JEE Main January 2024

Question Paper With Text Solution

27 January | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JANUARY 2024 | 27TH JANUARY SHIFT-2

SECTION – A

Question ID : 533543477

1. The integral $\int \frac{\left(x^8 - x^2\right)dx}{\left(x^{12} + 3x^6 + 1\right)\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)}$ is equal to :

(1) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/3} + C$

(2) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{1/2} + C$

(3) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right) + C$

(4) $\log_e\left(\left|\tan^{-1}\left(x^3 + \frac{1}{x^3}\right)\right|\right)^{3} + C$

Ans. Official answer NTA (1)

Sol.

Question ID : 533543482

2. Let the image of the point (1,0,7) in the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ be the point (α, β, γ) Then which one of the following points lies on the line passing through (α, β, γ) and making angles $\frac{2\pi}{3}$ and $\frac{3\pi}{4}$ with y-axis and z-axis respectively and an acute angle with x-axis ?

(1) $\left(1, 2, 1-\sqrt{2}\right)$ (2) $\left(3, -4, 3+2\sqrt{2}\right)$ (3) $\left(1, -2, 1+\sqrt{2}\right)$ (4) $\left(3, 4, 3-2\sqrt{2}\right)$

Ans. Official answer NTA (4)

Sol.

Question ID : 533543486

3. Considering only the principal values of inverse trigonometric functions, the number of positive real values of x satisfying $\tan^{-1}(x) + \tan^{-1}(2x) = \frac{\pi}{4}$ is :

(1) 1 (2) 2 (3) 0 (4) more than 2

Ans. Official answer NTA (1)

Sol.

Question ID : 533543468

4. Let A and B be two finite sets with m and n elements respectively. The total number of subsets of the set A is 56 more than the total number of subsets of B. Then the distance of the point P(m, n) from the point Q(–2, –3) is:

(1) 10 (2) 4 (3) 8 (4) 6

Ans. Official answer NTA (1)

Sol.

Question ID : 533543467

5. Let $f : R - \left\{\frac{-1}{2}\right\} \to R$ and $g : R - \left\{\frac{-5}{2}\right\} \to R$ be defined as $f(x) = \frac{2x+3}{2x+1}$ and $g(x) = \frac{|x|+1}{2x+5}$. Then, the domain of the function fog is :

(1) $R - \left\{-\frac{7}{4}\right\}$ (2) R (3) $R - \left\{-\frac{5}{2}\right\}$ (4) $R - \left\{-\frac{5}{2}, -\frac{7}{4}\right\}$

Ans. Official answer NTA (3)

Sol.

Question ID : 533543483

6. Let the position vectors of the vertices A, B and C of a triangle be $2\hat{i}+2\hat{j}+\hat{k}$, $\hat{i}+2\hat{j}+2\hat{k}$ and $2\hat{i}+\hat{j}+2\hat{k}$ respectively. Let l_1, l_2 and l_3 be be the lengths of perpendiculars drawn from the ortho center of the triangle on the sides AB, BC and CA respectively, then $l_1^2+l_2^2+l_3^2$ equal :

(1) $\frac{1}{5}$ (2) $\frac{1}{2}$ (3) $\frac{1}{4}$ (4) $\frac{1}{3}$

Ans. Official answer NTA (2)

Sol.

Question ID : 533543481

7. Let R be the interior region between the lines $3x-y+1=0$ and $x+2y-5=0$ containing the origin. The set of all values of a, for which the points $\left(a^2, a+1\right)$ lie in R, is :

(1) $(-3,-1)\cup\left(-\frac{1}{3},1\right)$ (2) $(-3,-1)\cup\left(\frac{1}{3},1\right)$

(3) $(-3,0)\cup\left(\frac{1}{3},1\right)$ (4) $(-3,0)\cup\left(\frac{2}{3},1\right)$

Ans. Official answer NTA (3)

Sol.

Question ID : 533543479

8. If $y=y(x)$ is the solution curve of the differential equation $\left(x^2-4\right)dy-\left(y^2-3y\right)dx=0, x>2, y(4)=\frac{3}{2}$ and the slope of the curve is never zero, then the value of $y(10)$ equals :

(1) $\frac{3}{1+2\sqrt{2}}$ (2) $\frac{3}{1+(8)^{1/4}}$ (3) $\frac{3}{1-2\sqrt{2}}$ (4) $\frac{3}{1-(8)^{1/4}}$

Ans. Official answer NTA (2)

Sol.

Question ID : 533543472

9. The 20^{th} term from the end of the progression $20, 19\frac{1}{4}, 18\frac{1}{2}, 17\frac{3}{4}, \ldots, -129\frac{1}{4}$ is :

(1) -110 (2) -180 (3) -115 (4) -100

Ans. Official answer NTA (3)

Sol.

