JEE Main June 2022

Question Paper With Text Solution

27 June | Shift-2

MATHEMATICS

MATRIX

JEE Main & Advanced | XI-XII Foundation| VI-X Pre-Foundation

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JEE MAIN JUNE 2022 | 27TH JUNE SHIFT-2

SECTION – A

Question ID : 191

Complex Number

1. The number of points of intersection of $|z-(4+3i)| = 2$ and $|z| + |z-4| = 6, z \in C$, is :

$|z-(4+3i)| = 2$ तथा $|z| + |z-4| = 6, z \in C$ के प्रतिच्छेदन बिन्दुओं की संख्या है :

(1) 0 (2) 1 (3) 2 (4) 3

Ans. Official Answer NTA (3)

Sol. $|z-(4+3i)| = 2$ is a circle $(x-4)^2 + (y-3)^2 = 2^2$

$|z| + |z-4| = 6$ is an ellipse with focus (0, 0) & (4, 0)

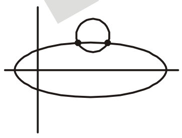
and $2a = 6$ $2ae = 4$ centre (2, 0)

$\Rightarrow a = 3$ $e = \frac{2}{3}$ $\Rightarrow b^2 = 5$

$$\frac{(x-2)^2}{9} + \frac{y^2}{5} = 1$$

Lower extremity of vertical diameter (4, 1)

put in ellipse $S_1 = \frac{(4-2)^2}{9} + \frac{1^2}{5} - 1 = \frac{-16}{45} < 0$

$\Rightarrow$ lies inside ellipse $\Rightarrow$ 

Question ID : 192

Methods of Differentiation

2. Let $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a \in R$. Then the sum of squares of all the values of a, for which

$2f'(10) - f'(5) + 100 = 0$, is :

माना $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a \in R$ है। तो a के सभी मानों, जिनके लिए $2f'(10) - f'(5) + 100 = 0$ है, के वर्गों का

योगफल है :

(1) 117 (2) 106 (3) 125 (4) 136

Ans. Official Answer NTA (3)

Sol. $f(x) = a(a^2 + ax) + 1(a^2x + ax^2)$

$f(x) = a^3 + 2a^2x + ax^2$

$f'(x) = 2a^2 + 2ax$

$2f'(10) - f'(5) + 100 = 0$

$2[2a^2 + 20a] - (2a^2 + 10a) + 100 = 0$

$2a^2 + 30a + 100 = 0$

$a^2 + 15a + 50 = 0$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= 225 - 100 = 125$

Question ID : 193

Complex Number

3. Let for some real number α and β, $a = \alpha - i\beta$. If the system of equations $4ix + (1 + i)y = 0$ and $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$ has more than one solution then $\frac{\alpha}{\beta}$ is equal to :

माना α तथा β के किसी वास्तविक मानों के लिए $a = \alpha - i\beta$ है। यदि समीकरण निकाय $4ix + (1 + i)y = 0$ तथा $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$ के एक से अधिक हल हैं, तो $\frac{\alpha}{\beta}$ बराबर है :

(1) $-2+\sqrt{3}$ (2) $2-\sqrt{3}$ (3) $2+\sqrt{3}$ (4) $-2-\sqrt{3}$

Ans. Official Answer NTA (2)

Sol. $\frac{i}{2\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right)} = \frac{1+i}{\alpha + i\beta}$

$\alpha i + i^2\beta = (1+i)(-1+i\sqrt{3})$

$-\beta + \alpha i = (-1-\sqrt{3}) + (\sqrt{3}-1)i$

$\beta = 1+\sqrt{3}$, $\alpha = \sqrt{3}-1$

$$\frac{\alpha}{\beta}=\frac{\sqrt{3}-1}{\sqrt{3}+1}=2-\sqrt{3}$$

Question ID : 194

Matrices

4. Let A and B be two 3 × 3 matrices such that AB = I and $|A|=\frac{1}{8}$. Then |adj (B adj (2A))| is equal to :

माना A तथा B दो 3 × 3 आव्यूह हैं तथा AB = I और $|A|=\frac{1}{8}$ हैं। तो |adj (B adj (2A))| बराबर है :

(1) 16 (2) 32 (3) 64 (4) 128

Ans. Official Answer NTA (3)

Sol. $|AB| = 1$

$|A|\,|B| = 1$

$\Rightarrow |B| = 8$

$$\left|\text{adj}\left(B\,\text{adj}(2A)\right)\right| = \left|B\,\text{adj}(2A)\right|^2$$
$$= |B|^2\,|\text{adj}(2A)|^2$$
$$= |B|^2\left(|2A|^2\right)^2$$
$$= |B|^2\,|2A|^4$$
$$= |B|^2\,(2^3)^4\,|A|^4$$
$$= 8^2.8^4.\left(\frac{1}{8}\right)^4$$
$$= 8^2 = 64$$

Question ID : 195

Sequence & progression

5. Let $S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+\ldots\ldots$ Then 4S is equal to :

माना $S=2+\frac{6}{7}+\frac{12}{7^2}+\frac{20}{7^3}+\frac{30}{7^4}+\ldots\ldots$ है। तो 4S बराबर है :

