JEE Main June 2022

Question Paper With Text Solution

27 June | Shift-1

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JUNE 2022 | 27TH JUNE SHIFT-1

SECTION – A

Question ID : 101361

Complex number

1. The area of the polygon, whose vertices are the non-real roots of the equation $\overline{z} = iz^2$ is :

उस बहुभुज का क्षेत्रफल, जिसके शीर्ष समीकरण $\overline{z} = iz^2$ के अवास्तविक मूल हैं, है :

(1) $\frac{3\sqrt{3}}{4}$ (2) $\frac{3\sqrt{3}}{2}$ (3) $\frac{3}{2}$ (4) $\frac{3}{4}$

Ans. Official Answer NTA (1)

Sol.
$\overline{z} = iz^2$

$|\overline{z}| = |iz^2|$

$|z| = |z^2|$

$|z|(|z|-1) = 0$

$|z| = 0$ OR $|z| = 1$

$z = 0$ OR $z\overline{z} = 1$

$z\overline{z} = iz^3$

$iz^3 = 1$

$z^3 = -i$

$z^3 = e^{i\left(-\frac{\pi}{2}+2k\pi\right)}$

$z = e^{i\left(\frac{-\frac{\pi}{2}+2k\pi}{3}\right)}$

$k = 0 \quad z_0 = e^{-i\frac{\pi}{6}} = \left(\frac{\sqrt{3}}{2}, \frac{-1}{2}\right)$

$k = 1 \quad z_1 = e^{i\frac{\pi}{2}} = (0,1)$

$k = 2 \quad z_2 = e^{i7\frac{\pi}{6}} = \left(\frac{-\sqrt{3}}{2}, -\frac{1}{2}\right)$

$$\text{Area} = \frac{1}{2}\begin{vmatrix} \sqrt{\frac{3}{2}} & -\frac{1}{2} & 1 \\ 0 & 1 & 1 \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 1 \end{vmatrix}$$

$$= \frac{3\sqrt{3}}{4}$$

Question ID : 101362

Determinant

2. Let the system of linear equations $x + 2y + z = 2$, $\alpha x + 3y - z = \alpha$, $-\alpha x + y + 2z = -\alpha$ be inconsistent. Then α is equal to :

माना रैखिक समीकरण निकाय $x + 2y + z = 2$, $\alpha x + 3y - z = \alpha$, $-\alpha x + y + 2z = -\alpha$ असंगत है। तो α बराबर है :

(1) $\frac{5}{2}$ (2) $-\frac{5}{2}$ (3) $\frac{7}{2}$ (4) $-\frac{7}{2}$

Ans. Official Answer NTA (4)

Sol. $x + 2y + z = 2$

$\alpha x + 3y - z = \alpha$

$-\alpha x + y + 2z = -\alpha$

for in consistent

$\Delta = 0$

$$\begin{vmatrix} 1 & 2 & 1 \\ \alpha & 3 & -1 \\ -\alpha & 1 & 2 \end{vmatrix} = 0$$

$\alpha = -7/2$

Question ID : 101363

Sequence & progression

3. If $x = \sum_{n=0}^{\infty} a^n$, $y = \sum_{n=0}^{\infty} b^n$, $z = \sum_{n=0}^{\infty} c^n$, where a, b, c are in A.P. and $|a| < 1$, $|b| < 1$, $|c| < 1$, $abc \neq 0$, then :

(1) x, y, z are in A.P (2) x, y, z are in G.P.

(3) $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ are in A.P. (4) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1-(a+b+c)$

यदि $x=\sum_{n=0}^{\infty}a^n y=\sum_{n=0}^{\infty}b^n, z=\sum_{n=c}^{\infty}c^n$ हैं, जहाँ a, b, c एक A.P. में हैं तथा $|a|<1, |b|<1, |c|<1, abc \neq 0$ है, तो :

(1) x, y, z एक A.P. में हैं (2) x, y, z एक G.P. में हैं

(3) $\frac{1}{x},\frac{1}{y},\frac{1}{z}$ एक A.P. में हैं (4) $\frac{1}{x}+\frac{1}{y}+\frac{1}{z}=1-(a+b+c)$

Ans. Official Answer NTA (3)

Sol. $x=\sum_{n=0}^{\infty}a^n=1+a+a^2+a^3........\infty=\frac{1}{1-a}$

$y=\sum_{n=0}^{\infty}b^n=1+b+b^2+b^3........\infty=\frac{1}{1-b}$

$z=\sum_{n=0}^{\infty}c^n=1+c+c^2........\infty=\frac{1}{1-c}$

$a, b, c \rightarrow$ A.P.
$1-a, 1-b, 1-c \rightarrow$ A.P.

$\frac{1}{x},\frac{1}{y},\frac{1}{z} \rightarrow$ A.P.

