JEE Main July 2022

Question Paper With Text Solution

28 July | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JULY 2022 | 28TH JULY SHIFT-2

SECTION – A

Question ID : 154771545261

Function

1. Let $S=\left\{x \in[-6,3]-\{-2,2\}: \frac{|x+3|-1}{|x|-2} \geq 0\right\}$ and $T = \{x \in Z : x^2 - 7|x| + 9 \leq 0\}$. Then the number of elements in $S \cap T$ is :

माना $S=\left\{x \in[-6,3]-\{-2,2\}: \frac{|x+3|-1}{|x|-2} \geq 0\right\}$ तथा $T = \{x \in Z : x^2 - 7|x| + 9 \leq 0\}$ हैं। तो $S \cap T$ में अवयवों की संख्या है :

(1) 7 (2) 5 (3) 4 (4) 3

Ans. Official Answer NTA (4)

Sol. $S \cap T = \{-5, -4, 3\}$

Question ID : 154771545261

Quadratic Equation

2. Let α, β be the roots of the equation $x^2-\sqrt{2}x+\sqrt{6}=0$ and $\frac{1}{\alpha^2}+1, \frac{1}{\beta^2}+1$ be the roots of the equation $x^2 + ax + b = 0$. Then the roots of the equation $x^2 - (a + b - 2)x + (a + b + 2) = 0$ are :

(1) non-real complex numbers (2) real and both negative

(3) real and both positive (4) real and exactly one of them is positive

माना समीकरण $x^2-\sqrt{2}x+\sqrt{6}=0$ के मूल α, β हैं तथा समीकरण $x^2+ax+b=0$ के मूल $\frac{1}{\alpha^2}+1, \frac{1}{\beta^2}+1$ हैं। तो समीकरण $x^2 - (a + b - 2)x + (a + b + 2) = 0$ के मूल:

(1) अवास्तविक सम्मिश्र संख्याऐं है (2) वास्तविक तथा दोनों ऋणात्मक हैं

(3) वास्तविक तथा दोनों धनात्मक हैं (4) वास्तविक तथा उनकें से ठीक एक धनात्मक है

Ans. Official Answer NTA (2)

Sol. $a=\frac{-1}{\alpha^2}-\frac{1}{\beta^2}-2$

$b=\frac{1}{\alpha^2}+\frac{1}{\beta^2}+1+\frac{1}{\alpha^2\beta^2}$

$$a+b=\frac{1}{(\alpha\beta)^2}-1=\frac{1}{6}-1=-\frac{5}{6}$$

$$x^2-\left(-\frac{5}{6}-2\right)x+\left(2-\frac{5}{6}\right)=0$$

$6x^2+17x+7=0$

$x=-\frac{7}{3}, x=-\frac{1}{2}$ are the roots

Both roots are real and negative.

Question ID : 154771545263

Matrices

3. Let A and B be any two 3×3 symmetric and skew symmetric matrices respectively. Then which of the following is NOT true?

(1) A^4-B^4 is a symmetric matrix (2) $AB-BA$ is a symmetric matrix

(3) B^5-A^5 is a skew-symmetric matrix (4) $AB-BA$ is a skew-symmetric matrix

माना A तथा B क्रमशः कोई भी दो 3×3 की सममित तथा विषम सममित आव्यूह हैं। तो निम्न में से कौनसा सत्य नहीं है ?

(1) A^4-B^4 एक सममित आव्यूह है (2) $AB-BA$ एक सममित आव्यूह है

(3) B^5-A^5 एक विषम सममित आव्यूह है (4) $AB-BA$ एक विषम सममित आव्यूह है

Ans. Official Answer NTA (3)

Sol. Given that $A^T=A, B^T=-B$

(A) $C=A^4-B^4$

$C^T=(A^4-B^4)=(A^4)^T-(B^4)^T=A^4-B^4=C$

(B) $C=AB-BA$

$C^T=(AB-BA)^T=(AB)^T-(BA)^T$

$=B^TA^T-A^TB^T=-BA+AB=C$

(C) $C=B^5-A^5$

$C^T=(B^5-A^5)^T=(B^5)^T-(A^5)^T=-B^5-A^5$

(D) $C=AB+BA$

$C^T=(AB+BA)^T=(AB)^T+(BA)^T$

$=-BA-AB=-C$

$\therefore$ Option C is not true.

Question ID : 154771545264

Function

4. Let $f(x) = ax^2 + bx + c$ be such that $f(1) = 3$, $f(-2) = \lambda$ and $f(3) = 4$. If $f(0) + f(1) + f(-2) + f(3) = 14$, then λ is equal to :

माना $f(x) = ax^2 + bx + c$ है, जिसके लिए $f(1) = 3$, $f(-2) = \lambda$ तथा $f(3) = 4$ हैं। यदि $f(0) + f(1) + f(-2) + f(3) = 14$ है, तो λ बराबर है :

(1) -4 (2) $\frac{13}{2}$ (3) $\frac{23}{2}$ (4) 4

Ans. Official Answer NTA (4)