(1) $\left(\frac{7}{3}\right)^2$ (2) $\frac{7^3}{3^2}$ (3) $\left(\frac{7}{3}\right)^3$ (4) $\frac{7^2}{3^3}$

Ans. Official Answer NTA (3)

Sol. $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + ...$

$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + ...$

$\frac{6S}{7} = \frac{2}{1} + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + ...$

$$\frac{6S}{7} = \frac{2}{1-\frac{1}{7}} + \frac{(2)\left(\frac{1}{7}\right)}{\left(1-\frac{1}{7}\right)^2}$$

$\frac{6S}{7} = \frac{14}{6} + \frac{14}{36}$

$\frac{6S}{7} = \frac{14}{36} \times 7$

$S = 2 \times \frac{7^3}{6^3}$

$4S = \left(\frac{7}{3}\right)^3$

Question ID : 196

Sequence & progression

6. If a_1, a_2, a_3......and b_1, b_2, b_3.....are A.P., and $a_1 = 2$, $a_{10} = 3$, $a_1b_1 = 1 = a_{10}b_{10}$, then a_4b_4 is equal to :

यदि a_1, a_2, a_3...... तथा b_1, b_2, b_3..... समांतर श्रेढ़ीं (A.P.) हैं तथा $a_1 = 2$, $a_{10} = 3$, $a_1b_1 = 1 = a_{10}b_{10}$ हैं, तो a_4b_4 बराबर है:

(1) $\frac{35}{27}$ (2) 1 (3) $\frac{27}{28}$ (4) $\frac{28}{27}$

Ans. Official Answer NTA (4)

Sol. $a_{10} - a_1 = 1$ $b_1 = \frac{1}{2}$ $b_{10} = \frac{1}{3}$

$d = \frac{1}{9}$ $9D = \frac{1}{3} - \frac{1}{2}$ $\Rightarrow D = \frac{-1}{54}$

$a_4 = a_1 + 3d$

$= 2 + \frac{1}{3}$

$= \frac{7}{3}$

$b_4 = b_1 + 3D$

$b_4 = \frac{1}{2} - \frac{1}{18}$

$b_4 = \frac{4}{9}$

$a_4 b_4 = \frac{28}{27}$

Question ID : 197

Definite Integration

7. If m and n respectively are the number of local maximum and local minimum points of the function $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$, then the ordered pair (m, n) is equal to :

यदि फलन $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t} dt$ के स्थानीय उच्चतम तथा स्थानीय निम्नतम बिन्दुओं की संख्या क्रमशः m तथा n है, तो क्रमित युग्म (m, n) बराबर है :

(1) (3, 2) (2) (2, 3) (3) (2, 2) (4) (3, 4)

Ans. Official Answer NTA (2)

Sol. $f'(x) = \frac{x^4 - 5x^2 + 4}{2 + e^{x^2}} . 2x$

$= \frac{2x(x^2 - 1)(x^2 - 4)}{e^{x^2} + 2}$

$= \frac{2(x-1)(x-2)x(x+1)(x+2)}{e^{x^2} + 2}$

−		+		−		+		−		+
	−2		−1		0		1		2	

Minima at x = – 2, 0, 2
maxima at x = – 1, 1
m = 2, n = 3 (2, 3)

Question ID :198

Definite Integration

8. Let f be a differentiable function in $\left(0,\frac{\pi}{2}\right)$. If $\int_{\cos x}^{1} t^2 f(t)dt = \sin^3 x + \cos x$ then $\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

माना $\left(0,\frac{\pi}{2}\right)$ में f एक अवकलनीय फलन है। यदि $\int_{\cos x}^{1} t^2 f(t)dt = \sin^3 x + \cos x$ है, तो $\frac{1}{\sqrt{3}} f'\left(\frac{1}{\sqrt{3}}\right)$ बराबर है :

(1) $6-9\sqrt{2}$ (2) $6-\frac{9}{\sqrt{2}}$ (3) $\frac{9}{2}-6\sqrt{2}$ (4) $\frac{9}{\sqrt{2}}-6$

Ans. Official Answer NTA (2)

Sol. $\int_{\cos x}^{1} t^2 f(t)\,dt = \sin^2 x + \cos x$

Differentiate both side

$-\cos^2 x\, f(\cos x)(-\sin x) = 3\sin^2 x \cos x - \sin x$

$\Rightarrow f(\cos x) = \frac{(3\sin x \cos x - 1)}{\cos^2 x}$

$\Rightarrow f(\cos x) = 3\tan x - \sec^2 x$

Again differentiating wrt 'x'

$f'(\cos x)(-\sin x) = 3\sec^2 x - 2\sec^2 x \tan x$

put $\cos x = \frac{1}{\sqrt{3}} \Rightarrow \sin x = \frac{\sqrt{2}}{\sqrt{3}}$

$\frac{-\sqrt{2}}{\sqrt{3}} f\left(\frac{1}{\sqrt{3}}\right) = 3.(3) - 2(3)\sqrt{2}$

