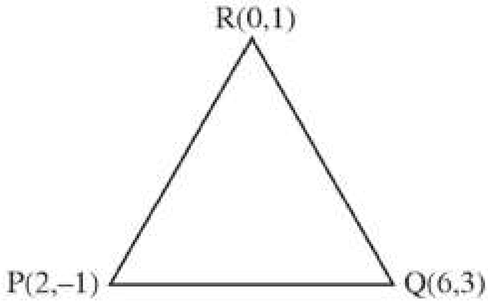
$(x . 3)^2 = 2x - 3$

$x^2 - 8x + 12 = 0$

$(x - 2)(x - 6) = 0$

x = 2or 6

y = –1or 3



$MPQ = \frac{1}{4} = 1$

$MQR = \frac{2}{6} = \frac{1}{3}$

$MPR = \frac{2}{6} = \frac{1}{3}$

$MPR = \frac{2}{-2} = -1$

$M_{PQ} \times M_{PR} = - \Rightarrow PQ \perp PR$

Orthocentre = P (2, –1)

Question ID : 100612

Circle

12. Let C be the centre of the circle $x^2 + y^2 - x + 2y = \frac{11}{4}$ and P be a point on the circle. A line passes through the point C, makes an angle of $\frac{\pi}{4}$ with the line CP and intersects the circle at the points Q and R. Then the area of the triangle PQR (in unit^2) is :

माना वृत्त $x^2 + y^2 - x + 2y = \frac{11}{4}$ को केन्द्र C है तथा वृत्त पर एक बिन्दु P है। एक रेखा, बिन्दु C से होकर जाती है, रेखा CP से $\frac{\pi}{4}$ को कोण बनाती है तथा वृत्त को बिन्दुओं Q तथा R पर काटती है। तो त्रिभुज PQR का क्षेत्रफल (वर्ग इकाई में) है :

(1) 2 (2) $2\sqrt{2}$ (3) $8\sin\left(\frac{\pi}{8}\right)$ (4) $8\cos\left(\frac{\pi}{8}\right)$

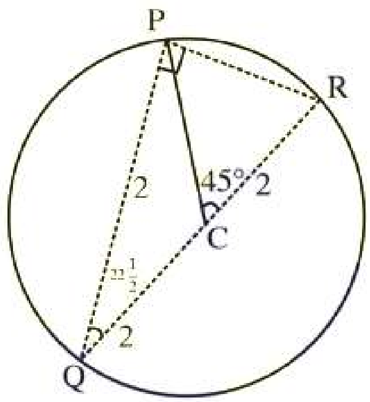
Ans. Official Answer NTA (2)

Sol. $x^2 + y^2 - x + 2y = \frac{11}{4}$

$\left(x - \frac{1}{2}\right)^2 + (y+1)^2 = (2)^2$

Or Δ PQR

$PR = QK \sin 2 \geq \frac{1}{3}$



$= 4.6 \sin\frac{\pi}{8}$

$PQ = QR\cos 22\frac{1}{2}$

$= 4\cos\frac{\pi}{8}$

As $\Delta PQR = \frac{1}{2} PR \times PQ$

$= \frac{1}{2}\left(4^2 \sin\frac{\pi}{6}\right)\left(4\cos\frac{\pi}{8}\right)$

$= 4\sin\frac{\pi}{4} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$

Question ID : 100613

Binomial Theorem

13. The remainder when $7^{2022} + 3^{2022}$ is divided by 5 is :

$7^{2022} + 3^{2022}$ को 6 से विभाजित करने पर शेषफल है :

(1) 0 (2) 2 (3) 3 (4) 4

Ans. Official Answer NTA (3)

Sol. $7^{2022} + 3^{2022}$

$= (49)^{1011} + (9)^{1011}$

$= (50-1)^{1011} + (10-1)^{1011}$

$= 5\lambda - 1 + 5K - 1$

$= 5m - 2$

Remainder $= 5 - 2 = 3$

Question ID : 100614

Matrices

14. Let the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and the matrix $B_0 = A^{49} + 2A^{98}$. If $B_n = \text{Adj}(B_{n-1})$ for all $n \geq 1$, then $\det(B_4)$ is equal to :

माना आव्यूह $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ तथा आव्यूह $B_0 = A^{49} + 2A^{98}$ हैं। यदि सभी $n \geq 1$ के लिए $B_n = \text{Adj}(B_{n-1})$ है, तो $\det(B_4)$ बराबर है :

(1) 3^{28} (2) 3^{30} (3) 3^{32} (4) 3^{36}

Ans. Official Answer NTA (3)