Sol. $f(0) + 3 + \lambda + 4 = 14$

$\therefore f(0) = 7 - \lambda = c$

$f(1) = a + b + c = 3$ .....(i)

$f(3) = 9a + 3b + c = 4$ .....(ii)

$f(-2) = 4a - 2b + c = 1$ .....(iii)

(ii) – (iii)

$a + b = \frac{4-\lambda}{4}$ put in equation (i)

$\frac{4-\lambda}{5} + 7 - \lambda = 3$

$6\lambda = 24; \quad \lambda = 4$

Question ID : 154771545265

Continuity & Differentiability

5. The function $f : R \to R$ defined by $f(x) = \lim\limits_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1 + x^{2n+1} - x^{2n}}$ is continuous for all x in :

$f(x) = \lim\limits_{n\to\infty} \frac{\cos(2\pi x) - x^{2n}\sin(x-1)}{1 + x^{2n+1} - x^{2n}}$ द्वारा परिभाषित फलन $f : R \to R$ किस समुच्चय के सभी बिन्दु x पर संतत है :

(1) $R - \{-1\}$ (2) $R - \{-1, 1\}$ (3) $R - \{1\}$ (4) $R - \{0\}$

Ans. Official Answer NTA (2)

Sol. n should be given as a naturel number.

$$f(x=\begin{cases}\frac{-\sin(x-1)}{x-1} & x<-1\\ -(\sin 2+1) & x=-1\\ \cos 2\pi x & -1<x<1\\ \frac{-\sin(x-1)}{x-1} & x>1\end{cases}$$

f(x) is discontinuous at $x=-1$ and $x=1$

Question ID : 154771545266

Monotonocity

6. The function $f(x)=xe^{x(1-x)}, x\in R$, is :

(1) increasing in $\left(-\frac{1}{2},1\right)$ (2) decreasing in $\left(\frac{1}{2},2\right)$

(3) increasing in $\left(-1,-\frac{1}{2}\right)$ (4) decreasing in $\left(-\frac{1}{2},\frac{1}{2}\right)$

फलन $f(x)=xe^{x(1-x)}, x\in R$, :

(1) $\left(-\frac{1}{2},1\right)$ में वर्धमान है (2) $\left(\frac{1}{2},2\right)$ में ह्रासमान है

(3) $\left(-1,-\frac{1}{2}\right)$ में वर्धमान है (4) $\left(-\frac{1}{2},\frac{1}{2}\right)$ में ह्रासमान है

Ans. Official Answer NTA (1)

Sol. $f(x)=x\,e^{x(1-x)}$

$f'(x)=-e^{x(1-x)}(2x+1)(x-1)$

f(x) is increasing in $\left(-\frac{1}{2},1\right)$

Question ID : 154771545267

Maxima & Minima

7. The sum of the absolute maximum and absolute minimum values of the function $f(x)=\tan^{-1}(\sin x-\cos x)$ in the interval $[0,\pi]$ is :

अंतराल $[0,\pi]$ में, फलन $f(x)=\tan^{-1}(\sin x-\cos x)$ के निरपेक्ष उच्चतम तथा निरपेक्ष निम्न मानों का योग है :

(1) 0 (2) $\tan^{-1}\left(\frac{1}{\sqrt{2}}\right)-\frac{\pi}{4}$ (3) $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)-\frac{\pi}{4}$ (4) $\frac{-\pi}{12}$

Ans. Official Answer NTA (3)

Sol. $f(x)=\tan^{-1}(\sin x-\cos x)$

$$f'(x)=\frac{\cos x+\sin x}{(\sin x-\cos x)^2+1}=0$$

$$\therefore x=\frac{3\pi}{4}$$

x	0	$\frac{3\pi}{4}$	π
$f(x)$	$-\frac{\pi}{4}$	$\tan^{-1}\sqrt{2}$	$\frac{\pi}{4}$

$$\therefore \begin{matrix}(f(x))_{max}=\tan^{-1}\sqrt{2}\\ (f(x))_{min}=-\frac{\pi}{4}\end{matrix}$$

$$\text{sum}=\tan^{-1}\sqrt{2}-\frac{\pi}{4}$$

$$=\cos^{-1}\frac{1}{\sqrt{3}}-\frac{\pi}{4}$$

Question ID : 154771545268

Methods of Differentiation

8. Let $x(t)=2\sqrt{2}\cot t\sqrt{\sin 2t}$ and $y(t)=2\sqrt{2}\sin t\sqrt{\sin 2t}$, $t\in\left(0,\frac{\pi}{2}\right)$. Then $\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ at $t=\frac{\pi}{4}$ is equal to :

माना $x(t)=2\sqrt{2}\cot t\sqrt{\sin 2t}$ तथा $y(t)=2\sqrt{2}\sin t\sqrt{\sin 2t}$, $t\in\left(0,\frac{\pi}{2}\right)$ हैं। तो $\frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$ पर $t=\frac{\pi}{4}$ बराबर है :

(1) $\frac{-2\sqrt{2}}{3}$ (2) $\frac{2}{3}$ (3) $\frac{1}{3}$ (4) $\frac{-2}{3}$

Ans. Official Answer NTA (4)