$\frac{1}{\sqrt{3}} f\left(\frac{1}{\sqrt{3}}\right) = \frac{3\left(3-2\sqrt{2}\right)}{-\sqrt{2}}$

$= 6 - \frac{9}{\sqrt{2}}$

Question ID : 199

Definite Integration

9. The integral $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}}dx$, where [.] denotes the greatest integer function, is equal to :

समाकलन $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}}dx$, जहाँ [.] महत्तम पूर्णांक फलन है, बराबर है :

(1) $1+6\ \log_e\left(\frac{6}{7}\right)$ (2) $1-6\log_e\left(\frac{6}{7}\right)$

(3) $\log_e\left(\frac{7}{6}\right)$ (4) $1-7\log_e\left(\frac{6}{7}\right)$

Ans. Official Answer NTA (1)

Sol. $\int_0^1 \frac{1}{7^{\left[\frac{1}{x}\right]}}dx$

$$=-\int_0^1 \frac{dx}{7^{\left[\frac{1}{x}\right]}}$$

$$-\left[\int_1^{\frac{1}{2}} \frac{1}{7^1}dx+\int_{\frac{1}{2}}^{\frac{1}{3}} \frac{1}{7^2}dx+\int_{\frac{1}{3}}^{\frac{1}{4}} \frac{1}{7^3}dx+......\right]$$

$$-\left[\frac{1}{7}\left(\frac{1}{2}-1\right)+\frac{1}{7^2}\left(\frac{1}{3}-\frac{1}{2}\right)+\frac{1}{7^3}\left(\frac{1}{4}-\frac{1}{3}\right)+......\right]$$

$$\left(\frac{1}{7}+\frac{1}{2.7^2}+\frac{1}{3.7^3}+.......\right)-\left(\frac{1}{7.2}+\frac{1}{7^2.3}+\frac{1}{7^3.4}+.........\right)$$

$$-\ell n\left(1-\frac{1}{7}\right)+7\left(\ell n\left(1-\frac{1}{7}\right)-\frac{1}{7}\right)$$

$$=6\ \ell n\frac{6}{7}+1$$

Question ID : 1910

Differential Equation

10. If the solution curve of the differential equation $((\tan^{-1} y) - x)dy = (1 + y^2)dx$ passes through the point (1, 0), then the abscissa of the point on the curve whose ordinate is tan(1), is :

यदि अवकल समीकरण $((\tan^{-1} y) - x)dy = (1 + y^2)dx$ का हल वक्र बिन्दु (1, 0) से होकर जाता है, तो वक्र पर उस बिन्दु, जिसकी कोटि tan(1) है, का भुज है :

(1) 2e (2) 2/e (3) 2 (4) 1/e

Ans. Official Answer NTA (2)

Sol. $\tan^{-1} y - x = \left(1 + y^2\right)\frac{dx}{dy}$

$\Rightarrow \frac{dx}{dy} = \frac{\tan^{-1} y}{1+y^2} - \frac{x}{1+y^2}$

$\Rightarrow \frac{dx}{dy} + \frac{x}{1+y^2} = \frac{\tan^{-1} y}{1+y^2}$

LDE in 'x'

$\text{I.F.} = e^{\int \frac{dy}{1+y^2}} = e^{\tan^{-1} y}$

$x\left(e^{\tan^{-1} y}\right) = \int \frac{\tan^{-1} y}{1+y^2} . e^{\tan^{-1} y} dy + c$

$xe^{\tan^{-1} y} = \left(\tan^{-1} y\right)e^{\tan^{-1} y} - e^{\tan^{-1} y} + c$

put $x = 1, y = 0$

$1 = 0 - 1 + c$

$\Rightarrow c = 2$

put $y = \tan 1$

$xe^1 = e - e + 2$

$x = \frac{2}{e}$

Question ID : 1911

Parabola

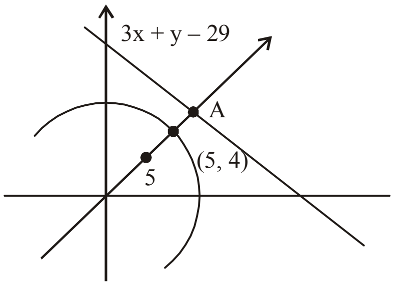
11. If the equation of the parabola, whose vertex is at (5, 4) and the directtrix is $3x + y - 29 = 0$, is $x^2 + ay^2 + bxy + cx + dy + k = 0$, then $a + b + c + d + k$ is equal to :

एक परवलय का शीर्ष (5, 4) तथा नियता $3x + y - 29 = 0$ हैं। यदि इसका समीकरण $x^2 + ay^2 + bxy + cx + dy + k = 0$ है, तो $a + b + c + d + k$ बराबर है :

(1) 575 (2) – 575 (3) 576 (4) – 576

Ans. Official Answer NTA (4)

Sol.