Question ID : 101364

Differential Equation

4. Let $\frac{dy}{dx}=\frac{ax-by+a}{bx+cy+a}$, where a,b,c are constants, represent a circle passing through the point (2, 5). Then the shortest distance of the point (11, 6) from the circle is :

माना $\frac{dy}{dx}=\frac{ax-by+a}{bx+cy+a}$, a, b, c अचर हैं, बिन्दु (2, 5) से होकर जाने वाले वृत्त को निरुपित करता है। तो बिन्दु (11, 6) की इस वृत्त से न्यूनतम दूरी है :

(1) 10 (2) 8 (3) 7 (4) 5

Ans. Official Answer NTA (2)

Sol. $\frac{dy}{dx}=\frac{ax-by+a}{bx+cy+a}$

$bxdy+cydy+ady=axdx-bydx+adx$

$b\int(xdy+ydx)+\int cydy+\int ady=\int axdx+\int adx$

$b\,xy + \frac{cy^2}{2} + ay = \frac{ax^2}{2} + ax + k$

for a circle b = 0

c = –a

$\frac{a}{2}x^2 + \frac{a}{2}y^2 + ax - ay + k = 0$

$x^2 + y^2 + 2x - 2y + k = 0$

(2, 5) k = –23

$x^2 + y^2 + 2x - 2y - 23 = 0$

centre (–1, 1) $r = \sqrt{1+1+23} = 5$

Shortest distance from p(11, 6) = |op – r|

$= \left|\sqrt{(12)^2 + (5)^2} - 5\right|$

= | 13 – 5 |

= 8

Question ID : 101365

Limit

5. Let a be an integer such that $\lim_{x\to 7}\frac{18-[1-x]}{[x-3a]}$ exists, where [t] is greatest integer ≤ t. Then a is equal to :

माना a एक पूर्णांक है जिसके लिए $\lim_{x\to 7}\frac{18-[1-x]}{[x-3a]}$, जहाँ [t] महत्तम पूर्णांक ≤ t है, का अस्तित्व है, तो a बराबर है :

(1) – 6 (2) – 2 (3) 2 (4) 6

Ans. Official Answer NTA (1)

Sol. $\lim_{x\to 7}\frac{18-[1-x]}{[x-3a]}$

$\lim_{x\to 7}\frac{18-1-[x]}{[x]-3a}$

$\lim_{x\to 7}\frac{17-[-x]}{[x]-3a}$

$$\text{R.H.L.} = \frac{17-(-7)}{6-3a} = \frac{24}{6-3a}$$

$$\text{L.H.L.} = \frac{17-(-8)}{7-3a} = \frac{25}{7-3a}$$

L.H.L. = R.H.L.

$$\frac{24}{6-3a} = \frac{25}{7-3a}$$

$3a = -18$

$a = -6$

Question ID : 101366

Monotonocity

6. The number of distinct real roots of $x^4 - 4x + 1 = 0$ is :

समीकरण $x^4 - 4x + 1 = 0$ के भिन्न वास्तविक मूलों की संख्या है :

(1) 4 (2) 2 (3) 1 (4) 0

Ans. Official Answer NTA (2)

Sol. $x^4 - 4x + 1 = 0$

$f(x) = x^4 - 4x + 1$

$f'(x) = 4x^3 - 4$

$f'(x) = 4(x^3 - 1) = 4(x-1)(x^2 + x + 1)$

So f(x) will have a minima at x = 1

$f(x) = 1 - 4 + 1 = -2$

so there will be 2 roots

Question ID : 101367

Maxima & Minima

7. The lengths of the sides of a triangle are $10 + x^2$, $10 + x^2$ and $20 - 20x^2$. If for $x = k$, the area of the triangle is maximum, then $3k^2$ is equal to :

एक त्रिभुज की भुजाओं की लंबाई $10 + x^2$, $10 + x^2$ तथा $20 - 20x^2$ है। यदि $x = k$ के लिए त्रिभुज का क्षेत्रफल अधिकतम है, तो $3k^2$ बराबर है :

(1) 5 (2) 8 (3) 10 (4) 12

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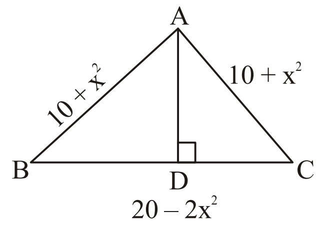
Ans. Official Answer NTA (3)

Sol. $BD = 10 - x^2$



$$AD = \sqrt{\left(10 + x^2\right) - \left(10 - x^2\right)^2}$$

$$AD = \sqrt{(10 + 10)\left(x^2 + x^2\right)}$$

$$AD = \sqrt{40}x$$

$$\text{Area} = \frac{1}{2} AD \times BC = \frac{1}{2}\sqrt{40}x\left(20 - 2x^2\right)$$

$$A = \sqrt{40}\left(10x - x^3\right)$$

$$\frac{dA}{dx} = 0$$

$$\sqrt{40}\left(10 - 3x^2\right) = 0$$

$$x^2 = \frac{10}{3}$$

$3x^2 = 10$

$3k^2 = 10$

Question ID : 101368

Methods of Differentiation

8. If $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$, $|y| < 2$, then :

यदि $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$, $|y| < 2$ हैं, तो :

(1) $x^2y'' + xy' - 25y = 0$ (2) $x^2y'' - xy' - 25y = 0$

(3) $x^2y'' - xy' + 25y = 0$ (4) $x^2y'' + xy' + 25y = 0$

Ans. Official Answer NTA (4)

Sol. $\cos^{-1}\left(\frac{y}{2}\right) = \log_e\left(\frac{x}{5}\right)^5$

$\frac{y}{2} = \cos\left(5\ell n\left(\frac{x}{5}\right)\right)$

$y' = -2\sin\left(5\ell n\left(\frac{x}{5}\right)\right)\times\frac{5}{x}$

$xy' = -10\sin\left(5\ell n\left(\frac{x}{5}\right)\right)$

diff. w.r.t. x

$xy'' + y' = -10\cos\left(5\ell n\left(\frac{x}{5}\right)\right)\times\frac{5}{x}$

$x^2y'' + xy' = -50\cos\left(5\ell n\frac{x}{5}\right)$

$x^2y'' + xy' = -25y$

$x^2y'' + xy' + 25y = 0$

Question ID : 101369

Indefinite Integration

9. If $\int\frac{(x^2+1)e^x}{(x+1)^2}dx = f(x)e^x + C$, where C is a constant, then $\frac{d^3f}{dx^3}$ at $x = 1$ is equal to :