Question ID : 533543480

10. Let e_1 be the eccentricity of the hyperbola $\frac{x^2}{16}-\frac{y^2}{9}=1$ and e_2 be the eccentricity of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a>b$, which passes through the foci of the hyperbola. If $e_1e_2=1$ then the length of the chord of the ellipse parallel to the x-axis and passing through $(0, 2)$ is :

(1) $\frac{10\sqrt{5}}{3}$ (2) $4\sqrt{5}$ (3) $3\sqrt{5}$ (4) $\frac{8\sqrt{5}}{3}$

Ans. Official answer NTA (1)

Sol.

Question ID : 533543473

11. If $2\tan^2\theta-5\sec\theta=1$ has exactly 7 solutions in the interval $\left[0,\frac{n\pi}{2}\right]$, for the least value of $n\in N$, then $\sum_{k=1}^{n}\frac{k}{2^k}$ is equal to :

(1) $\frac{1}{2^{14}}\left(2^{15}-15\right)$ (2) $1-\frac{15}{2^{13}}$ (3) $\frac{1}{2^{13}}\left(2^{14}-15\right)$ (4) $\frac{1}{2^{15}}\left(2^{14}-14\right)$

Ans. Official answer NTA (3)

Sol.

Question ID : 533543474

12. If $\lim_{x\to0}\frac{3+\alpha\sin x+\beta\cos x+\log_e(1-x)}{3\tan^2 x}=\frac{1}{3}$, then $2\alpha-\beta$ is equal to :

(1) 7 (2) 1 (3) 5 (4) 2

Ans. Official answer NTA (3)

Sol.

Question ID : 533543470

13. The values of α, for which $\begin{vmatrix} 1 & \frac{3}{2} & \alpha+\frac{3}{2} \\ 1 & \frac{1}{3} & \alpha+\frac{1}{3} \\ 2\alpha+3 & 3\alpha+1 & 0 \end{vmatrix} = 0$ lie in the interval :

(1) $(0, 3)$ (2) $\left(-\frac{3}{2}, \frac{3}{2}\right)$ (3) $(-2, 1)$ (4) $(-3, 0)$

Ans. Official answer NTA (4)

Sol.

Question ID : 533543476

14. Let $g(x) = 3f\left(\frac{x}{3}\right) + f(3-x)$ and $f''(x) > 0$ for all $x \in (0, 3)$. If g is decreasing in $(0, \alpha)$ and increasing in $(\alpha, 3)$, then 8α is :

(1) 20 (2) 0 (3) 24 (4) 18

Ans. Official answer NTA (4)

Sol.

Question ID : 533543478

15. For $0 < a < 1$, the value of the integral $\int_0^{\pi} \frac{dx}{1-2a\cos x + a^2}$ is :

(1) $\frac{\pi}{1-a^2}$ (2) $\frac{\pi^2}{\pi+a^2}$ (3) $\frac{\pi^2}{\pi-a^2}$ (4) $\frac{\pi}{1+a^2}$

Ans. Official answer NTA (1)

Sol.

Question ID : 533543469

16. If α, β are the roots of the equation $x^2 - x - 1 = 0$ and $S_n = 2023\alpha^n + 2024\beta^n$, then :

(1) $S_{11} = S_{10} + S_{12}$ (2) $2S_{12} = S_{11} + S_{10}$ (3) $S_{12} = S_{11} + S_{10}$ (4) $2S_{11} = S_{12} + S_{10}$

Ans. Official answer NTA (3)

Sol.

Question ID : 533543485

17. An urn contains 6 white and 9 black balls. Two successive draws of 4 balls are made without replacement. The probability, that the first draw gives all white balls and the second draw gives all black balls, is :

(1) $\frac{3}{715}$ (2) $\frac{5}{715}$ (3) $\frac{3}{256}$ (4) $\frac{5}{256}$

Ans. Official answer NTA (1)

Sol.

Question ID : 533543484

18. The position vectors of the vertices A, B and C of a triangle are $2\hat{i} - 3\hat{j} + 3\hat{k}$, $2\hat{i} + 2\hat{j} + 3\hat{k}$ and $-\hat{i} + \hat{j} + 3\hat{k}$ respectively. Let l denotes the length of the angle bisector AD of $\angle BAC$ where D is on the line segment BC, then $2l^2$ equals :

(1) 49 (2) 42 (3) 50 (4) 45

Ans. Official answer NTA (4)

Sol.