$$\frac{x-5}{3}=\frac{y-4}{1}=\frac{-(15+4-29)}{3^2+1^2}$$

$$\frac{x-5}{3}=\frac{y-4}{1}=1$$

$x = 8, y = 5$

$A(8, 5)$

$S(x,y)\ \ \frac{x+8}{2}=5 \qquad \frac{y+5}{2}=4$

$x = 2 \qquad y = 3$

$S(2, 3) \qquad PS = PM$

$\Rightarrow PS^2 = PM^2$

$$(x-2)^2+(y-3)^2=\left(\frac{3x+y-29}{\sqrt{10}}\right)^2$$

$\Rightarrow x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$

$\Rightarrow a + b + c + d + k = -576$

Question ID : 1912

Circle

12. The set of value of k, for which the circle $C : 4x^2 + 4y^2 - 12x + 8y + k = 0$ lies inside the fourth quadrant and the point $\left(1,-\frac{1}{3}\right)$ lies on or inside the circle C, is :

(1) an empty set (2) $\left(6,\frac{65}{9}\right]$ (3) $\left[\frac{80}{9},10\right)$ (4) $\left(9,\frac{92}{9}\right]$

k के मानों, जिनके लिए वृत्त $C : 4x^2 + 4y^2 - 12x + 8y + k = 0$ चतुर्थ चतुर्थांश के अंतर्गत है तथा बिन्दु $\left(1,-\frac{1}{3}\right)$ वृत्त C पर

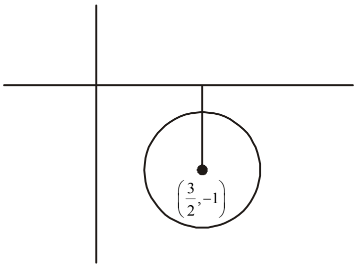
या इसके अंतर्गत है, का समुच्चय है :

(1) एक रिक्त समूह (2) $\left(6, \frac{65}{9}\right]$ (3) $\left[\frac{80}{9}, 10\right)$ (4) $\left(9, \frac{92}{9}\right]$

Ans. Official Answer NTA (4)

Sol. $x^2 + y^2 - 3x + 2y + \frac{k}{4} = 0$

centre $\left(\frac{3}{2}, -1\right)$



$r < 1$

$\sqrt{\left(\frac{3}{2}\right)^2 + (1)^2 - \frac{k}{4}} < 1$

$\frac{13}{4} - \frac{k}{4} < 1$

$k > 9$

& $S_1 \le 0$ $\left(1, -\frac{1}{3}\right)$

$1 + \frac{1}{9} - 3\ \frac{-2}{3} + \frac{k}{4} \le 0$

$\frac{k}{4} \le \frac{23}{9}$

$k \le \frac{92}{9}$

$\Rightarrow k \in \left(9, \frac{92}{9}\right]$

Question ID : 1913

3D Geometry

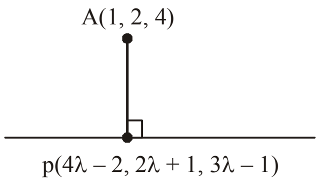
13. Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{3}$ be P. Then the distance of P from the plane $3x+4y+12z+23=0$ is :

माना बिन्दु (1, 2, 4) से रेखा $\frac{x+2}{4}=\frac{y-1}{2}=\frac{z+1}{3}$ पर लंब का पाद P है। तो P की समतल $3x+4y+12z+23=0$ से दूरी है :

(1) 5 (2) 50/13 (3) 4 (4) 63/13

Ans. Official Answer NTA (1)

Sol.



$\overrightarrow{AP}\cdot\left(4\hat{i}+2\hat{j}+3\hat{k}\right)=0$

$4(4\lambda-3)+2(2\lambda-1)+3(3\lambda-5)=0$

$29\lambda = 29$

$\lambda = 1$

P(2, 3, 2)

$d=\left|\frac{6+12+24+23}{\sqrt{3^2+4^2+12^2}}\right|=\frac{65}{13}=5$

Question ID : 1914

3D Geometry

14. The shortest distance between the line $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ and $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$, is :

रेखाओं $\frac{x-3}{2}=\frac{y-2}{3}=\frac{z-1}{-1}$ तथा $\frac{x+3}{2}=\frac{y-6}{1}=\frac{z-5}{3}$ के बीच न्यूनतम दूरी है :

(1) $\frac{18}{\sqrt{5}}$ (2) $\frac{22}{3\sqrt{5}}$ (3) $\frac{46}{3\sqrt{5}}$ (4) $6\sqrt{3}$

Ans. Official Answer NTA (1)

Sol. $SD = \left|\frac{(6\hat{i}-4\hat{j}-4\hat{k}).(10\hat{i}-8\hat{j}-4\hat{k})}{\sqrt{10^2+8^2+4^2}}\right|$ $\quad \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix}$

$= \frac{60+32+16}{2\sqrt{45}}$ $\quad = 10\hat{i}-8\hat{j}-4\hat{k}$

$= \frac{54}{3\sqrt{5}} = \frac{18}{\sqrt{5}}$

Question ID : 1915

Vecotrs

15. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonals of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute $|\vec{a}|=1$, and $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$. If $c = 2\sqrt{2}(\vec{a}\times\vec{b}) - 2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is :