यदि $\int\frac{(x^2+1)e^x}{(x+1)^2}dx = f(x)e^x + C$ है, जहाँ C एक अचर है, तो $x = 1$ पर $\frac{d^3f}{dx^3}$ बराबर है :

(1) $-\frac{3}{4}$ (2) $\frac{3}{4}$ (3) $-\frac{3}{2}$ (4) $\frac{3}{2}$

Ans. Official Answer NTA (2)

Sol. $g(x) = \left(\frac{x^2+1}{(x+1)^2}\right)e^x = \left(\frac{x^2+x+1-x}{(x+1)^2}\right)e^x$

$g(x) = \left(\frac{x}{x+1} + \frac{1-x}{(x+1)^2}\right)e^x$

$$g(x)=\left(\frac{x}{x+1}+\frac{1}{(x+1)^2}-\left(\frac{x+1-1}{(x+1)^2}\right)\right)e^x$$

$$g(x)=\left(\frac{x}{x+1}+\frac{1}{(x+1)^2}-\frac{1}{1+x}+\frac{1}{(1+x)^2}\right)e^x dx$$

$$\int g(x).dx=e^x\left(\frac{x}{1+x}-\frac{1}{1+x}\right)+C$$

$$f(x)=\frac{x-1}{1+x}$$

$$f'(x)=\frac{(x+1)-(x-1)}{(x+1)^2}=\frac{2}{(x+1)^2}$$

$$f''(x)=\frac{-4}{(x+1)^3}$$

$$f'''(x)=\frac{12}{(x+1)^4}$$

$$f'''(x)=\frac{12}{16}=\frac{3}{4}$$

Question ID : 101370

Definite Integration

10. The value of the integral $\int_{-2}^{2}\frac{|x^3+x|}{(e^{x|x|}+1)}dx$ is equal to :

समाकलन $\int_{-2}^{2}\frac{|x^3+x|}{(e^{x|x|}+1)}dx$ का मान बराबर है :

(1) $5e^2$ (2) $3e^{-2}$ (3) 4 (4) 6

Ans. Official Answer NTA (4)

Sol. $I = \int_{-2}^{2} \frac{\left|x^3 + x\right|}{\left(e^{x|x|} + 1\right)} dx$ ——(1)

apply king rule

$I = \int_{-2}^{2} \frac{\left|-x^3 - x\right|}{e^{-x|x|} + 1} dx$

$I = \int_{-2}^{2} \frac{\left|x^3 + x\right|}{e^{-x|x|} + 1} dx$ ——(2)

(1) + (2)

$2I = \int_{-2}^{2} \left|x^3+x\right| \left(e^{\frac{1}{x|x|+1}} + \frac{1}{e^{-x|x|} + 1} \right) dx$

$2I = \int_{-2}^{2} \left|x^3+x\right| \left(\frac{e^{x|x|} + 1}{e^{x|x|} + 1} \right) dx$

$2I = \int_{-2}^{2} \left|x^3 + x\right| dx$

$2I = 2\int_{0}^{2} \left|x^3 + x\right| dx$

$I = \int_{0}^{2} \left(x^3 + x\right) dx$

$I = \left(\frac{x^4}{4} + \frac{x^2}{2} \right)_0^2 = 4 + 2 = 6$

Question ID : 101371

Differential Equation

11. If $\frac{dy}{dx} + \frac{2^{x-y}\left(2^y - 1\right)}{2^x - 1} = 0$, x, y > 0, y(1) = 1, then y(2) is equal to :

यदि $\frac{dy}{dx} + \frac{2^{x-y}\left(2^y - 1\right)}{2^x - 1} = 0$, x, y > 0, y(1) = 1 हैं, तो y(2) बराबर है :

(1) $2 + \log_2 3$ (2) $2 + \log_2 2$ (3) $2 - \log_2 2$ (4) $2 - \log_2 3$

Ans. Official Answer NTA (4)

Sol. $\frac{dy}{dx} = \frac{-2^{x-y}\left(2^y - 1\right)}{2^x - 1}$

$$\frac{dy}{dx}=\frac{-2^x.2^{-y}\left(2^y-1\right)}{2^x-1}$$

$$\frac{dy}{dx}=\frac{-2^x\left(1-2^{-y}\right)}{2^x-1}$$

$$\int\frac{dy}{1-2^{-y}}=-\int\frac{2^x}{2^x-1}dx$$

$$\int\frac{2^y dx}{2^y-1}=-\int\frac{2^x}{2^x-1}dx$$

Let $2^y-1=t$ $\quad$ $2^x-1=u$

$2^y\ \ell n2\ dy=dt$ $\quad$ $2^x\ ln2\ dx=du$

$$\int\frac{1}{t}dt=-\int\frac{1}{u}du$$

$ln\ t=-ln\ u+ln\ c$

$$t=\frac{c}{u}$$

$$2^y-1=\frac{c}{2^x-1}$$

Now $\quad x=1,\quad y=1$

$\Rightarrow c=1$

$$2^y-1=\frac{1}{2^x-1}$$

Now put $x=2$

$$2^y-1=\frac{1}{4-1}$$

$$2^y=1+\frac{1}{3}=\frac{4}{3}$$

$$y=\log_2\frac{4}{3}=2-\log_2 3$$

Question ID : 101372

Straight Line

12. In an isosceles triangle ABC, the vertex A is (6, 1) and the equation of the base BC is $2x+y=4$. Let the point B lie on the line $x+3y=7$. If (α,β) is the centroid of ΔABC, then $15(\alpha+\beta)$ is equal to :