Question ID : 533543471

19. Let $\alpha = \frac{(4!)!}{(4!)^{3!}}$ and $\beta = \frac{(5!)!}{(5!)^{4!}}$. Then :

(1) $\alpha \notin N$ and $\beta \in N$ (2) $\alpha \in N$ and $\beta \in N$

(3) $\alpha \notin N$ and $\beta \notin N$ (4) $\alpha \in N$ and $\beta \notin N$

Ans. Official answer NTA (2)

Sol.

Question ID : 533543475

20. Consider the function f : (0, 2) → R defined by $f(x)=\frac{x}{2}+\frac{2}{x}$ and the function g(x) defined by

$$g(x)=\begin{cases}\min(f(t)), & 0<t\le x \text{ and } 0<x\le 1\\ \frac{3}{2}+x, & 1<x<2\end{cases}$$. Then :

(1) g is continuous but not differentiable at x=1 (2) g is continuous and differentiable for all x ∈ (0, 2)

(3) g is not continuous for all x ∈ (0, 2) (4) g is neither continuous nor differentiable at x = 1

Ans. Official answer NTA (1)

Sol.

SECTION – B

Question ID : 533543490

21. The coefficient of x^{2012} in the expansion of $(1-x)^{2008}(1+x+x^2)^{2007}$ is equal to _____.

Ans. Official answer NTA (0)

Sol.

Question ID : 533543487

22. Let the complex numbers α and $\frac{1}{\overline{\alpha}}$ lie on the circle $|z-z_0|^2=4$ and $|z-z_0|^2=16$ respectively, where $z_0=1+i$. Then, the value of $100|\alpha|^2$ is _____.

Ans. Official answer NTA (20)

Sol.

Question ID : 533543489

23. If the sum of squares of all real values of α, for which the lines 2x – y + 3 = 0, 6x + 3y + 1 = 0 and ax + 2y – 2 = 0 do not form a triangle is p, then the greatest integer less than or equal to p is _____.

Ans. Official answer NTA (32)

Sol.

Question ID : 533543493

24. If the solution curve, of the differential equation $\frac{dy}{dx}=\frac{x+y-2}{x-y}$ passing through the point (2, 1) is

$\tan^{-1}\left(\frac{y-1}{x-1}\right)-\frac{1}{\beta}\log_e\left(\alpha+\left(\frac{y-1}{x-1}\right)^2\right)=\log_e|x-1|$, then $5\beta+\alpha$ is equal to _____.

Ans. Official answer NTA (11)

Sol.

Question ID : 533543488

25. Let A be a 2×2 real matrix and I be the identity matrix of order 2. If the roots of the equation $|A-xI|=0$ be -1 and 3, then the sum of the diagonal elements of the matrix A^2 is_____.

Ans. Official answer NTA (10)

Sol.

Question ID : 533543491

26. Let $f(x)=\int_0^x g(t)\log_e\left(\frac{1-t}{1+t}\right)dt$, where g is a continuous odd function.

If $\int_{-\pi/2}^{\pi/2}\left(f(x)+\frac{x^2\cos x}{1+e^x}\right)dx=\left(\frac{\pi}{\alpha}\right)^2-\alpha$, then α is equal to _____.

Ans. Official answer NTA (2)

Sol.

Question ID : 533543494

27. Consider a circle $(x-\alpha)^2+(y-\beta)^2=50$ where $\alpha, \beta>0$. If the circle touches the line $y+x=0$ at the point P, whose distance from the origin is $4\sqrt{2}$, then $(\alpha+\beta)^2$ is equal to _____.

Ans. Official answer NTA (100)

Sol.

Question ID : 533543496

28. The mean and standard deviation of 15 observations were found to be 12 and 3 respectively. On rechecking it was found that an observation was read as 10 in place of 12 . If μ and σ^2 denote the mean and variance of the correct observations respectively, then $15(\mu + \mu^2 + \sigma^2)$ is equal to _____.

Ans. Official answer NTA (2521)

Sol.

Question ID : 533543495

29. The line $\frac{x-2}{2} = \frac{y}{-2} = \frac{z-7}{16}$ and $\frac{x+3}{4} = \frac{y+2}{3} = \frac{z+2}{1}$ intersect at the point P. If the distance of P from the line $\frac{x+1}{2} = \frac{y-1}{3} = \frac{z-1}{1}$ is l, then $14l^2$ is equal to _____.

Ans. Official answer NTA (108)

Sol.

Question ID : 533543492

30. If the area of the region $\left\{(x, y) : 0 \le y \le \min\left\{2x, 6x - x^2\right\}\right\}$ is A, then 12 A is equal to _____.

Ans. Official answer NTA (304)

Sol.