Sol. $x = 2\sqrt{2}\cos t\sqrt{\sin 2t}$

$$\frac{dx}{dt} = \frac{2\sqrt{2}\cos 3t}{\sqrt{\sin 2t}}$$

$$y(t) = 2\sqrt{2}\sin t\sqrt{\sin 2t}$$

$$\frac{dy}{dt} = \frac{2\sqrt{2}\sin 3t}{\sqrt{\sin 2t}}$$

$$\frac{dy}{dx} = \tan 3t$$

$$\frac{dy}{dx} = -1 \text{ at } t = \frac{\pi}{4}$$

$$\frac{d^2y}{dx^2} = \frac{3}{2\sqrt{2}}\sec^3 3t.\sqrt{\sin 2t} = -3 \text{ at } t = \frac{\pi}{4}$$

$$\therefore \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}} = \frac{1+1}{-3} = -\frac{2}{3}$$

Question ID : 154771545269

Definite Integration

9. Let $I_n(x) = \int_0^x \frac{1}{\left(t^2+5\right)^n}dt$, n = 1, 2, 3, ... Then :

माना $I_n(x) = \int_0^x \frac{1}{\left(t^2+5\right)^n}dt$, n = 1, 2, 3, ... है। तो :

(1) $50I_6 - 9I_5 = xI_5'$

(2) $50I_6 - 11I_5 = xI_5'$

(3) $50I_6 - 9I_5 = I_5'$

(4) $50I_6 - 11I_5 = I_5'$

Ans. Official Answer NTA (1)

Sol. $$I_n(x) = \int_0^x \frac{dt}{\left(t^2+5\right)^n}$$

Applying integral by parts

$$I_n(x) = \left[\frac{t}{\left(t^2+5\right)^n}\right]_0^x - \int_0^x n\left(t^2+5\right)^{-n-1}.2t^2$$

$$I_n(x)=\frac{x}{\left(x^2+5\right)^n}+2n\int_0^x\frac{t^2}{\left(t^2+5\right)^{n+1}}dt$$

$$I_n(x)=\frac{x}{\left(x^2+5\right)^n}+2n\int_0^x\frac{\left(t^2+5\right)-5}{\left(t^2+5\right)^{n+1}}dt$$

$$I_n(x)=\frac{x}{\left(x^2+5\right)^n}+2n\,I_n(x)-10n\;I_{n+1}(x)$$

$$10n\,I_{n+1}(x)+(1-2n)I_n(x)=\frac{x}{\left(x^2+5\right)^n}$$

Put $n=5$

Question ID : 154771545270

Area Under Curve

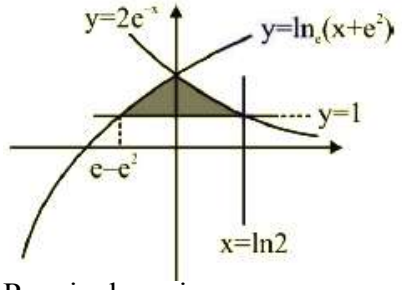
10. The area enclosed by the curve $y=\log_e(x+e^2)$, $x=\log_e\left(\frac{2}{y}\right)$ and $x=\log_e 2$, above the line $y=1$ is :

रेखा $y=1$ के ऊपर वक्रों $y=\log_e(x+e^2)$, $x=\log_e\left(\frac{2}{y}\right)$ तथा $x=\log_e 2$ से घिरे क्षेत्र का क्षेत्रफल है :

(1) $2+e-\log_e 2$ (2) $1+e-\log_e 2$ (3) $e-\log_e 2$ (4) $1+\log_e 2$

Ans. Official Answer NTA (2)

Sol.



Required area is

$$=\int_{e-e^2}^{0}\ell n\left(x+e^2\right)-1dx+\int_0^{\ell n2}2e^{-x}-1dx=1+e-\ell n2$$

Question ID : 154771545271

Differential Equation

11. Let $y=y(x)$ be the solution curve of the differential equation $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$, $x>1$ passing through

the point $\left(2,\sqrt{\frac{1}{3}}\right)$. Then $\sqrt{7}\ y(8)$ is :

माना अवकल समीकरण $\frac{dy}{dx}+\frac{1}{x^2-1}y=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$, $x>1$ का हल वक्र $y=y(x)$ बिन्दु $\left(2,\sqrt{\frac{1}{3}}\right)$ से होकर जाता है। तो $\sqrt{7}\ y(8)$ होगा :

(1) $11+6\log_e 3$ (2) 19 (3) $12-2\log_e 3$ (4) $19-6\log_e 3$

Ans. Official Answer NTA (4)

Sol. $\frac{dy}{dx}+\frac{1}{x^2-1}y\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$,

$\frac{dy}{dx}+Py=Q$

$I.F.=e^{\int Pdx}=\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}$

$y\left(\frac{x-1}{x+1}\right)^{\frac{1}{2}}=\int\left(\frac{x-1}{x+1}\right)^{1}dx$