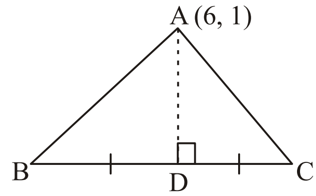
एक समद्विबाहु त्रिभुज ABC का शीर्ष A बिन्दु (6, 1) पर है तथा आधार BC का समीकरण $2x+y=4$ है। माना बिन्दु B, रेख

$x + 3y = 7$ पर स्थित है। यदि ΔABC का केन्द्रक (α, β) है, तो $15(\alpha + \beta)$ बराबर है :

(1) 39 (2) 41 (3) 51 (4) 63

Ans. Official Answer NTA (3)

Sol. Equation of BC $\Rightarrow 2x + y = 4$ ———(1)



B lie on line $x + 3y = 7$ ———(2)

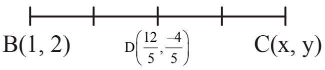
from (1) & (2) B(1, 2)

Foot of perpendicular A on line BC

$$\frac{x-6}{2} = \frac{y-1}{1} = -\left(\frac{12+1-4}{5}\right)$$

$$\frac{x-6}{2} = \frac{y-1}{1} = \frac{-9}{5}$$

$$x = \frac{12}{5} \qquad y = \frac{-4}{5}$$



$$\frac{x+1}{2} = \frac{12}{5} \qquad \frac{y+2}{2} = \frac{-4}{5}$$

$$x = \frac{19}{5} \qquad y = \frac{-18}{5}$$

Centroid $\alpha = \dfrac{6+1+\frac{19}{5}}{3} = \dfrac{54}{15}$

$$\beta = \frac{1+2-\frac{18}{5}}{3} = \frac{-3}{15}$$

$$15|\alpha+\beta| = 51$$

Question ID : 101373

Ellipse

13. Let the ecentricity of an ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b, e = \frac{1}{4}$. If this ellipse passes through the point $\left(-4\sqrt{\frac{2}{5}}, 3\right)$, then $a^2 + b^2$ is qual to :

माना दीर्घवृत्त $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$, $a > b$ की उत्केन्द्रता $\frac{1}{4}$ है। यदि यह दीर्घवृत्त बिन्दु $\left(-4\sqrt{\frac{2}{5}},3\right)$ से होकर जाता है, तो $a^2 + b^2$ बराबर है :

(1) 29 (2) 31 (3) 32 (4) 34

Ans. Official Answer NTA (2)

Sol. $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$e=\frac{1}{4}=\sqrt{1-\frac{b^2}{a^2}}$

$\frac{1}{16}=\frac{a^2-b^2}{a^2}$

$b^2=\frac{15}{16}a^2$

Now point $\left(-4\sqrt{\frac{2}{5}},3\right)$

$\frac{16\times\frac{2}{5}}{a^2}+\frac{9}{b^2}=1$

$\frac{32}{5a^2}+\frac{9\times16}{15a^2}=1$

$5a^2 = 80$

$a^2 = 16$

$b^2=\frac{15}{16}\times16=15$

$a^2 + b^2 = 31$

Question ID : 101374

3D Geometry

14. If two straight lines whose direction cosines are given by the relations $l+m-n=0, 3l^2+m^2+cnl=0$ are parallel, then the positive value of c is :

यदि दो सरल रेखाएँ, जिनकी दिक्कोज्याएँ $l+m-n=0, 3l^2+m^2+cnl=0$ द्वारा दी गई हैं, समांतर हैं, तो c का धनात्मक मान है :

(1) 6 (2) 4 (3) 3 (4) 2

Ans. Official Answer NTA (1)

Sol. $l + m - n = 0$

$3l^2 + m^2 + nlc = 0$ ——(1)

$m = n - l$

$m^2 = n^2 + l^2 - 2nl$ ——(2)

From (1)

$3\ell^2 + \left(n^2 + \ell^2 - 2n\ell\right) + n\ell c = 0$

divide by n^2

$3\frac{\ell^2}{n^2} + 1 + \frac{\ell^2}{n^2} - \frac{2\ell}{n} + \frac{\ell c}{n} = 0$

$4\frac{\ell^2}{n^2} + \frac{\ell}{n}(c-2) + 1 = 0$

lines are parallel so

$\frac{\ell_1}{\ell_2} = \frac{m_1}{m_2} = \frac{n_1}{n_2}$

$\Rightarrow \frac{\ell_1}{n_1} = \frac{\ell_2}{n_2} \Rightarrow$ equal roots

$\Rightarrow D = 0$

$(c-2)^2 - 4(4)(1) = 0$

$c = 6$ OR -2

$c = 6$

Question ID : 101375

Vectors

15. Let $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ and $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$. Then the number of vectors $\vec{b}$ such that $\vec{b} \times \vec{c} = \vec{a}$ and $|\vec{b}| \in \{1,2,.....,10\}$ is :

माना $\vec{a} = \hat{i} + \hat{j} - \hat{k}$ तथा $\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$ हैं। तो सदिशों $\vec{b}$ की संख्या, जिनके लिए $\vec{b} \times \vec{c} = \vec{a}$ तथा $|\vec{b}| \in \{1,2,.....,10\}$ हैं, है :

(1) 0 (2) 1 (3) 2 (4) 3

Ans. Official Answer NTA (1)

