JEE Main January 2024

Question Paper With Text Solution

27 January | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

Office : Piprali Road, Sikar (Raj.) | Ph. 01572-241911
Website : www.matrixedu.in ; Email : smd@matrixacademy.co.in

JEE MAIN JANUARY 2024 | 27TH JANUARY SHIFT-1

SECTION – A

Question ID : 533543386

1. If the shortest distance of the parabola $y^2 = 4x$ from the centre of the circle $x^2 + y^2 - 4x - 16y + 64 = 0$ is d, then d^2 is equal to :

(1) 16 (2) 20 (3) 36 (4) 24

Ans. Official answer NTA (2)

Sol.

Question ID : 533543389

2. Let x = x(t) and y = y(t) be solutions of the differential equations $\frac{dx}{dt} + ax = 0$ and $\frac{dy}{dt} + by = 0$ respectively, a, b ∈ R. Given that x(0) = 2; y(0) = 1 and 3y(1) = 2x(1), the value of t, for which x(t) = y(t), is :

(1) $\log_3 4$ (2) $\log_{\frac{2}{3}} 2$ (3) $\log_4 3$ (4) $\log_{\frac{4}{3}} 2$

Ans. Official answer NTA (2)

Sol.

Question ID:533543383

3. The number of common terms in the progressions 4,9,14,19,....., upto 25th term and 3,6,9,12,....., upto 37th term is

यकिं

(1) 7 (2) 9 (3) 8 (4) 5

Ans. Official answer NTA (1)

Sol.

Question ID:533543391

4. The portion of the lijne 4x + 5y = 20 in the first quadrant is trisected by the lines L_1 and L_2 passing through the origin. The tangent of an angle between the lines L_1 and L_2 is :

(1) $\frac{8}{5}$ (2) $\frac{2}{5}$ (3) $\frac{30}{41}$ (4) $\frac{25}{41}$

Ans. Official answer NTA (3)

Sol.

Question ID:533543385

5. If $a = \lim\limits_{x\to 0} \dfrac{\sqrt{1+\sqrt{1+x^4}}-\sqrt{2}}{x^4}$ and $b = \lim\limits_{x\to 0} \dfrac{\sin^2 x}{\sqrt{2}-\sqrt{1+\cos x}}$, then the value of ab^3 is :

(1) 36 (2) 25 (3) 32 (4) 30

Ans. Official answer NTA (3)

Sol.

Question ID:533543380

6. Consider the matrix $f(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

Given below are two statements :
Statement I : f(–x) is the inverse of the matrix f(x).
Statement II : f(x) f(y) = f(x + y).

In the light of the above statements, choose the correct answer from the options given below

(1) Both statement I and statement II are false.

(2) Statement I is true but statement II is false.

(3) Both statement I and statement II are true.

(4) Statement I is false but statement II is true.

Ans. Official answer NTA (3)

Sol.

Question ID : 533543382

7. If A denotes the sum of all the coefficients in the expansion of $(1 - 3x + 10x^2)^n$ and B denotes the sum of all the coefficients in the expansion of $(1 + x^2)^n$, then :

(1) $B = A^3$ (2) $3A = B$ (3) $A = B^3$ (4) $A = 3B$

Ans. Official answer NTA (3)

Sol.

Question ID : 533543387

8. If $\int_0^1 \frac{1}{\sqrt{3+x}+\sqrt{1+x}}dx = a + b\sqrt{2} + c\sqrt{3}$, where a, b, c are rational numbers, then $2a + 3b - 4c$ is equal to :

(1) 10 (2) 7 (3) 4 (4) 8

Ans. Official answer NTA (4)

Sol.

Question ID : 533543384

9. Consider the function.

$$f(x) = \begin{cases} \frac{a\left(7x-12-x^2\right)}{b\left|x^2-7x+12\right|} & , \quad x < 3 \\ 2^{\frac{\sin(x-3)}{x-[x]}} & , \quad x > 3 \\ b & , \quad x = 3 \end{cases}$$

where [x] denotes the greatest integer less than or equal to x. If S denotes the set of all ordered pairs (a, b) such that f(x) is continuous at $x = 3$, then the number of elements in S is :

(1) 1 (2) 2 (3) Infinitely many (4) 4

Ans. Official answer NTA (1)

Sol.