$=x-2\log_e|x+1|+C$

Curve passes through $\left(2,\frac{1}{\sqrt{3}}\right)$

$\Rightarrow C=2\log_e 3-\frac{5}{3}$

at $x=8$,

$\sqrt{7}y(8)=19-6\log_e 3$

Question ID : 154771545272

Differential Equation

12. The differential equation of the family of circles passing through the points (0, 2) and (0, –2) is :

बिन्दुओं (0, 2) तथा (0, –2) से होकर जाने वाले वृत्त कुल का अवकल समीकरण है :

(1) $2xy\frac{dy}{dx}+\left(x^2-y^2+4\right)=0$ (2) $2xy\frac{dy}{dx}+\left(x^2+y^2-4\right)=0$

(3) $2xy\frac{dy}{dx}+\left(y^2-x^2+4\right)=0$ (4) $2xy\frac{dy}{dx}-\left(x^2-y^2+4\right)=0$

Ans. Official Answer NTA (1)

Sol. Equation of circle passing through (0, –2) and (0, 2) is

$x^2+\left(y^2-4\right)+\lambda x=0,\left(\lambda \in R\right)$

Divided by x we get

$$\frac{x^2+\left(y^2-4\right)}{x}+\lambda=0$$

Differentiating with respect to x

$$\frac{x\left[2x+2y.\frac{dy}{dx}\right]-\left[x^2+y^2-4\right].1}{x^2}=0$$

$$\Rightarrow 2xy.\frac{dy}{dx}+\left(x^2-y^2+4\right)=0$$

Question ID : 154771545273

Circle

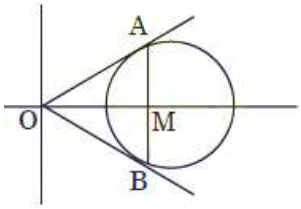
13. Let the tangents at two points A and B on the circle $x^2+y^2-4x+3=0$ meet at origin O(0, 0). Then the area of the triangle OAB is :

माना वृत्त $x^2+y^2-4x+3=0$ के दो बिन्दुओं A तथा B पर स्पर्श रेखाएँ मूल बिन्दु O(0, 0) पर मिलती हैं। तो त्रिभुज OAB का क्षेत्रफल है :

(1) $\frac{3\sqrt{3}}{2}$ (2) $\frac{3\sqrt{3}}{4}$ (3) $\frac{3}{2\sqrt{3}}$ (4) $\frac{3}{4\sqrt{3}}$

Ans. Official Answer NTA (2)

Sol.



$C:\left(x-2\right)^2+y^2=1$

Equation of chord AB : $2x=3$

$OA=OB=\sqrt{3}$

$$AM=\frac{\sqrt{3}}{2}$$

Area of triangle $OAB=\frac{1}{2}\left(2AM\right)\left(OM\right)$

$= \frac{3\sqrt{3}}{4}$ sq. units

Question ID : 154771545274

Hyperbola

14. Let the hyperbola $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$ pass through the point $\left(2\sqrt{2}, -2\sqrt{2}\right)$. A parabola is drawn whose focus is same as the focus of H with positive abscissa and the directrix of the parabola passes through the other focus of H. If the length of the latus rectum of the parabola is e times the length of the latus rectum of H, where e is the eccentricity of H, then which of the following points lies on the parabola?

माना अतिपरवलय $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, बिन्दु $\left(2\sqrt{2}, -2\sqrt{2}\right)$ से होकर जाता है। एक परवलय खींचा जाता है जिसकी नाभि, H की धनात्मक भुज वाली नाभि पर है तथा परवलय की नियता H की दूसरी नाभि से होकर जाती है। यदि परवलय की नाभि लंब जीवा की लंबाई, H की नाभि लंब की जीवा की लंबाई e गुना है, जहाँ e, H की उत्केन्द्रता है, तो निम्न में से कौनसा बिन्दु परवलय पर है ?

(1) $\left(2\sqrt{3}, 3\sqrt{2}\right)$ (2) $\left(3\sqrt{3}, -6\sqrt{2}\right)$ (3) $\left(\sqrt{3}, -\sqrt{6}\right)$ (4) $\left(3\sqrt{6}, 6\sqrt{2}\right)$

Ans. Official Answer NTA (2)

Sol. $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

Foci : S (ae, 0), S' (–ae, 0)
Foot of directrix of parabola is (–ae, 0)
Focus of parabola is (ae, 0)
Now, semi latus rectum of parabola = |SS'| = 2ae

Given, $4ae = e\left(\frac{2b^2}{a}\right)$

$\Rightarrow b^2 = 2a^2$ .....(1)

Given, $\left(2\sqrt{2}, -2\sqrt{2}\right)$ lies on H

$\Rightarrow \frac{1}{a^2} - \frac{1}{b^2} = \frac{1}{8}$ .....(2)

From (1) and (2)
$a^2 = 4, b^2 = 8$

$\because b^2 = a^2\left(e^2 - 1\right)$

$\therefore e = \sqrt{3}$

$\Rightarrow$ Equation of parabola is $y^2 = 8\sqrt{3}x$

Question ID : 154771545275

3D Geometry

15. Let the line $\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$ and $\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$ be coplanar and P be the plane containing these two lines. Then which of the following points does NOT lie no P?