माना एक समांतर चतुर्भुज, जिसका क्षेत्रफल $2\sqrt{2}$ है, के विकर्णों के अनुदिश सदिश $\vec{a}$ तथा $\vec{b}$ हैं। माना $\vec{a}$ तथा $\vec{b}$ के बीच कोण, न्यून कोण है, $|\vec{a}|=1$ तथा $|\vec{a}.\vec{b}| = |\vec{a}\times\vec{b}|$ हैं। यदि $c = 2\sqrt{2}(\vec{a}\times\vec{b}) - 2\vec{b}$ है, तो $\vec{b}$ तथा $\vec{c}$ के बीच एक कोण है :

(1) $\frac{\pi}{4}$ (2) $-\frac{\pi}{4}$ (3) $\frac{5\pi}{6}$ (4) $\frac{3\pi}{4}$

Ans. Official Answer NTA (4)

Sol. $\cos\theta = \sin\theta$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}$

$\frac{1}{2}|\vec{a}\times\vec{b}| = 2\sqrt{2}$

$|\vec{a}||\vec{b}|\sin\theta = 4\sqrt{2}$

$|\vec{b}|.\frac{1}{\sqrt{2}} = 4\sqrt{2}$

$|\vec{b}| = 8$

$|\vec{c}| = \sqrt{8|\vec{a}\times\vec{b}|^2 + 4|\vec{b}|^2}$

$\sqrt{8(4\sqrt{2})^2 + 4(8)^2}$

$= \sqrt{32 \times 8 + 32 \times 8}$

$= 16\sqrt{2}$

$\vec{c} = 2\sqrt{2}\left(\vec{a} \times \vec{b}\right) - 2\vec{b}$

$\vec{b}.\vec{c} = 0 - 2\vec{b}.\vec{b}$

$|\vec{c}| \cos\theta = -2|\vec{b}|$

$\cos\theta = \frac{-2(8)}{16\sqrt{2}}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

$\Rightarrow \theta = \frac{3\pi}{4}$

Question ID : 1916

Statistics

16. The mean and variance of the data 4, 5, 6, 6, 7, 8, x, y, where x < y, are 6 and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to :

आंकड़ों 4, 5, 6, 6, 7, 8, x, y, जहाँ x < y है, के माध्य तथा प्रसरण क्रमशः 6 तथा $\frac{9}{4}$ हैं। तो $x^4 + y^2$ बराबर है :

(1) 162 (2) 320 (3) 674 (4) 420

Ans. Official Answer NTA (2)

Sol. $\frac{36 + x + y}{8} = 6$ $\Rightarrow \frac{9}{4} = \frac{16+25+36+36+49+64+x^2+y^2}{8} - (6)^2$

$x + y = 12$ $\Rightarrow x^2 + y^2 = 306 - 226$

$\Rightarrow x^2 + y^2 = 80$

$x = 4, y = 8$

$x^4 + y^2 = 256 + 64$ $\Rightarrow 320$

Question ID : 1917

Area Under Curve

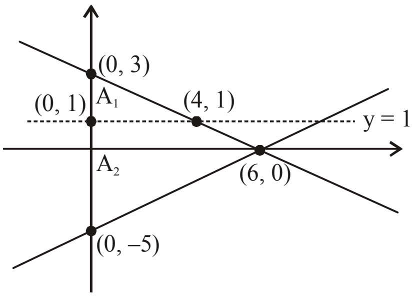
17. If a point A (x,y) lies in the region bounded by the y-axis, straight lines $2y + x = 6$ and $5x - 6y = 30$, then the probability that $y < 1$ is :

यदि y-अक्ष, सरल रेखाओं $2y + x = 6$ तथा $5x - 6y = 30$ से घिरे (bounded) क्षेत्र में एक बिन्दु $A(x, y)$ है, तो $y < 1$ होने की प्रायिकता है :

(1) $\frac{1}{6}$ (2) $\frac{5}{6}$ (3) $\frac{2}{3}$ (4) $\frac{6}{7}$

Ans. Official Answer NTA (2)

Sol.



$$P(y<1) = \frac{A_2}{A_1 + A_2}$$

$$= 1 - \frac{A_1}{A_1 + A_2}$$

$$= 1 - \frac{(2)(4)}{6 \times 8}$$

$$1 - \frac{1}{6} \quad = \frac{5}{6}$$

Question ID : 1918

ITF

18. The value of $\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is :

$\cot\left(\sum_{n=1}^{50} \tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ का मान है :

(1) $\frac{26}{25}$ (2) $\frac{25}{26}$ (3) $\frac{50}{51}$ (4) $\frac{52}{51}$

Ans. Official Answer NTA (1)

Sol. $\cot \sum \tan^{-1} \frac{(n+1)-n}{1+n(n+1)}$

$\cot \sum_{n=1}^{50} \tan^{-1}(n+1) - \tan^{-1} n$

$\cot\left(\tan^{-1} 51 - \tan^{-1} 1\right)$

$\cot \tan^{-1} \frac{50}{52} = \frac{52}{50} = \frac{26}{25}$

Question ID : 1919

Trigonometric Ratio and Identities

19. $\alpha = \sin 36^o$ is a root of which of the following equation ?