Question ID : 533543388

10. If (a, b) be the orthocentre of the triangle whose vertices are (1,2), (2,3) and (3,1), and $I_1 = \int_a^b x \sin\left(4x - x^2\right)dx$,

$I_2 = \int_a^b \sin\left(4x - x^2\right)dx$, then $36\frac{I_1}{I_2}$ is equal to :

(1) 72 (2) 88 (3) 66 (4) 80

Ans. Official answer NTA (1)

Sol.

Question ID : 533543396

11. Let $a_1, a_2, \ldots\ldots, a_{10}$ be 10 observations such that $\sum_{k=1}^{10} a_k = 50$ and $\sum_{\forall k<j} a_k \cdot a_j = 1100$. Then the standard deviation of $a_1, a_2, \ldots\ldots, a_{10}$ is equal to :

(1) $\sqrt{115}$ (2) 10 (3) $\sqrt{5}$ (4) 5

Ans. Official answer NTA (3)

Sol.

Question ID : 533543393

12. The distance of the point (7, –2, 11) from the line $\frac{x-6}{1} = \frac{y-4}{0} = \frac{z-8}{3}$ along the line $\frac{x-5}{2} = \frac{y-1}{-3} = \frac{z-5}{6}$ is :

(1) 18 (2) 12 (3) 14 (4) 21

Ans. Official answer NTA (3)

Sol.

Question ID : 533543394

13. If the shortest distance between the lines $\frac{x-4}{1} = \frac{y+1}{2} = \frac{z}{-3}$ and $\frac{x-\lambda}{2} = \frac{y+1}{4} = \frac{z-2}{-5}$ is $\frac{6}{\sqrt{5}}$, then the sum of all possible values of λ is :

(1) 5 (2) 7 (3) 10 (4) 8

Ans. Official answer NTA (4)

Sol.

Question ID : 533543390

14. Four distinct points (2k, 3k), (1,0), (0,1) and (0,0) lie ion a circle for k equal to :

(1) $\frac{3}{13}$ (2) $\frac{1}{13}$ (3) $\frac{5}{13}$ (4) $\frac{2}{13}$

Ans. Official answer NTA (3)

Sol.

Question ID : 533543392

15. The length of the chord of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$, whose mid point is $\left(1,\frac{2}{5}\right)$, is equal to :

(1) $\frac{\sqrt{1691}}{5}$ (2) $\frac{\sqrt{2009}}{5}$ (3) $\frac{\sqrt{1541}}{5}$ (4) $\frac{\sqrt{1741}}{5}$

Ans. Official answer NTA (1)

Sol.

Question ID : 533543395

16. Let $\vec{a}=\hat{i}+2\hat{j}+\hat{k}$, $\vec{b}=3\left(\hat{i}-\hat{j}+\hat{k}\right)$. Let $\vec{c}$ be the vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}\cdot\vec{c}=3$. Then $\vec{a}\cdot\left(\left(\vec{c}\times\vec{b}\right)-\vec{b}-\vec{c}\right)$ is equal to:

(1) 36 (2) 24 (3) 32 (4) 20

Ans. Official answer NTA (2)

Sol.

Question ID : 533543377

17. Let S = {1,2,3,....,10}. Suppose M is the set of all the subsets of S, then the relation $R=\left\{(A,B):A\cap B\neq\phi;A,B\in M\right\}$ is :

(1) symmetric only

(2) symmetric and transitive only

(3) symmetric and reflexive only

(4) reflexive only

Ans. Official answer NTA (1)

Sol.

Question ID : 533543378

18. The function $f : N - \{1\} \to N$; is defined by f(n) = the highest prime factor of n, is :

(1) neither one-one nor onto (2) onto only

(3) one-one only (4) both one-one and onto

Ans. Official answer NTA (1)

Sol.

Question ID : 533543379

19. If $S = \{z \in C : |z - i| = |z + i| = |z - 1|\}$, then n(S) is :

(1) 2 (2) 0 (3) 3 (4) 1

Ans. Official answer NTA (4)

Sol.