Sol. $A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

$a \leftrightarrow R_2$

$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$

$R_2 \leftrightarrow R_3$

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$

$B_0 = A^{49} + 2A^{98}$
$= A + 2I$
$B_n = \text{Adj}\,(B_n - 1)$
$B_4 = \text{Adj}(\text{Adj}(\text{Adj}(\text{Adj}B_0))$

$= |B_0|^{(n-1)^4}$

$= |B_0|^{16}$

$B_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{bmatrix}$

$= 2(4 - 0) - 1(0 - 1)$
$= 9$

$B_4\ (9)^{16} = (3)^{32}$

Question ID : 100615

Complex number

15. Let $S_1 = \left\{z_1 \in C : |z_1 - 3| = \frac{1}{2}\right\}$ and $S_2 = \{z_2 \in C : |z_2 - |z_2 + 1|| = |z_2 + |z_2 - 1||\}$. Then, for $z_1 \in S_1$ and $z_2 \in S_2$, the least value of $|z_2 - z_1|$ is :

माना $S_1 = \left\{z_1 \in C : |z_1 - 3| = \frac{1}{2}\right\}$ तथा $S_2 = \{z_2 \in C : |z_2 - |z_2 + 1|| = |z_2 + |z_2 - 1||\}$ हैं। तो $z_1 \in S_1$ तथा $z_2 \in S_2$ के लिए $|z_2 - z_1|$ का निम्नतम मान है :

(1) 0 (2) $\frac{1}{2}$ (3) $\frac{3}{2}$ (4) $\frac{5}{2}$

Ans. Official Answer NTA (3)

Sol. $|z_2 + |z_2 - 1||^2 = |z_2 - |z_2 + 1||^2$

$\Rightarrow |z_2 + |z_2 - 1||\left(\bar{z}_2 + |z_2 - 1|\right) = \left(z_2 - |z_2 + 1|\right)\left(\bar{z}_2 - (z_2 + 1)\right)$

$\Rightarrow z_2 \left|\bar{z}_2 + 12_2 - 1\right| - \left(\bar{z}_2 - |z_2 + 1|\right) + \bar{z}_2\left(|z_2 - 1| + |z_2 + 1|\right)$

$= |z_2 + 1|^2 = |z_2 - 1|^2$

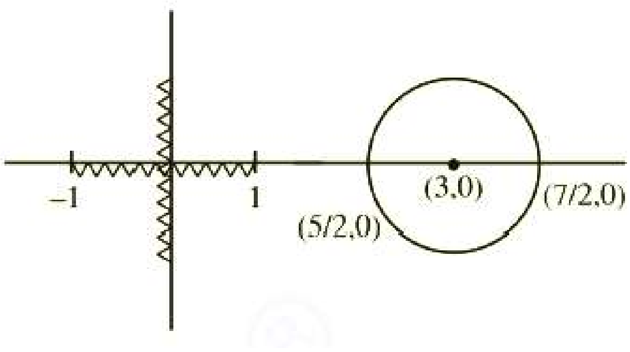
$\Rightarrow \left[z_2 + \bar{z}_2\right)\left(|z_2 - 1|\right) + \left(z_2 + 1\right) = 2\left(z_2 + \bar{z}_2\right)$

$\Rightarrow \left(z_2 + \bar{z}_2\right)\left(|z_2 - 1| + |z_2 + 1| - 2\right) = 0$

$\therefore z_2 + \bar{z}_2 = 0$ or $|z_2 - 1| + |z_2 + 1| - 2 = 0$

$\therefore z_2$ lie on imaginary axis. Or on real axis within [–1, 1]

Also $|z_1 - 3| = \frac{1}{2}$ lie on circle having centre 3 and radius $\frac{1}{2}$.



Clearly $|z_1 - z_2|$ min $= \frac{5}{2} - 1 = \frac{3}{2}$

Question ID : 100616

3D Geometry

16. The foot of the perpendicular from a point on the circle $x^2 + y^2 = 1, z = 0$ to the plane $2x + 3y + z = 6$ lies on which one of the following curves?

वृत्त $x^2 + y^2 = 1, z = 0$ के एक बिन्दु से समतल $2x + 3y + z = 6$ पर डाले गए लंब का पाद निम्न में से किस वक्र पर है ?