माना रेखाएँ $\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$ तथा $\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$ सह–तलीय हैं तथा P वह समतल है जिस में यह दोनों रेखाएँ स्थित हैं। तो निम्न में से कौनसा बिन्दु, समतल P पर नहीं है ?

(1) (0, –2, –2) (2) (–5, 0, –1) (3) (3, –1, 0) (4) (0, 4, 5)

Ans. Official Answer NTA (4)

Sol. Given, $L_1:\frac{x-1}{\lambda}=\frac{y-2}{1}=\frac{z-3}{2}$

and $L_2:\frac{x+26}{-2}=\frac{y+18}{3}=\frac{z+28}{\lambda}$

are coplanar

$$\Rightarrow\begin{vmatrix}27 & 20 & 31\\ \lambda & 1 & 2\\ -2 & 3 & \lambda\end{vmatrix}=0$$

$\Rightarrow \lambda = 3$

Now, normal of plane P, which contains L_1 and L_2

$$=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 3 & 1 & 2\\ -2 & 3 & 3\end{vmatrix}$$

$=-3\hat{i}-13\hat{j}+11\hat{k}$

$\Rightarrow$ Equation of required plane P :

$3x + 13y – 11z + 4 = 0$

(0, 4, 5) does not lie on plane P.

Question ID : 154771545276

3D Geometry

16. A plane P is parallel to two lines whose direction ratios are –2, 1, –3 and –1, 2, –2 and it contains the point(2, 2, –2). Let P intersect the co-ordinate axes at the points A, B, C making the intercepts α, β, γ. If V is the volume of the tetrahedron OABC, where O is the origin, and $p = \alpha + \beta + \gamma$, then the ordered pair (V, p) is equal to :

एक समतल P, दो रेखाओं, जिनके दिक् अनुपात –2, 1, –3 तथा –1, 2, –2 हैं, के समान्तर है तथा बिन्दु (2, 2, –2)

समतल P पर है। माना P निर्देशांक अक्षों को बिन्दुओं A, B, C पर काटता है तथा अंतःखण्ड α, β, γ बनाता है। यदि चतुष्फलक OABC का आयतन V है, जहाँ O मूल बिन्दु है, तथा $p = \alpha + \beta + \gamma$ है, तो क्रमित युग्म (V, p) बराबर है:

(1) (48, –13) (2) (24, –13) (3) (48, 11) (4) (24, –5)

Ans. Official Answer NTA (2)

Sol. Normal of plane P :

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -2 & 1 & -3 \\ -1 & 2 & -2 \end{vmatrix} = 4\hat{i} - \hat{j} - 3\hat{k}$$

Equation of plane P which passes through (2, 2, –2) is $4x - 7 - 3z - 12 = 0$

Now, A (3, 0, 0), B (0, – 12, 0), C (0, 0, – 4)

$\Rightarrow \alpha = 3, \beta = -12, \gamma = -4$

$\Rightarrow p = \alpha + \beta + \gamma = -13$

Now, volume of tetrahedron OABC

$$V = \left| \frac{1}{6} \overrightarrow{OA} . \left(\overrightarrow{OB} \times \overrightarrow{OC} \right) \right| = 24$$

(V, p) = (24, –13)

Question ID : 154771545277

Vectors

17. Let S be the set of all $a \in R$ for which the angle between the vectors $\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$ and $\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}, (b > 1)$ is acute. Then S is equal to :

माना सभी $a \in R$, जिनके लिए सदिशों $\vec{u} = a(\log_e b)\hat{i} - 6\hat{j} + 3\hat{k}$ तथा $\vec{v} = (\log_e b)\hat{i} + 2\hat{j} + 2a(\log_e b)\hat{k}, (b > 1)$ के बीच का कोण न्यून कोण है, का समुच्चय S है। तो S बराबर है:

(1) $\left(-\infty, -\frac{4}{3}\right)$ (2) Φ (3) $\left(-\frac{4}{3}, 0\right)$ (4) $\left(\frac{12}{7}, \infty\right)$

Ans. Official Answer NTA (3)

Sol. For angle to be acute

$\vec{u} . \vec{v} > 0$

$\Rightarrow a(\log_e b)^2 - 12 + 6a(\log_e b) > 0$

$\forall b > 1$

let $\log_e b = t \Rightarrow t > 0$ as $b > 1$

$y = at^2 + 6at - 12$ & $y > 0, \forall t > 0$

$\Rightarrow a \in \phi$

Question ID : 154771545278

Heights & Distances

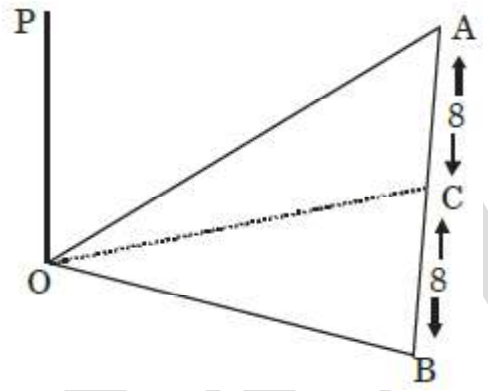
18. A horizontal park is in the shape of a triangle OAB with AB = 16. A vertical lamp post OP is erected at the point O such that $\angle PAO = \angle PBO = 15°$ and $\angle PCO = 45°$, where C is the midpoint of AB, Then $(OP)^2$ is equal to :