Question ID : 533543381

20. ${}^{n-1}C_r = (k^2 - 8)\,{}^nC_{r+1}$ if and only if : :

(1) $2\sqrt{2} < k \le 3$ (2) $2\sqrt{3} < k < 3\sqrt{3}$

(3) $2\sqrt{3} < k \le 3\sqrt{2}$ (4) $2\sqrt{2} < k < 2\sqrt{3}$

Ans. Official answer NTA (1)

Sol.

SECTION – B

Question ID : 533543399

21. Let for a differentiable function $f : (0, \infty) \to R$, $f(x) - f(y) \geq \log_e\left(\frac{x}{y}\right) + x - y$, $\forall\, x, y \in (0, \infty)$. Then $\sum_{n=1}^{20} f'\left(\frac{1}{n^2}\right)$ is equal to _____.

Ans. Official answer NTA (2890)

Sol.

Question ID : 533543402

22. Let the area of the region $\{(x, y) : x - 2y + 4 \geq 0, x + 2y^2 \geq 0, x + 4y^2 \leq 8, y \geq 0\}$ be $\frac{m}{n}$, where m and n are coprime numbers. Then m + n is equal to _____.

Ans. Official answer NTA (119)

Sol.

Question ID : 533543397

23. If a satisfies the equation $x^2 + x + 1 = 0$ and $(1 + \alpha)^7 = A + B\alpha + C\alpha^2$, A, B, C ≤ 0, then $5(3A - 2B - C)$ is equal to _____.

Ans. Official answer NTA (5)

Sol.

Question ID : 533543398

24. Let $A = \begin{bmatrix} 2 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$, $B = [B_1, B_2, B_3]$, where B_1, B_2, B_3 are column matrics, and $AB_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $AB_2 = \begin{bmatrix} 2 \\ 3 \\ 0 \end{bmatrix}$, $AB_3 = \begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}$. If $\alpha = |B|$ and β is the sum of all the diagonal elements of B, then $\alpha^3 + \beta^3$ is equal to _____.

Ans. Official answer NTA (28)

Sol.

Question ID : 533543400

25. If $8 = 3 + \frac{1}{4}(3+p) + \frac{1}{4^2}(3+2p) + \frac{1}{4^3}(3+3p) + ...\infty$, then the value of p is _____.

Ans. Official answer NTA (9)

Sol.

Question ID : 533543401

26. Let $f(x) = x^3 + x^2 f'(1) + x f''(2) + f'''(3)$, $x \in R$. Then $f'(10)$ is equal to _____.

Ans. Official answer NTA (202)

Sol.

Question ID : 533543405

27. A fair die is tossed repeatedly until a six is obtained. Let X denote the number of tosses required and let a = P(X = 3), b = P(X ≥ 3) and c = P(X ≥ 6x > 3). Then $\frac{b+c}{a}$ is equal to _____.

Ans. Official answer NTA (12)

Sol.

Question ID : 533543404

28. The least positive integral value of α, for which the angle between the vectors $\alpha\hat{i} - 2\hat{j} + 2\hat{k}$ and $\alpha\hat{i} - 2\alpha\hat{j} - 2\hat{k}$ is acute, is _____.

Ans. Official answer NTA (5)

Sol.

Question ID : 533543406

29. Let the set of all $a \in R$ such that the equation $\cos 2x + a \sin x = 2a - 7$ has a solution be $[p, q]$ and $r = \tan 9^\circ - \tan 27^\circ - \frac{1}{\cot 63^\circ} + \tan 81^\circ$, then pqr is equal to _____.

Ans. Official answer NTA (48)

Sol.

Question ID : 533543403

30. If the solution of the differential equation $(2x + 3y - 2)\,dx + (4x + 6y - 7)\,dy = 0$, $y(0) = 3$, is $\alpha x + \beta y + 3\log_e |2x + 3y - \gamma| = 6$, then $\alpha + 2\beta + 3\gamma$ is equal to _____.

Ans. Official answer NTA (29)

Sol.