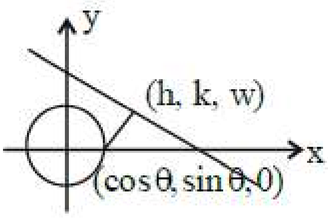
(1) $(6x + 5y - 12)^2 + 4(3x + 7y - 8)^2 = 1, z = 6 - 2x - 3y$

(2) $(5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1, z = 6 - 2x - 3y$

(3) $(6x + 5y - 14)^2 + 9(3x + 5y - 7)^2 = 1, z = 6 - 2x - 3y$

(4) $(5x + 6y - 14)^2 + 9(3x + 7y - 8)^2 = 1, z = 6 - 2x - 3y$

Ans. Official Answer NTA (2)

Sol. 

$$\frac{h-\cos\theta}{2} = \frac{k-\sin\theta}{3} = \frac{w-0}{1}$$

$$= \frac{-1(2\cos\theta + 3\sin\theta - 6)}{14}$$

$$h = \cos\frac{-2(2\cos\theta + 3\sin\theta - 6)}{14}$$

$$= \frac{10\cos\theta - 6\sin\theta + 12}{14}$$

$$k = \sin\theta - \frac{3}{14}(2\cos\theta + 3\sin\theta - 6)$$

$$k = \frac{5\sin\theta - 6\cos\theta + 18}{14}$$

Elementaary $\sin\theta$ and $\cos\theta$

$$(5h + 6k - 12)^2 + 4(3h + 5k - 9)^2 = 1$$

Question ID : 100617

Sequence & progression

17. If the minimum value of $f(x) = \frac{5x^2}{2} + \frac{\alpha}{x^5}$, $x > 0$, is 14, then the value of α is equal to :

यदि $f(x)=\frac{5x^2}{2}+\frac{\alpha}{x^5}$, $x>0$, का निम्नतम मान 14 है, तो α का मान बराबर है :

(1) 32 (2) 64 (3) 128 (4) 256

Ans. Official Answer NTA (3)

Sol. $\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{\alpha}{2x^5}+\frac{\alpha}{2x^5}$

$\geq 7\left(\frac{\alpha^2}{2^7}\right)^{\frac{1}{7}}$

$\frac{7.(\alpha)^{2/7}}{2}=14$

$(\alpha^2)^{1/7} = 2^2$

$\alpha = (2^2)^{7/2} = 2^7$

$\alpha = 128$

Question ID : 100618

Sequence & progression

18. Let α, β and γ be three positive real numbers. Let $f(x) = \alpha x^5 + \beta x^3 + \gamma x$, $x \in R$ and $g : R \to R$ be such that $g(f(x)) = x$ for all $x \in R$. If $a_1, a_2, a_3, \ldots\ldots, a_n$ be in arithmetic progression with mean zero, then the value of $f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right)$ is equal to :

माना α, β तथा γ तीन धनात्मक वास्तविक संख्याऐं है। माना $f(x) = \alpha x^5 + \beta x^3 + \gamma x$, $x \in R$ तथा $g : R \to R$ इस प्रकार हैं कि सभी $x \in R$ के लिए $g(f(x)) = x$ है। यदि $a_1, a_2, a_3, \ldots\ldots, a_n$ एक समांतर श्रेढ़ी में में है, जिनका माध्य शून्य है, तो $f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right)$ का मान बराबर है :

(1) 0 (2) 3 (3) 9 (4) 27

Ans. Official Answer NTA (1)

Sol. Consider a case when $\alpha = \beta = 0$ then

$f(x) = yx$

$g(x)=\frac{x}{y}$

$\frac{1}{n}\sum_{i=1}^{n} f(a_i) \Rightarrow \frac{y}{n}(a_1+a_2+\ldots..+a_n)$

$= 0$

$\Rightarrow f(g(0)) \Rightarrow f(0)$

$\Rightarrow 0$

Question ID : 100619

Binomial Theorem

19. Consider the sequence $a_1, a_2, a_3, ...$ such that $a_1 = 1, a_2 = 2$ and $a_{n+2} = \frac{2}{a_{n+1}} + a_n$ for n = 1, 2, 3, ... If

$$\left(\frac{a_1 + \frac{1}{a_2}}{a_3}\right) \cdot \left(\frac{a_2 + \frac{1}{a_3}}{a_4}\right) \cdot \left(\frac{a_3 + \frac{1}{a_4}}{a_5}\right) .... \left(\frac{a_{30} + \frac{1}{a_{31}}}{a_{32}}\right) = 2^{\alpha}\left({}^{61}C_{31}\right)$$, then α is equal to :

अनुक्रम $a_1, a_2, a_3, ...$ का विचार कीजिए जिसके लिए $a_1 = 1, a_2 = 2$ तथा $a_{n+2} = \frac{2}{a_{n+1}} + a_n$, n = 1, 2, 3, ...