एक क्षैतिज पार्क एक त्रिभुज OAB के आकार का है, जिसमें AB = 16 है। एक ऊर्ध्वाधर बिजली का खंभा OP बिन्दु O पर खड़ा है, $\angle PAO = \angle PBO = 15°$ तथा $\angle PCO = 45°$ हैं, जहाँ AB का मध्य बिन्दु C है, तो $(OP)^2$ बराबर है :

(1) $\frac{32}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (2) $\frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$ (3) $\frac{16}{\sqrt{3}}\left(\sqrt{3}-1\right)$ (4) $\frac{16}{\sqrt{3}}\left(2-\sqrt{3}\right)$

Ans.

Sol.



$$\frac{OP}{OA} = \tan 15^\circ$$

$$\Rightarrow OA = OP \cot 15^\circ$$

$$\frac{OP}{OC} = \tan 45^\circ \Rightarrow OP = OC$$

Now, $OP = \sqrt{OA^2 - 8^2}$

$$\Rightarrow OP^2 = (OP)^2 \cot^2 15^\circ - 64$$

$$\Rightarrow OP^2 = \frac{32}{\sqrt{3}}\left(2-\sqrt{3}\right)$$

Question ID : 154771545279

Probability

19. Let A and B be two events such that $P(B|A)=\frac{2}{5}$, $P(A|B)=\frac{1}{7}$ and $P(A\cap B)=\frac{1}{9}$. Consider

(S1) $P(A'\cup B)=\frac{5}{6}$,

(S2) $P(A'\cap B')=\frac{1}{18}$

Then :

(1) Both (S1) and (S2) are true (2) Both (S1) and (S2) are false

(3) Only (S1) is true (4) Only (S2) is true

माना दो घटनाओं A तथा B के लिए $P(B|A)=\frac{2}{5}$, $P(A|B)=\frac{1}{7}$ तथा $P(A\cap B)=\frac{1}{9}$ हैं:

(S1) $P(A'\cup B)=\frac{5}{6}$,

(S2) $P(A'\cap B')=\frac{1}{18}$

तब :

(1) (S1) तथा (S2) दो सत्य हैं (2) (S1) तथा (S2) दोनों असत्य हैं

(3) केवल (S1) सत्य है (4) केवल (S2) सत्य है

Ans. Official Answer NTA (1)

Sol. $P(A|B)=\frac{1}{7}\Rightarrow\frac{P(A\cap B)}{P(B)}=\frac{1}{7}$

$\Rightarrow P(B)=\frac{7}{9}$

$P(B|A)=\frac{2}{5}\Rightarrow\frac{P(A\cap B)}{P(A)}=\frac{2}{5}$

$\Rightarrow P(A)=\frac{5}{18}$

Now, $P(A'\cup B)=1-P(A\cup B)+P(B)$

$=1-P(A)+P(A\cap B)=\frac{5}{6}$

$P(A'\cap B)=1-P(A\cup B)$

$=1-P(A)-P(B)+P(A\cap B)=\frac{1}{18}$

$\Rightarrow$ Both (S1) and (S2) are true.

Question ID : 154771545280

Mathematical Reasoning

20. Let

p : Ramesh listens to music.

q : Ramesh is out of his village.

r : It is Sunday.

s : It is Saturday.

Then the statement "Ramesh listens to music only if he is in his village and it is Sunday or Saturday" can be expressed as :

माना

p : रमेश संगीत सुनता है

q : रमेश अपने गाँव से बाहर है

r : आज रविवार है

s : आज शनिवार है

कथन " रमेश संगीत सुनता है केवल यदि वह अपने गाँव में है तथा आज रविवार या शनिवार है" को किस से व्यक्त कर सकते हैं ?

(1) $((\sim q) \wedge (r \vee s)) \Rightarrow p$

(2) $(q \wedge (r \vee s)) \Rightarrow p$

(3) $p \Rightarrow (q \wedge (r \vee s))$

(4) $p \Rightarrow ((\sim q) \wedge (r \vee s))$

Ans. Official Answer NTA (4)

Sol. $p \equiv$ Ramesh listens to music

$\sim q \equiv$ He is in village.

$r \vee s \equiv$ Saturday or sunday

$p \Rightarrow \left((\sim q) \wedge (r \vee s)\right)$

SECTION – B

Question ID : 154771545281

Sequence & progression

21. Let the coefficient of the middle terms in the expansion of $\left(\frac{1}{\sqrt{6}}+\beta x\right)^4$, $(1-3\beta x)^2$ and $\left(1-\frac{\beta}{2}x\right)^6$, $\beta > 0$, respectively form the first three terms of an A.P. If is the common difference of this A.P., then $50-\frac{2d}{\beta^2}$ is equal to _____.