निम्न में से किस समीकरण का मूल $\alpha = \sin 36^o$ है ?

(1) $16x^4 - 10x^2 - 5 = 0$

(2) $16x^4 + 20x^2 - 5 = 0$

(3) $16x^4 - 20x^2 + 5 = 0$

(4) $16x^4 - 10x^2 + 5 = 0$

Ans. Official Answer NTA (3)

Sol. $\sin 72^\circ = \sin 108^\circ$

$\sin 2(36^\circ) = \sin 3(36^\circ)$

$2\alpha \cos 36^\circ = 3\alpha - 4\alpha^3$

$4\cos^2 36^\circ = \left(3 - 4\alpha^2\right)^2$

$4 - 4\alpha^2 = 9 + 16\alpha^4 - 24\alpha^2$

$16\alpha^4 - 20\alpha^2 + 5 = 0$

Question ID : 1920

Mathematical Reasoning

20. Which of the following statement is a tautology?

निम्न में से कौनसा कथन पुनरुक्ति है?

(1) $((\sim q)\wedge p)\wedge q$ (2) $((\sim q)\wedge p)\wedge(p\wedge(\sim p))$

(3) $((\sim q)\wedge p)\vee(p\vee(\sim p))$ (4) $(p\wedge q)\wedge(\sim(p\wedge q))$

Ans. Official Answer NTA (3)

Sol. (1) $(\sim q \wedge q) \wedge p \equiv F$

(2) $(\sim q \wedge p) \wedge F \equiv F$

(3) $(\sim q \wedge p) \vee T \equiv T$

(4) $p \wedge q \wedge \sim p \wedge \sim q \equiv F$

SECTION – B

Question ID : 1921

Function

21. let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Define f: $S \to S$ as $f(n)=\begin{cases}2n, \text{if } n=1,2,3,4,5\\ 2n-11, \text{if } n=6,7,8,9,10\end{cases}$. Let $g : S \to S$ be function such that $fog(n)=\begin{cases}n+1, \text{if } n \text{ is odd}\\ n-1, \text{if } n \text{ is even}\end{cases}$. Then g(10) ((g(1) + g(2) +g(3) + g(4) + g(5)) is equal to ______.

माना S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10} है। f: $S \to S$ को $f(n)=\begin{cases}2n, \text{if } n=1,2,3,4,5\\ 2n-11, \text{if } n=6,7,8,9,10\end{cases}$ द्वारा परिभाषित कीजिए। माना एक फलन $g : S \to S$ के लिए $fog(n)=\begin{cases}n+1, \text{यदि } n \text{ विषम है}\\ n-1, \text{यदि } n \text{ सम है}\end{cases}$ तो

g(10) ((g(1) + g(2) +g(3) + g(4) + g(5)) बराबर है ______.

Ans. Official Answer NTA (190)

Sol. $g(n)=\begin{cases}f^{-1}(n+1) & , \quad n \in \text{odd}\\ f^{-1}(n-1) & , \quad n \in \text{even}\end{cases}$

$$f^{-1}(n)=\begin{cases}\dfrac{n}{2} & , \quad n=2,4,6,8,10\\ \dfrac{n+11}{2} & , \quad n=1,3,5,7,9\end{cases}$$

$$g(n)=\begin{cases}\dfrac{n+1}{2} & , \quad n\in \text{odd}\\ \dfrac{n+10}{2} & , \quad n\in \text{even}\end{cases}$$

$g(10) = 10,$ $g(1) = 1,$ $g(2) = 6,$ $g(3) = 2,$ $g(4) = 7$ $g(5) = 3$

$\Rightarrow$ 190 Ans.

Question ID : 1922

Quadratic Equation

22. Let α, β be the roots of the equation $x^2 - 4\lambda x + 5 = 0$ and α, γ be the roots of the equation $x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+7+3\lambda\sqrt{3}=0$, $\lambda > 0$. If $\beta+\gamma=3\sqrt{2}$, then $(\alpha+2\beta+\gamma)^2$ is equal to _____.

माना समीकरण $x^2 - 4\lambda x + 5 = 0$ के मूल α, β हैं तो समीकरण $x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+7+3\lambda\sqrt{3}=0$ के मूल α, γ हैं, $\lambda > 0$ है। यदि $\beta+\gamma=3\sqrt{2}$ है, तो $(\alpha+2\beta+\gamma)^2$ बराबर है _____.