हैं। यदि $$\left(\frac{a_1 + \frac{1}{a_2}}{a_3}\right) \cdot \left(\frac{a_2 + \frac{1}{a_3}}{a_4}\right) \cdot \left(\frac{a_3 + \frac{1}{a_4}}{a_5}\right) .... \left(\frac{a_{30} + \frac{1}{a_{31}}}{a_{32}}\right) = 2^{\alpha}\left({}^{61}C_{31}\right)$$ है, तो α बराबर है :

(1) –30 (2) –31 (3) –60 (4) –61

Ans. Official Answer NTA (3)

Sol. $a_{n+2}\, a_{n+1} - a_{n+1} - a_n = 2$

Series will satisfy

$a_1a_2,\ a_2a_3,\ a_3a_4,\ a_4a_5$

1.2 2.2 2.3 2.4

$$\frac{a_n + \frac{1}{a_{n+1}}}{a_{n+2}} = \frac{a_{n+2} - \frac{1}{a_{n+1}}}{a_{n+2}}$$

$$= 1 - \frac{1}{a_{n+1}a_{n+2}}$$

$$= 1 - \frac{1}{2(r+1)}$$

$$= \frac{2r+1}{2(r+1)}$$

now proof is given by

$= \prod_{r=1}^{30} \frac{(2r+1)}{2(r+1)}$

$= \frac{(1.3.5.......61)}{2^{30}.(2.3......31)}$

$\Rightarrow \frac{(1.3.5.......61)}{\lfloor 31 . 2^{30}} \times \frac{2^{30} + \lfloor 30}{2^{30} + \lfloor 30}$

$= \frac{\lfloor 61}{2^{60} \lfloor 31 \lfloor 30}$

$\alpha = -60$

Question ID : 100620

Differential Equation

20. The minimum value of the twice differentiable function $f(x) = \int_0^x e^{x-t} f'(t) dt - (x^2 - x + 1)e^x$, $x \in R$, is:

दो बार अवकलनीय फलन $f(x) = \int_0^x e^{x-t} f'(t) dt - (x^2 - x + 1)e^x$, $x \in R$ का निम्नतम मान है :

(1) $-\frac{2}{\sqrt{e}}$ (2) $-2\sqrt{e}$ (3) $-\sqrt{e}$ (4) $\frac{2}{\sqrt{e}}$

Ans. Official Answer NTA (1)

Sol. $f(x) = e^x . \int_0^x \frac{f'(t)}{e^t} dt$

$f'(x) = e^x . \int_0^x \frac{f'(t)}{e^t} dt + e^x . \frac{f'(x)}{e^x}$

$-\left[(2x-1).e^x + (x^2 - x + 1).e^x\right]$

$\int_0^x \frac{f'(t)}{e^t} dt = x^2 + x$

$\frac{f'(x)}{e^x} = 2x + 1$

$f'(x) = (2x+1).e^x$

$f'(x) = 0 \Rightarrow x = -\frac{1}{2}$

$f(x) = (2x+1).e^x - 2e^x + C$

$|f(0) = -1$

$-1 = 1 - 2 + C$

$C = 0$

$f(x) = e^x(2x - 1)$

$f\left(-\frac{1}{2}\right) = \frac{-2}{\sqrt{e}}$

SECTION – B

Question ID : 100621

P & C

21. Let S be the set of all passwords which are six to eight characters long, where each character is either an alphabet from {A, B, C, D, E} or a number from {1, 2, 3, 4, 5} with the repetition of characters allowed. If the number of passwords in S whose at least one character is number from {1, 2, 3, 4, 5} is $\alpha \times 5^6$, then α is equal to _________.

माना छः से आठ चिन्ह लंबे सभी संकेत–शब्दों {A, B, C, D, E} से एक अक्षर या {1, 2, 3, 4, 5} से एक अंक है तथा जिनमें चिन्हों की पुनरावृत्ति की अनुमति है, का समुच्चय S है यदि S में उन संकेत–शब्दों, जिनका कम से कम एक चिन्ह {1, 2, 3, 4, 5} में से एक अंक है, कि संख्या $\alpha \times 5^6$ है, तो α बराबर है ________ ।

Ans. Official Answer NTA (7073)

Sol. Required no. = Total – no character from {1, 2, 3, 4, 5}

$= (10^6 - 5^6) + (10^7 - 5^7) + (10^8 - 5^8)$

$= 10^6(1 + 10 + 100) - 5^6(1 + 5 + 25)$

$= 10^6 \times 111 - 5^6 \times 31$

$= 2^6 \times 5^6 \times 111 - 5^6 \times 31$

$= 5^6(2^6 \times 111 - 31)$

$= 5^6 \times \underbrace{7073}_{\alpha}$

$\therefore \alpha = 7073$

Question ID : 100622

3D Geometry

22. Let P(–2, –1, 1) and $Q\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ be the vertices of the rhombus PRQS. If the direction ratios of the diagonal RS are $\alpha, -1, \beta$, where both α and β are integers of minimum absolute values, then $\alpha^2 + \beta^2$ is equal to _________.