Sol. $\vec{a} = \hat{i} + \hat{j} - \hat{k}$

$\vec{c} = 2\hat{i} - 3\hat{j} + 2\hat{k}$

$\Rightarrow \vec{b} \times \vec{c} \,||\, \vec{a}$

$\Rightarrow c \perp \vec{a}$

$\Rightarrow \vec{a}.\vec{c} = 0$

$\left(\hat{i}+\hat{j}-\hat{k}\right).\left(2\hat{i}-3\hat{j}+2\hat{k}\right)$

$2-3-2\neq 0$

which is coutradiction

So no of vectors $\vec{b}=0$

Question ID : 101376

Probability

16. Five numbers $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ are randomly selected from numbers 1,2,3,.....,18 and are arranged in the increasing order $(\chi_1 < \chi_2 < \chi_3 < \chi_4 < \chi_5)$. The probability that $\chi_2 = 7$ and $\chi_4 = 11$ is :

संख्याओं 1, 2, 3, ...,18 से पाँच संख्याएँ $\chi_1, \chi_2, \chi_3, \chi_4, \chi_5$ यादृच्छया चुनी जाती हैं तथा उन्हें वर्धमान क्रम $(\chi_1 < \chi_2 < \chi_3 < \chi_4 < \chi_5)$ में व्यवस्थित किया जाता है। $\chi_2 = 7$ तथा $\chi_4 = 11$ होने की प्रायिकता है :

(1) $\frac{1}{136}$ (2) $\frac{1}{72}$ (3) $\frac{1}{68}$ (4) $\frac{1}{34}$

Ans. Official Answer NTA (3)

Sol. $x_1 < x_2 < x_3 < x_4 < x_5$

Total no. of ways = ${}^{18}C_5$

Now

$x_2 = 7, x_4 = 11$

$x_1 < 7 < x_3 < 11 < x_5$

favourable ways = $6 \times 3 \times 7$
(x_1 x_2 x_5)

$$P(E) = \frac{6\times3\times7}{18_{C_5}} = \frac{1}{68}$$

Question ID : 101377

Probability

17. Let X be a random variable having binomial distribution B(7,p). If P(X=3)=5P(X=4), then the sum of the mean and the variance of X is :

माना एक यादृच्छिक चर X का द्विपद बंटन B(7, p) है। यदि P(X = 3) = 5P(X = 4) है, तो X के माध्य तथा प्रसरण का योगफल है :

(1) $\frac{105}{16}$ (2) $\frac{7}{16}$ (3) $\frac{77}{36}$ (4) $\frac{49}{16}$

Ans. Official Answer NTA (3)

Sol. B(7, P)

n = 7

P(X = 3) = 5P(X = 4)

$^7C_3\ P^3(1-P)^4 = 5\ ^7C_4\ P^4(1-P)^3$

(1 – P) = 5P

1 = 6P

$P = \frac{1}{6}$ $\qquad$ $q = \frac{5}{6}$

$\text{mean} = np = 7 \times \frac{1}{6} = \frac{7}{6}$

$\text{vaniane} = npq = 7 \times \frac{1}{6} \times \frac{5}{6} = \frac{35}{36}$

$\text{sum} = \frac{7}{6} + \frac{35}{36} = \frac{77}{36}$

Question ID : 101378

Trigonometric Ratio and Identities

18. The value of $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ is equal to :

$\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$ का मान बराबर है :

(1) -1 (2) $-\frac{1}{2}$ (3) $-\frac{1}{3}$ (4) $-\frac{1}{4}$

Ans. Official Answer NTA (2)

Sol. $\cos\left(\frac{2\pi}{7}\right) + \cos\left(\frac{4\pi}{7}\right) + \cos\left(\frac{6\pi}{7}\right)$

$$= \frac{\sin\left(3 \times \frac{\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)} \cos\left(\frac{\frac{2\pi}{7} + \frac{6\pi}{7}}{2}\right)$$

$$= \frac{\sin\left(\frac{3\pi}{7}\right)}{\sin\left(\frac{\pi}{7}\right)} \cos\left(\frac{4\pi}{7}\right)$$

$$=\frac{\sin\frac{3\pi}{7}}{\sin\left(\frac{\pi}{7}\right)}\cos\left(\pi-\frac{3\pi}{7}\right)$$

$$\Rightarrow\frac{-2\sin\frac{3\pi}{7}\cos\frac{3\pi}{7}}{2\sin\frac{\pi}{7}}$$

$$\Rightarrow\frac{-\sin\frac{6\pi}{7}}{2\sin\frac{\pi}{7}}=\frac{-\sin\frac{\pi}{7}}{2\sin\frac{\pi}{7}}=-\frac{1}{2}$$

Question ID : 101379

ITF

19. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ is equal to :

$\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$ बराबर है :

(1) $\frac{11\pi}{12}$ (2) $\frac{17\pi}{12}$ (3) $\frac{31\pi}{12}$ (4) $-\frac{3\pi}{4}$

Ans. Official Answer NTA (1)

Sol. $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)+\cos^{-1}\left(\cos\frac{7\pi}{6}\right)+\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

$\sin^{-1}\left(\sin\frac{\pi}{3}\right)+\cos^{-1}\left(\cos\left(\pi+\frac{\pi}{6}\right)\right)+\tan^{-1}\left(\tan\left(\pi-\frac{\pi}{4}\right)\right)$

$\sin^{-1}\left(\sin\frac{\pi}{3}\right)+\cos^{-1}\left(-\cos\frac{\pi}{6}\right)+\tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