Ans. Official Answer NTA (98)

Sol. $\alpha + \beta = 4\lambda$ _______(1)

$\alpha + \gamma = 3\sqrt{2}+2\sqrt{3}$ _______(2)

$\beta+\gamma=3\sqrt{2}$ _______(3)

(1) + (2) + (3)

$$\alpha+\beta+\gamma=\frac{4\lambda+6\sqrt{2}+2\sqrt{3}}{2}$$

$$=2\lambda+3\sqrt{2}+\sqrt{3}$$

$\Rightarrow \gamma=3\sqrt{2}+\sqrt{3}-2\lambda$

$\beta=2\lambda-\sqrt{3}$

$\alpha=2\lambda+\sqrt{3}$

$\alpha+2\beta+\gamma=4\lambda+3\sqrt{2}$

$\alpha\beta = 5$

$\left(2\lambda+\sqrt{3}\right)\left(2\lambda-\sqrt{3}\right)=5$

$4\lambda^2 - 3 = 5$

$\lambda^2 = 2$

$\lambda=\sqrt{2}$

$\alpha+2\beta+\gamma=4\sqrt{2}+3\sqrt{2}$

$=\left(7\sqrt{2}\right)^2$

$= 98$

Question ID : 1923

Matrices

23. Let A be a matrix of order 2 × 2, whose entries are from the set {0, 1, 2, 3, 4, 5}. If the sume of all entries of A is a prime number p, $2 < p < 8$, then the number of such matrices A is ______.

माना 2 × 2 कोटि का एक आव्यूह A है, जिसके अवयव समुच्चय {0, 1, 2, 3, 4, 5} में से हैं। यदि A के सभी अवयवों का योग एक अभाज्य संख्या p, $2 < p < 8$ है, तो ऐसे आव्यूहों की संख्या है ______

Ans. Official Answer NTA (180)

Sol. $a + b + c + d = 3$

a, b, c, d, = {0, 1, 2, 3}

No. of ways = ${}^{3+4-1}C_{4-1}$ $= {}^6C_3 = 20$

$a + b + c + d = 5$

a, b, c, d, ∈ {0, 1, 2, 3, 4, 5}

No. of ways = ${}^{4+5-1}C_{4-1} = {}^8C_3$

$= 56$

$a + b + c + d = 7$

a, b, c, d = {0, 1, 2, 3, 4, 5}

Coeff of x^7 in $(1 + x + ... + x^5)^4$

x^7 in $\left(\frac{1-x^6}{1-x}\right)^4$

x^7 in $\left(1-x^6\right)^4\left(1-x\right)^{-4}$

x^7 in $\left(1-x\right)^{-4}-4+x^1$ in $\left(1-x\right)^{-4}$

${}^{4+7-1}C_{4-1} - 4 \times {}^{4+1-1}C_{4-1}$

${}^{10}C_3 - 4.{}^4C_3$

$120 - 16 = 104$

Hence total no. of ways = 20 + 56 + 104 = 180

Question ID : 1924

Binomial Theorem

24. If the sum of coefficients of all positive powers of x, in the Binomial expansion of $\left(x^n+\frac{2}{x^5}\right)^7$ is 939, then the sum of all the possible integral value of n is _____.

यदि $\left(x^n+\frac{2}{x^5}\right)^7$ के द्विपद प्रसार में x की सभी धनात्मक घातों के गुणांकों का योग 939 है, तो n के सभी संभव पूर्णांक मानों का योग है _____.

Ans. Official Answer NTA (57)

Sol. $\left(x^n+\frac{2}{x^5}\right)^7 = {}^7C_0x^{7n} + {}^7C_1 2^1 x^{6n-5} + {}^7C_2.2^2x^{5n-10} + {}^7C_3 2^3 x^{4n-15} + {}^7C_4 2^4 x^{3n-20}$

$+ {}^7C_5 2^5 x^{2n-25} + {}^7C_6 2^6 x^{n-30} + {}^7C_7 2^7 x^{-35}$

${}^7C_0 = 1$ ${}^7C_2(2^2) = 84$ ${}^7C_4(2^4) = 560$

${}^7C_1\ (2) = 14$ ${}^7C_3(2^3) = 280$

Sum = 939

$3n - 20 \geq 0$ & $2n - 25 < 0$

$n \geq \frac{20}{3}$ $n < 12.5$

$\Rightarrow n = 7, 8, 9, 10, 11, 12$

Sum = 57

Question ID : 1925

Limit

25. Let [t] denote the greatest integer $\leq$ t and {t} denote the fractional part of t. The integral value of α for which the left hand limit of the function $f(x)=[1+x]+\frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$ at $x=0$ is equal to $\alpha-\frac{4}{3}$, is _______.

माना [t] महत्तम पूर्णांक $\leq$ t है, तथा t का भिन्नात्मक भाग {t} है। α का वह पूर्णांक मान, जिसके लिए फलन $f(x)=[1+x]+\frac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$ की $x = 0$ पर बाएँ पक्ष की सीमा $\alpha-\frac{4}{3}$ है, है _______.