माना P(–2, –1, 1) तथा $Q\left(\frac{56}{17}, \frac{43}{17}, \frac{111}{17}\right)$ एक समचतुर्भुज PRQS के शीर्ष हैं। यदि विकर्ण RS के दिक्–अनुपात

$\alpha, -1, \beta$ हैं, जहाँ α तथा β दोनों निम्नतम निरपेक्ष मान के पूर्णांक हैं, तो $\alpha^2 + \beta^2$ बराबर है ________।

Ans. Official Answer NTA (450)

Sol. $RS \equiv (\alpha, -1, \beta)$

DR of $PQ \equiv \left(\frac{56}{17}+2, \frac{43}{17}+1, \frac{111}{17}-1\right)$

$\equiv \left(\frac{90}{17}, \frac{60}{17}, \frac{94}{17}\right)$

$\frac{90}{17}\alpha + \frac{60}{17}(-1) + \frac{94}{17}\beta = 0$

$90\alpha + 94\beta = 60$

$\beta = \frac{60-90\alpha}{94}$

$\beta = \frac{30(2-3\alpha)}{94}$

$\beta = -30\frac{(3\alpha-2)}{94}$

$\frac{\beta}{-15} = \frac{3\alpha-2}{47}$

$\Rightarrow \beta = -15, \alpha = -15$

$\alpha^2 + \beta^2 = 225 + 225$

$= 450$

Question ID : 100623

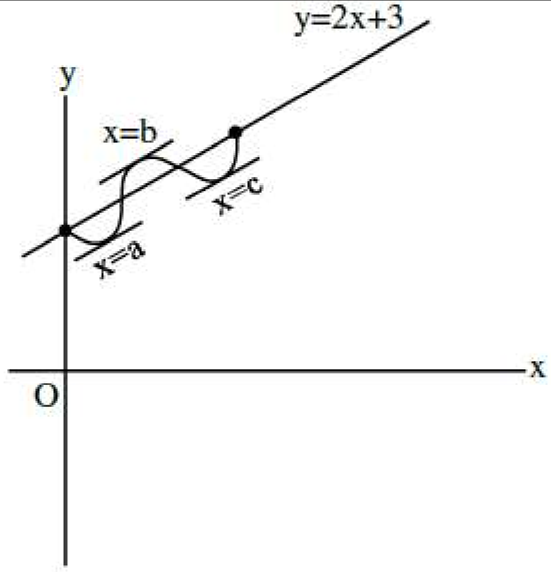
Monotonocity

23. Let $f : [0, 1] \to R$ be a twice differentiable function in (0, 1) such that f(0) = 3 and f(1) = 5. If the line $y = 2x + 3$ intersects the graph of f at only two distinct points in (0, 1), then the least number of points $x \in (0, 1)$, at which $f''(x) = 0$, is _________.

माना $f : [0, 1] \to R$, अंतराल (0, 1) में दो बार अवकलनीय है तथा $f(0) = 3$ हैं। यदि रेखा $f(1) = 5$ हैं। यदि रेखा $y = 2x + 3$, f के ग्राफ को (0, 1) में केवल दो भिन्न बिन्दुओं पर काटती है, तो बिन्दुओं $x \in (0, 1)$ की न्यूनतम संख्या, जिन पर $f''(x) = 0$ है, है________।

Ans. Official Answer NTA (2)

Sol.



$f'(a) = f'(b) = f'(c) = 2$
$\Rightarrow f''(x)$ is zero
for atleast $x_1 \in (a, b)$ & $x_2 \in (b, c)$

Question ID : 100624

Definite Integration

24. If $\int_0^{\sqrt{3}} \frac{15x^3}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}} dx = \alpha\sqrt{2}+\beta\sqrt{3}$, where α, β are integers, then $\alpha+\beta$ is equal to _________.