$\frac{\pi}{3}+\pi-\frac{\pi}{6}-\frac{\pi}{4}$

$=11\frac{\pi}{12}$

Question ID : 101380

Mathematical Reasoning

20. The boolean expression $(\sim(p \wedge q)) \vee q$ is equivalent to :

बूलीय व्यंजक $(\sim(p \wedge q)) \vee q$ निम्न में से किसके तुल्य है? :

(1) $q \rightarrow (p \wedge q)$ (2) $p \rightarrow q$ (3) $p \rightarrow (p \rightarrow q)$ (4) $p \rightarrow (p \vee q)$

Ans. Official Answer NTA (4)

Sol. $(\sim(p \wedge q)) \vee q$

$(\sim p \vee \sim q) \vee q$

$\sim p \vee (\sim q \vee q)$

$\sim p \vee T$

T

Now check by option which one is a tautology

Option (4) $p \rightarrow (p \vee q)$

p	q	$p \vee q$	$p \rightarrow p \vee q$
T	T	T	T
T	F	T	T
F	T	T	T
F	F	F	T

SECTION – B

Question ID : 101381

Function

21. Let $f : R \rightarrow R$ be a function defined by $f(x) = \frac{2e^{2x}}{e^{2x}+e}$. Then $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+....+f\left(\frac{99}{100}\right)$ is equal to __________.

माना फलन $f : R \rightarrow R$, $f(x) = \frac{2e^{2x}}{e^{2x}+e}$ द्वारा परिभाषित है। तो $f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)+....+f\left(\frac{99}{100}\right)$ बराबर है __________

Ans. Official Answer NTA (99)

Sol. $f(x) = \frac{2e^{2x}}{e^{2x}+e}$

$f(1-x) = \frac{2e^{2-2x}}{e^{2-2x}+e}$

$$f(1-x)=\frac{2e^{2-2x}}{e^{2-2x}+e}=\frac{(2e)}{(e+e^{2x})}$$

$$f(x)+f(1-x)=\frac{2e^{2x}}{e^{2x}+e}+\frac{2e}{e^{2x}+e}=2$$

$$f\left(\frac{1}{100}\right)+f\left(\frac{2}{100}\right)+f\left(\frac{3}{100}\right)..........f\left(\frac{99}{100}\right)$$

$$f\left(\frac{1}{100}\right)+f\left(\frac{99}{100}\right)=2$$

$$f\left(\frac{2}{100}\right)+f\left(\frac{98}{100}\right)=2$$

$$\vdots$$

$$f\left(\frac{49}{100}\right)+f\left(\frac{51}{100}\right)=2$$

$$f\left(\frac{50}{100}\right)=f\left(\frac{1}{2}\right)$$

$$=49\times 2+f\left(\frac{1}{2}\right)=1$$

$$=98+1=99$$

Question ID : 101382
Quadratic Equation

22. If the sum of all the roots the equation $e^{2x}-11e^{x}-45e^{-x}+\frac{81}{2}=0$ is $\log_e p$, then p is equal to __________.

यदि समीकरण $e^{2x}-11e^{x}-45e^{-x}+\frac{81}{2}=0$ के सभी हलों का योगफल $\log_e p$ है, तो p बराबर है __________

Ans. Official Answer NTA (45)

Sol. $e^{2x}-11e^{x}-45e^{-x}+\frac{81}{2}=0$

Let $e^x = t$

$t^2 - 11t - 45\times\frac{1}{t}+\frac{81}{2}=0$

$2t^3 - 22t^2 + 81t - 90 = 0$ ⟨ t_1, t_2, t_3

$t_1t_2t_3 = \frac{90}{2} = 45$

$e^{x_1}.e^{x_2}.e^{x_3} = 45$

$e^{x_1+x_2+x_3} = 45$

$x_1 + x_2 + x_3 = \log_e 45$

So p = 45

Question ID : 101383

Matrices

23. The positive value of determinant of the matrix A, whose Adj (Adj (A)) = $\begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}$, is ________.

यदि Adj (Adj (A)) = $\begin{pmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{pmatrix}$ है, तो आव्यूह A के सारणिक का धनात्मक मान है ________

Ans. Official Answer NTA (14)

Sol. $\text{adj}(\text{adj}(A)) = \begin{bmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{bmatrix}$

$\det.\left(|A|' A\right) = \begin{vmatrix} 14 & 28 & -14 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$

$R_1 \to R_1 + R_2 + R_3$

$|A|^3 |A| = \begin{vmatrix} 28 & 28 & 28 \\ -14 & 14 & 28 \\ 28 & -14 & 14 \end{vmatrix}$

$C_1 \to C_1 - C_2, C_2 \to C_2 - C_3$

$|A|^4 = \begin{vmatrix} 0 & 0 & 28 \\ -28 & -14 & 28 \\ 42 & -28 & 14 \end{vmatrix}$

$|A|^4 = 28((28)^2 + 14 \times 42)$
$|A|^4 = 28 \times 14 \times 14\,(4 + 3)$
$|A|^4 = 4 \times 7 \times 2 \times 7 \times 2 \times 7 \times 7$
$|A| = 14$

Question ID : 101384
P & C

24. The number of ways, 16 identical cubes, of which 11 are blue and rest are red, can be placed in a row so that between any two red cubes there should be at least 2 blue cubes, is __________.