माना $\left(\frac{1}{\sqrt{6}}+\beta x\right)^4$, $(1-3\beta x)^2$ तथा $\left(1-\frac{\beta}{2}x\right)^6$, $\beta > 0$ के प्रसार में मध्य पदों के गुणांक क्रमशः एक A.P. के पहले तीन पद हैं। यदि इस A.P. का सार्व अंतर d है, तो $50-\frac{2d}{\beta^2}$ बराबर है _____।

Ans. Official Answer NTA (57)

Sol. ${}^4C_2 \times \frac{\beta^2}{6}, -6\beta, -{}^6C_3 \times \frac{\beta^3}{8}$ are in A.P

$$\beta^2 - \frac{5}{2}\beta^3 = -12\beta$$

$$\beta = \frac{12}{5} \text{ or } \beta = -2 \therefore \beta = \frac{12}{5}$$

$$d = -\frac{72}{5} - \frac{144}{25} = -\frac{504}{25}$$

$$\therefore 50 - \frac{2d}{\beta^2} = 57$$

Question ID : 154771545282

P & C

22. A class contains b boys and g girls. If the number of ways of selecting 3 boys and 2 girls from the class is 168, then b + 3 g is equal to _____.

एक कक्षा में b लड़के तथा g लड़कियाँ हैं। यदि इस कक्षा में से 3 लड़कियाँ चुनने के तरीकों की संख्या 168 है, तो b + 3g बराबर है_____।

Ans. Official Answer NTA (17)

Sol. ${}^bC_3 \times {}^gc_2 = 168$

$b(b-1)(b-2)\,(g)(g-1) = 8 \times 7 \times 6 \times 3 \times 2$

$b + 3\,g = 17$

Question ID : 154771545283

Ellipse

23. Let the tangents at the points P and Q on the ellipse $\frac{x^2}{2}+\frac{y^2}{4}=1$ meet at the point $R\left(\sqrt{2},2\sqrt{2}-2\right)$. If S is the focus of the ellipse on its negative major axis, then SP^2+SQ^2 is equal to _____.

माना दीर्घवृत्त $\frac{x^2}{2}+\frac{y^2}{4}=1$ के बिन्दुओं P तथा Q पर स्पर्श रेखाएँ बिन्दु $R\left(\sqrt{2},2\sqrt{2}-2\right)$ पर मिलती हैं। यदि दीर्घवृत्त के ऋणात्मक दीर्घ अक्ष पर नाभि पर S है, तो SP^2+SQ^2 बराबर है _____।

Ans. Official Answer NTA (13)

Sol. Ellipse is

$$\frac{x^2}{2}+\frac{y^2}{4}=1;\ e=\frac{1}{\sqrt{2}};S\equiv\left(0,-\sqrt{2}\right)$$

Chord of contact is

$$\frac{x}{\sqrt{2}}+\frac{\left(2\sqrt{2}-2\right)y}{4}=1$$

$$\Rightarrow \frac{x}{\sqrt{2}}=1-\frac{\left(\sqrt{2}-1\right)y}{2}$$ solving with ellipse

$$\Rightarrow y=0,\sqrt{2}\quad \therefore x=\sqrt{2},1$$

$$P\equiv\left(1,\sqrt{2}\right)\ Q\equiv\left(\sqrt{2},0\right)$$

$$\therefore (SP)^2+(SQ)^2=13$$

Question ID : 154771545284

Binomial Theorem

24. If $1+(2+{}^{49}C_1+{}^{49}C_2+.....+{}^{49}C_{49})({}^{50}C_2+{}^{50}C_4+.....+{}^{50}C_{50})$ is equal to $2^n.\ m$, where m is odd, then n + m is equal to _____.

यदि $1+(2+{}^{49}C_1+{}^{49}C_2+.....+{}^{49}C_{49})({}^{50}C_2+{}^{50}C_4+.....+{}^{50}C_{50})=2^n.\ m$ हैं, जहाँ m एक विषम संख्या हैं, तो n + m बराबर है _____।

Ans. Official Answer NTA (99)

Sol. $1+(1+2^{49})(2^{49}-1)=2^{98}$

$m=1,\ n=98$

$m+n=99$

Question ID : 154771545285

Parabola

25. Two tangent lines l_1 and l_2 are drawn from the point (2, 0) to the parabola $2y^2=-x$. If the lines l_1 and l_2 are also

tangent to the circle $(x-5)^2+y^2=r$, then 17r is equal to _____.

बिन्दु (2, 0) से परवलय $2y^2=-x$ पर दो स्पर्श रेखाएँ l_1 तथा l_2 खींची गई हैं। यदि रेखाएँ l_1 तथा l_2 वृत्त $(x-5)^2+y^2=r$ की 17r बराबर है _____।

Ans. Official Answer NTA (9)

Sol. $y^2=-\frac{x}{2}$

$y=mx-\frac{1}{8m}$

this tangent pass through (2, 0)

$m=\pm\frac{1}{4}$ i.e., one tangent is $x-4y-2=0$

$17r=9$

Question ID : 154771545286

Sequence & progression

26. If $\frac{6}{3^{12}}+\frac{10}{3^{11}}+\frac{20}{3^{10}}+\frac{40}{3^9}+.....+\frac{10240}{3}=2^n.m$, where m is odd, then m . n is equal to _____.