यदि $\int_0^{\sqrt{3}} \frac{15x^3}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}} dx = \alpha\sqrt{2}+\beta\sqrt{3}$ है, जहाँ α, β पूर्णांक है, तो $\alpha+\beta$ बराबर______ ।

Ans. Official Answer NTA (10)

Sol. Put $1 + x^2 = t^2$

$2x\, dx = 2t\, dt$

$X\, dx = t\, dt$

$\therefore \int_1^2 \frac{15(t^2-1)t\,dt}{\sqrt{t^2+t^3}}$

$15\int_1^2 \frac{t(t^2-1)}{t\sqrt{1+t}} dt$

Put $1 + t = u^2$
$dt = 2u\, du$

$15\int_{\sqrt{2}}^{\sqrt{3}} (u^4-2u^2) du$

$$30\left(\frac{u^5}{5}-\frac{2u^3}{3}\right)_{\sqrt{2}}^{\sqrt{3}}$$

$$30\left[\frac{1}{5}\left(\sqrt{3}^5-\sqrt{2}^5\right)-\frac{2}{5}\left(\sqrt{3}^3-\sqrt{2}^3\right)\right]$$

$$30\left[\frac{1}{5}\left(9\sqrt{3}-4\sqrt{2}\right)-\frac{2}{3}\left(3\sqrt{3}-2\sqrt{2}\right)\right]$$

$$30\left[-\frac{1}{5}+\sqrt{3}+\frac{8}{15}\sqrt{2}\right]$$

$$-6\sqrt{3}+16\sqrt{2}=\alpha\sqrt{2}+\beta\sqrt{3}$$

$$\alpha=16,\beta=-6$$

$$\therefore \alpha+\beta=10$$

Question ID : 100625

Matrices

25. Let $A=\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}$ and $B=\begin{bmatrix}\beta & 1\\ 1 & 0\end{bmatrix}$, $\alpha,\ \beta\in R$. Let α_1 be the value of α which satisfies $(A+B)^2=A^2+\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$ and α_2 be the value of α which satisfies $(A+B)^2=B^2$. Then $|\alpha_1-\alpha_2|$ is equal to ________.

माना $A=\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}$ तथा $B=\begin{bmatrix}\beta & 1\\ 1 & 0\end{bmatrix}$, $\alpha,\beta\in R$ हैं। माना $(A+B)^2=A^2+\begin{bmatrix}2 & 2\\ 2 & 2\end{bmatrix}$ को संतुष्ट करने वाला α का मान α_1 है तथा $(A+B)^2=B^2$ को संतुष्ट करने वाला α का मान α_2 हैं। तो $|\alpha_1-\alpha_2|$ बराबर है ______।

Ans. Official Answer NTA (2)

Sol. $$A+B=\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}$$

$$(A+B)^2=\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}\begin{bmatrix}\beta+1 & 0\\ 3 & \alpha\end{bmatrix}$$

$$=\begin{bmatrix}(\beta+1)^2 & 0\\ 3(\beta+1)+3\alpha & \alpha^2\end{bmatrix}$$

$$A^2=\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}\begin{bmatrix}1 & -1\\ 2 & \alpha\end{bmatrix}$$

$$=\begin{bmatrix}-1 & -1-\alpha \\ 2+2\alpha & \alpha^2-2\end{bmatrix}$$

$$\therefore \begin{bmatrix}1 & -\alpha+1 \\ 2\alpha+4 & \alpha^2\end{bmatrix}=\begin{bmatrix}(\beta+1)^2 & 0 \\ 3(\alpha+\beta+1) & \alpha^2\end{bmatrix}$$

$\alpha = 1 = \alpha_1$

$$B^2=\begin{bmatrix}\beta & 1 \\ 1 & 0\end{bmatrix}\begin{bmatrix}\beta & 1 \\ 1 & 0\end{bmatrix}$$

$$=\begin{bmatrix}\beta^2+1 & \beta \\ \beta & 1\end{bmatrix}=\begin{bmatrix}(\beta+1)^2 & 0 \\ 3(\beta+1)+3\alpha & \alpha^2\end{bmatrix}$$

$\therefore\ \beta = 0, \alpha = -1 = \alpha_2$

$|\alpha_1 - \alpha_2| = |1-(-1)| = 2$

Question ID : 100626

Sequence & progression

26. For $p, q \in R$, consider the real valued function $f(x) = (x - p)^2 - q$, $x \in R$ and $q > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $|f(a_i)| = 500$ for all $i = 1, 2, 3, 4$ then the absolute difference between the roots of $f(x) = 0$ is _________.