16 समरूप घन, जिनमें से 11 नीले हैं तथा शेष लाल हैं, को एक पंक्ति में इस प्रकार रखने के तरीकों, कि किन्हीं भी दो लाल घन के बीच कम–से–कम 2 नीले घन हों, की संख्या है __________

Ans. Official Answer NTA (56)

Sol. 5 – Red
11 – Blue
$\_x\,R\,\_y\,R\,\_z\,R\,\_w\,R\,\_t\,R\,\_u$
$x + y + z + w + t + u = 11$
when $x \geq 0, y \geq 2, z \geq 2, w \geq 2, t \geq 2, u \geq 0$
Now distribute 2 to each y, z, w, t
$x + y + z + w + t + u = 3$
$^{3+6-1}C_{6-1} = {}^{8}C_5 = 56$

Question ID : 101385
Binomial Theorem

25. If the coefficient of x^{10} in the binomial expansion of $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ is $5^k.l$, where $l, k \in N$ and l is co-prime to 5, then k is equal to __________.

यदि $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$ के द्विपद प्रसार में x^{10} का गुणांक $5^k.l$ है, जहाँ $l, k \in N$ तथा l तथा l, 5 सहअभाज्य (co-prime) हैं, तो k बराबर है __________

Ans. Official Answer NTA (5)

Sol. $\left(\frac{\sqrt{x}}{5^{\frac{1}{4}}} + \frac{\sqrt{5}}{x^{\frac{1}{3}}}\right)^{60}$

$^{60}C_r\left(\frac{x^{\frac{1}{2}}}{5^{\frac{1}{4}}}\right)^{60-r}\left(\frac{5^{\frac{1}{2}}}{x^{\frac{1}{3}}}\right)^r$

$^{60}C_r 5^{-\left(\frac{60-r}{4}\right)}.5^{\frac{r}{2}}.x^{\frac{60-r}{2}}.x^{-\frac{r}{3}}$

$^{60}C_r\ 5^{\frac{3r-60}{4}}.x^{\frac{180-5r}{6}}$

$5r = 120$
$r = 24$

$^{60}C_{24}\ 5^3 = \frac{60!}{24!36!}5^3$

Now exponent of 5 in 60! $= \left[\frac{60}{5}\right]+\left[\frac{60}{25}\right]+\left[\frac{60}{125}\right] = 12+2 = 14$

exponent of 5 in 24! $= \left[\frac{24}{5}\right]+\left[\frac{24}{25}\right] = 4+0 = 4$

exponent of 5 in 36! $= \left[\frac{36}{5}\right]+\left[\frac{36}{25}\right] = 7+1 = 8$

coeff of x^{10} will be = exponent of 5

$= 3 + 14 - 4 - 8$

$\Rightarrow 5$

Question ID : 101386

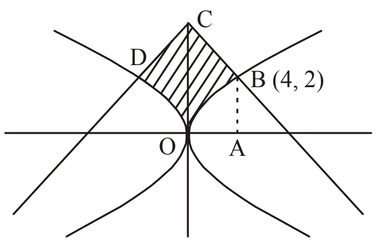
Area Under Curve

26. Let $A_1 = \{(x,y) : |x| \le y^2, |x| + 2y \le 8\}$ and $A_2 = \{(x,y) : |x| + |y| \le k\}$. If 27 (Area A_1) = 5 (Area A_2), then k is equal to __________.

माना $A_1 = \{(x,y) : |x| \le y^2, |x| + 2y \le 8\}$ तथा $A_2 = \{(x,y) : |x| + |y| \le k\}$ हैं। यदि 27 (क्षेत्रफल A_1) = 5 (क्षेत्रफल A_2) है, तो k बराबर है ___________

Ans. Official Answer NTA (6)

Sol. $A_1 = \{(x, y) : |x| \le y^2, |x| + 2y \le 8\}$



for point B $y^2 = x$ & $x + 2y = 8$

$\left(\frac{8-x}{2}\right)^2 = x$ $\qquad\qquad$ $y = \frac{8-x}{2}$

$x^2 - 16x + 64 = 4x$

$x^2 - 20x + 64 = 0$

$x = 4, y = 2$

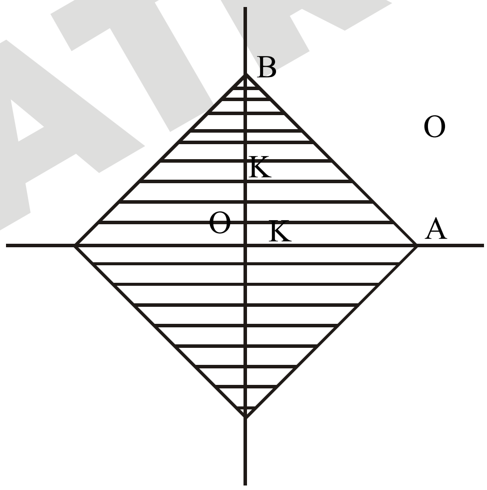
$$A_1 = 2\int_0^4 \left(\left(\frac{8-x}{2}\right) - \sqrt{x}\right)dx$$

$$A_1 = 2\left[\frac{8x}{2} - \frac{X^2}{4} - \frac{2x^{\frac{3}{2}}}{3}\right]_0^4$$

$$A_1 = \frac{40}{3}$$

$A_2 = \{(x, y) : |x| + |y| \le k\}$

$A_2 = 4(\text{Area of } \Delta OAB)$



$= 4 \times \frac{1}{2} k \times k$

$= 2k^2$

$27(A_1) = 5(A_2)$

$27 \times \frac{40}{3} = 5 \times 2k^2$

$k^2 = 36$

$k = 6$

Question ID : 1101387

Sequence & progression

27. If the sum of the first ten terms of the series $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+.....$ is $\frac{m}{n}$, where m and n are co-prime numbers, then m + n is equal to __________.