यदि $\frac{6}{3^{12}}+\frac{10}{3^{11}}+\frac{20}{3^{10}}+\frac{40}{3^9}+.....+\frac{10240}{3}=2^n.m$ है, जहाँ m एक विषम संख्या है, तो m . n बराबर है _____।

Ans. Official Answer NTA (12)

Sol. $\frac{6}{3^{12}}+10\left(\frac{1}{3^{11}}+\frac{2}{3^{10}}+\frac{2^2}{3^9}+\frac{2^3}{3^8}+.....+\frac{2^{10}}{3}\right)$

$\frac{6}{3^{12}}+\frac{10}{3^{11}}\left(\frac{6^{11}-1}{6-1}\right)$

$=2^{12}\,.\,1;\ m.n=12$

Question ID : 154771545287

Trigonometric Equation

27. Let $S=\left[-\pi,\frac{\pi}{2}\right)-\left\{-\frac{\pi}{2},-\frac{\pi}{4},-\frac{3\pi}{4},\frac{\pi}{4}\right\}$. Then the number of elements in the set $A=\left\{\theta\in S:\tan\theta\left(1+\sqrt{5}\tan(2\theta)\right)=\sqrt{5}-\tan(2\theta)\right\}$ is _____.

माना $S=\left[-\pi,\frac{\pi}{2}\right)-\left\{-\frac{\pi}{2},-\frac{\pi}{4},-\frac{3\pi}{4},\frac{\pi}{4}\right\}$ है। तो समुच्चय $A=\left\{\theta\in S:\tan\theta\left(1+\sqrt{5}\tan(2\theta)\right)=\sqrt{5}-\tan(2\theta)\right\}$ में अवयवों की संख्या है_____।

Ans. Official Answer NTA (5)

Sol. $\tan\theta+\sqrt{5}\tan 2\theta\tan\theta=\sqrt{5}-\tan 2\theta$

$\tan 3\theta=\sqrt{5}$

$\theta=\frac{n\pi}{3}+\frac{\alpha}{3};\ \tan\alpha=\sqrt{5}$

Five solution

Question ID : 154771545288

Complex Number

28. Let $z=a+ib, b\neq 0$ be complex numbers satisfying $z^2=\bar{z}.2^{1-|z|}$. Then the least value of $n\in N$, such that $z^n=(z+1)^n$, is equal to _____.

माना सम्मिश्र संख्याऐं $z=a+ib, b\neq 0$ समीकरण $z^2=\bar{z}.2^{1-|z|}$ को संतुष्ट करती हैं। तो $n\in N$ का निम्नतम मान, जिसके लिए $z^n=(z+1)^n$ है, बराबर है _____।

Ans. Official Answer NTA (6)

Sol. $\left|z^2\right|=\left|\bar{z}\right|.2^{1-|z|}\Rightarrow|z|=1$

$z^2=\bar{z}\Rightarrow z^3=1\therefore z=\omega \text{ or } \omega^2$

$\omega^n=(1+\omega)^n=\left(-\omega^2\right)^n$

Least natural value of n is 6.

Question ID : 154771545289

Probability

29. A bag contains 4 white and 6 black balls. Three balls are drawn at random from the bag. Let X be the number of white balls, among the drawn balls. If σ^2 is the variance of X, then $100\sigma^2$ is equal to _____.

एक थैले में 4 सफेद तथा 6 काली गेंद है। थैले में तीन गेंद यादृच्छया निकाली जाती हैं। माना निकाली गई गेंदों में सफेद गेंदों की संख्या X है। यदि X का प्रसरण σ^2 है, तो $100\sigma^2$ बराबर है _____।

Ans. Official Answer NTA (56)

Sol.

X	0	1	2	3
P(X)	$\frac{1}{6}$	$\frac{1}{2}$	$\frac{3}{10}$	$\frac{1}{30}$

$\sigma^2=\Sigma x^2P(x)-\left(\Sigma xP(x)\right)^2=\frac{56}{100}$

$100\sigma^2=56$

Question ID : 154771545290

Definite Integration

30. The value of the integral $\int_0^{\frac{\pi}{2}} 60\frac{\sin(6x)}{\sin x}dx$ is equal to _____.

समाकलन $\int_0^{\frac{\pi}{2}} 60\frac{\sin(6x)}{\sin x}dx$ का मान बराबर है _____।

Ans. Official Answer NTA (104)

Sol. $I = 60\int_0^{\pi/2}\left(\frac{\sin 6x-\sin 4x}{\sin x}+\frac{\sin 4x-\sin 2x}{\sin x}+\frac{\sin 2x}{\sin x}\right)dx$

$I = 60\int_0^{\pi/2}(2\cos 5x+2\cos 3x+2\cos x)dx$

$I = 60\left(\frac{2}{5}\sin 5x+\frac{2}{3}\sin 3x+2\sin x\right)\Big|_0^{\pi/2} = 104$