$p, q \in R, q > 0$, के लिए वास्तविक मान फलन $f(x) = (x - p)^2 - q$, $x \in R$ का विचार कीजिए। माना a_1, a_2, a_3 तथा a_4 एक धनात्मक सार्व अंतर की समांतर श्रेढ़ी में है तथा इनका माध्य p है। यदि $i = 1, 2, 3, 4$ के लिए $|f(a_i)| = 500$ है, तो $f(x) = 0$ के मूलों का निरपेक्ष अंतर है _________।

Ans. Official Answer NTA (50)

Sol. $f(x) = 0 \Rightarrow (x - p)^2 - q = 0$.

Roots are $p+\sqrt{q}, p-\sqrt{q}$ absolute difference between roots $2\sqrt{q}$.

Now, $|f(a_i)| = 500$

Let a_1, a_2, a_3, a_4 are a_1 $a + d$, $a + 2d$, $a + 3d$

$|f(a_4)| = 500$

$|(a_1 - p)^2 - q| = 500$

$\Rightarrow (a_1 - p)^2 - q = 500$

$\Rightarrow \frac{9}{4}d^2 - q = 500$ ....(1)

and $|f(a_1)|^2 = |f(a_2)|^2$

$((a_1 - p)^2 - q)^2 = ((a_2 - p)^2 - q)^2$

$\Rightarrow ((a_1 - p)^2 - (a_2 - p)^2)\ ((a_1 - p)^2 - q + (a_2 - p)^2 - q) = 0$

$\Rightarrow \frac{9}{4}d^2 - q + \frac{d^2}{4} - q = 0$

$2q = \frac{10d^2}{4} \Rightarrow q = \frac{5d^2}{4}$

$\Rightarrow d^2 = \frac{4q}{5}$

From equation (1) $\frac{9}{4}.\frac{4.q}{5} - q = 500$

$\frac{4q}{5} = 500$

and $2\sqrt{q} = 2 \times \frac{50}{2} = 50$

Question ID : 100627

Hyperbola

27. For the hyperbola $H : x^2 - y^2 = 1$ and the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b > 0$, let the

अतिपरवलय $H : x^2 - y^2 = 1$ तथा दीर्घवृत्त $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, $a > b > 0$ के लिए, माना

(1) eccentricity of E be reciprocal of the eccentricity of H, and

E की उत्केन्द्रता, H की उत्केन्द्रता की व्युत्क्रमणीय हैं, तथा

(2) the line $y = \sqrt{\frac{5}{2}}x + K$ be a common tangent of E and H.

रेखा $y = \sqrt{\frac{5}{2}}x + K$, E तथा H की एक उभयनिष्ठ स्पर्श रेखा है।

Then $4(a^2 + b^2)$ is equal to _________.

तो $4(a^2 + b^2)$ बराबर है _________.

Ans. Official Answer NTA (3)

Sol. $e_E = \sqrt{1 - \frac{b^2}{a^2}}, e_H = \sqrt{2}$

If $\Rightarrow e_E = \frac{1}{e_H}$

$\Rightarrow \frac{a^2 - b^2}{a^2} = \frac{1}{2}$

$2a^{2-2b} = a^2$

$a^2 = 2b^2$

and $y = \sqrt{\frac{5}{2}}x + k$ is tangent to ellipse then

$K^2 = a^2 \times \frac{5}{2} + b^2 = \frac{3}{2}$

$6b^2 = \frac{3}{2} \Rightarrow b^2 = \frac{1}{4}$ and $a^2 = \frac{1}{2}$

$\therefore\ 4.\left(a^2 + b^2\right) = 3$

Question ID : 100628

Statistics

28. Let $x_1, x_2, x_3, ..., x_{20}$ be in geometric progression with $x_1 = 3$ and the common ratio $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If $\overline{x}$ is the mean of new data, then the greatest integer less than or equal $\overline{x}$ to is _________.