यदि श्रेणी $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}+.....$ के प्रथम दस पदों का योगफल $\frac{m}{n}$ है, जहाँ m तथा n सहअभाज्य (co-prime) संख्याएँ हैं, तो m + n बराबर है __________

Ans. Official Answer NTA (276)

Sol. $\frac{1}{5}+\frac{2}{65}+\frac{3}{325}+\frac{4}{1025}+\frac{5}{2501}$

$T_r = \frac{r}{4r^4+1}$

$S_{10} = \sum_{r=1}^{10}\frac{r}{4r^4+1} = \frac{1}{4}\sum_{r=1}^{10}\left(\frac{1}{2r^2-2r+1}-\frac{1}{2r^2+2r+1}\right)$

$S_{10} = \frac{1}{4}\left(1-\frac{1}{221}\right)$

$S_{10} \Rightarrow \frac{220}{4\times 221} = \frac{55}{221} = \frac{m}{n}$

$m + n = 276$

Question ID : 101388

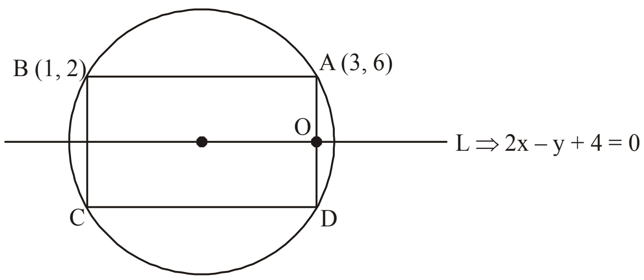
Circle

28. A rectangle R with end points of one of the its sides as (1, 2) and (3, 6) is inscribed in a circle. If the equation of diameter of the circle is $2x - y + 4 = 0$, then the area of R is __________.

एक आयत R, जिसकी एक भुजा के सिरे (1, 2) तथा (3, 6) हैं, एक वृत्त के अंतर्गत है। यदि वृत्त के एक व्यास का समीकरण $2x - y + 4 = 0$ है, तो R का क्षेत्रफल है __________

Ans. Official Answer NTA (16)

Sol.



Slope of AB $= \frac{6-2}{3-1} = \frac{4}{2} = 2$

Slope of line L = 2

So line AB and L are parallel

$AB = \sqrt{(3-1)^2 + (6-2)^2} = \sqrt{4+16} = -\sqrt{20}$

$AD = 2\ OA = 2\left|\frac{6-6+4}{\sqrt{4+1}}\right| = \frac{2\times 4}{\sqrt{5}} = \frac{8}{\sqrt{5}}$

Area $= \sqrt{20}\times\frac{8}{\sqrt{5}} = 2\sqrt{5}\times\frac{8}{\sqrt{5}} = 16$

Question ID : 101389

Parabola

29. A circle of radius 2 unit passes through the vertex and the focus of the parabola $y^2 = 2x$ and touches the parabola $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, where $\alpha > 0$. Then $(4\alpha - 8)^2$ is equal to __________.

इकाई त्रिज्या का वृत्त, परवलय $y^2 = 2x$ के शीर्ष तथा नाभि से होकर जाता है तथा परवलय $y = \left(x - \frac{1}{4}\right)^2 + \alpha$, जहाँ $\alpha > 0$ है, को स्पर्श करता है। तो $(4\alpha - 8)^2$ बराबर है __________

Ans. Official Answer NTA(63)

Sol.

Question ID : 101390

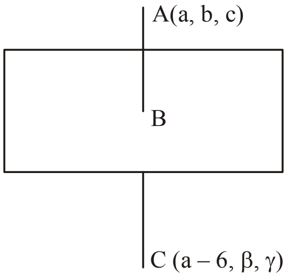
3D Geometry

30. Let the mirror image of the point (a, b, c) with respect to the plane $3x - 4y + 12z + 19 = 0$ be $(a - 6, \beta, \gamma)$. If $a + b + c = 5$, then $7\beta - 9\gamma$ is equal to __________.

माना बिन्दु (a, b, c) का समतल $3x - 4y + 12z + 19 = 0$ के सापेक्ष दर्पण प्रतिबिंब $(a - 6, \beta, \gamma)$ है। यदि $a + b + c = 5$, है, तो $7\beta - 9\gamma$ बराबर है __________

Ans. Official Answer NTA (137)

Sol. $3x - 4y + 12z + 19 = 0$



Direction ratio of $\overrightarrow{AC} = (-6, \beta - b, \gamma - c)$

D. R. of plane = (3, –4, 12)

So $\quad \frac{-6}{3} = \frac{\beta - b}{-4} = \frac{\gamma - c}{12}$

$\beta - b = 8 \qquad \gamma - c = -24$

$\beta = 8 + b \qquad \gamma = c - 24$

Now mid point of AC will lie on given plane

So $\quad B\left(\frac{a+a-6}{2}, \frac{b+\beta}{2}, \frac{c+\gamma}{2}\right)$

$= \left(a-3, \frac{b+\beta}{2}, \frac{c+\gamma}{2}\right)$

$3(a-3) - 4\left(\frac{b+\beta}{2}\right) + 12\left(\frac{C+\gamma}{2}\right) + 19 = 0$

$3a - 2b + 6c - 2\beta + 6\gamma + 10 = 0$

$3a - 2b + 6c - 2(8 + b) + 6(c - 24) + 10 = 0$

$3(5-b-c) - 2b + 6c - 2b + 6c - 150 = 0$

$-7b + 9c - 135 = 0$

$-7(\beta - 8) + 9(\gamma + 24) - 135 = 0$

$7\beta - 9\gamma = 137$