माना $x_1 = 3, x_2, x_3, ..., x_{20}$ एक गुणोत्तर श्रेढ़ी में हैं, जिसका सार्व अनुपात $\frac{1}{2}$ है। प्रत्येक x_i की जगह $(x_i - i)^2$. लेकर नये आँकड़े बनाए जाते हैं। यदि नये आँकड़े का माध्य $\overline{x}$ है तो महत्तम पूर्णांक $\leq \overline{x}$ है________।

Ans. Official Answer NTA (142)

Sol. $\Sigma x_0^1 = \dfrac{3\left(1-\left(\frac{1}{2}\right)\right)^{20}}{1\frac{-1}{2}} = 6\left(1-\frac{1}{2^{20}}\right)$

$= \sum\limits_{i=1}^{20}\left(x_{i=1}\right)^2$

$= \sum\limits_{i=1}^{20}\left(x_i\right)^2 + (i) - 2x_i i$

Now $= \sum\limits_{i=1}^{20}\left(x_i\right)^2 = \dfrac{9\left(1-\left(\frac{1}{4}\right)\right)^{20}}{1-\frac{1}{4}} = 12\left(1-\frac{1}{2^{40}}\right)$

$\sum\limits_{i=1}^{20} i^2 = \frac{1}{6} \times 20 \times 21 + 41 = 2870$

$$\sum_{i=1}^{20} x_i i = s = 3 + 2.3\frac{1}{2} + 3.3\frac{1}{2^2} + 4.3\frac{1}{2^3} + ...AGP$$

$$= 6\left(-\frac{22}{2^{20}}\right)$$

$$\bar{x} = \frac{12 - \frac{12}{2^{40}} + 2870 - 12\left(2 - \frac{22}{2^{20}}\right)}{20}$$

$$\bar{x} = \frac{2858}{20} + \left(\frac{-12}{2^{40}} + \frac{22}{2^{20}}\right) \times \frac{1}{20}$$

$$\left[\bar{x}\right] = 142$$

Question ID : 100629

Limits

29. $\lim\limits_{x\to 0}\left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)^{\frac{100}{x}}$ is equal to _________.

$\lim\limits_{x\to 0}\left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)^{\frac{100}{x}}$ बराबर है________।

Ans. Official Answer NTA (1)

Sol. $\lim\limits_{x\to 10}\left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)^{x}$

Form 1^∞

$$= e^{\lim\limits_{x\to 0}\left[\left(\frac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)}{(x+2)^3+2(x+2)^2+3\sin(x+2)}\right)-1\right]\times\frac{100}{x}}$$

$$= e^{\lim\limits_{x\to 0}\left[\frac{100}{x}\left(\frac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)-\left((x+2)^3+2(x+2)^2+3\sin(x+2)\right)}{(x+2)^3+2(x+2)^2+3\sin(x+2)}\right)\right]}$$

$$= e^{\lim\limits_{x\to 0}\frac{100}{x}\left[\left(\frac{(x+2\cos x)^3+(x+2)^3+2(x+2\cos x)^2-2(x+2)^2+3\sin(x+2\cos x)-3\sin(x+2)}{8+8+3\sin^2}\right)\right]}$$

$$= e^{\frac{100}{16+3\sin^2}\lim\limits_{x\to 0}\frac{3(x+2\cos x)^2\times(1+2\sin x)-3(x+2)^2-4(x+2\cos x)}{x(1-2\sin x)-4(x+2)+3\cos(x+2\cos x)\times(1-2\sin x)-3\cos(x+2)}}$$

$$= e^{\frac{100}{16+3\sin 2}\left(\frac{12-3(4)+8\times1-8+3\cos 2-3\cos 2}{1}\right)}$$

Using L'H rule.

$= e^{o} = 1$

Question ID : 100630

Quadratic Equation

30. The sum of all real value of x for which $\frac{3x^2-9x+17}{x^2+3x+10}=\frac{5x^2-7x+19}{3x^2+5x+12}$ is equal to _________.

x के सभी मानों, जिसके लिए $\frac{3x^2-9x+17}{x^2+3x+10}=\frac{5x^2-7x+19}{3x^2+5x+12}$ है, का योग बराबर है _________।

Ans. Official Answer NTA (6)

Sol. $\frac{3x^2-9x+17}{x^2+3x+10}=\frac{5x^2-7x+19}{3x^2+5x+12}$

$$\frac{x^2+3x+10+2x^2-12x+7}{x^2+3x+10}=\frac{3x^2+5x+12+2x^2-12x+7}{3x^2+5x+12}$$

$$1+\frac{2x^2-12x+7}{x^2+3x+10}=1+\frac{2x^2-12x+7}{3x^2+5x+12}$$

$$\left(2x^2-12x+7\right)\left(\frac{1}{x^2+3x+10}-\frac{1}{3x^2+5x+12}\right)=0$$

$2x^2 - 12x + 7 = 0$ OR $3x^2 + 5x + 12 = x^2 + 3x + 10$

$x=\frac{12\pm\sqrt{D}}{4}$ $\qquad$ $2x^2 + 2x + 2 = 0$

$x^2 + x + 1 = 0$

Sum or Roots = 6 $\qquad$ No solution.