Ans. Official Answer NTA (3)

Sol. $\lim\limits_{x\to 0}[1+x]+\dfrac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$

$\lim\limits_{x\to 0^-} 0+\dfrac{\alpha^{-2+1}-1-1}{-2+1}$

$\dfrac{\alpha^{-1}-2}{-1}=\alpha-\dfrac{4}{3}$

$2-\dfrac{1}{\alpha}=\alpha-\dfrac{4}{3}$

$\alpha+\dfrac{1}{\alpha}=\dfrac{10}{3}$

$3\alpha^2-10\alpha+3=0$

$\alpha=3,\dfrac{1}{3}$ Integral value $=3$

Question ID : 1926

Methods of Differentiation

26. If $y(x)=(x^x)^x$, $x>0$, then $\dfrac{d^2x}{dy^2}+20$ at $x=1$ is equal to _____.

यदि $y(x)=(x^x)^x$, $x>0$ है, तो $x=1$ पर $\dfrac{d^2x}{dy^2}+20$ का मान बराबर है _____.

Ans. Official Answer NTA (16)

Sol. $y=x^{x^2}$ at $x=1, y=1$

$\ln y=x^2\ln x$

$\dfrac{1}{y}\dfrac{dy}{dx}=2x\ell nx+x^2.\dfrac{1}{x}$

$\Rightarrow\dfrac{dx}{dy}=\dfrac{1}{y(2x\,\ell n\,x+x)}$

Diff wrt y

$\dfrac{d^2x}{dy^2}=\dfrac{-1}{y^2(2x\,\ell n\,x+x)^2}.\left[(2x\,\ell n\,x+x)+y\left(2x.\dfrac{1}{x}+2\ell n\,x+1\right)\dfrac{dx}{dy}\right]$

put $x=1, y=1$

$$\frac{d^2x}{dy^2}=\frac{-1}{(1)(0+1)^2}\left[(0+1)+(1)(2+0+1).\frac{1}{(1)(0+1)}\right]$$

$= -(1+3) = -4$

$$\Rightarrow \frac{d^2x}{dy^2}+20=-4+20=16$$

Question ID : 1927

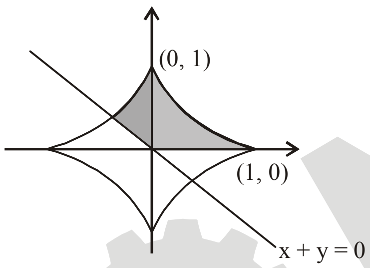
Area Under Curve

27. If the area of region $\left\{(x,y): x^{\frac{2}{3}}+y^{\frac{2}{3}}\le 1, x+y\ge 0, y\ge 0\right\}$ is A, then $\frac{256A}{\pi}$ is equal to ______.

यदि क्षेत्र $\left\{(x,y): x^{\frac{2}{3}}+y^{\frac{2}{3}}\le 1, x+y\ge 0, y\ge 0\right\}$ का क्षेत्रफल A है, तो $\frac{256A}{\pi}$ बराबर है ______.

Ans. Official Answer NTA (36)

Sol.



$$A=\frac{3}{2}\int_0^1\left(1-x^{\frac{2}{3}}\right)^{\frac{3}{2}}dx$$

put $x=\sin^3\theta$

$$A=\frac{3}{2}\int_0^{\frac{\pi}{2}}\left(1-\sin^2\theta\right)^{\frac{3}{2}}.3\sin^2\theta\cos\theta\, d\theta$$

$$A=\frac{3}{2}\int_0^{\frac{\pi}{2}}3\sin^2\theta\cos^4\theta\, d\theta$$

$$A=\frac{9}{2}\times\frac{(1.3)(1)}{(6)(4)(2)}.\frac{\pi}{2}$$

$$A=\frac{9\pi}{64}$$

$\Rightarrow \frac{64A}{\pi} = 9$

$\Rightarrow \frac{256A}{\pi} = 36$

Question ID : 1928

Differential Equation

28. Let y = y(x) be the solution of the differential equation $\left(1-x^2\right)dy = \left(xy + \left(x^3+2\right)\sqrt{1-x^2}\right)dx, -1 < x < 1$ and y(0) = 0. If $\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1-x^2}\, y(x)\, dx = k$, then k^{-1} is equal to ___________.

माना अवकल समीकरण $\left(1-x^2\right)dy = \left(xy + \left(x^3+2\right)\sqrt{1-x^2}\right)dx, -1 < x < 1$, y(0) = 0, का हल y = y(x) है। यदि $\int_{-\frac{1}{2}}^{\frac{1}{2}} \sqrt{1-x^2}\, y(x)\, dx = k$ है, तो k^{-1} बराबर है ___________

Ans. Official Answer NTA (320)

Sol. $\frac{dy}{dx} = \frac{xy}{1-x^2} + \frac{x^3+2}{\sqrt{1-x^2}}$

$\frac{dy}{dx} + \left(\frac{-x}{1-x^2}\right)y = \frac{x^3+2}{\sqrt{1-x^2}}$

I.F. = $e^{\int \frac{-x}{1-x^2}dx} = \sqrt{1-x^2}$

$y\left(\sqrt{1-x^2}\right) = \int \left(x^3+2\right)dx + c$

$\Rightarrow y\sqrt{1-x^2} = \frac{x^4}{4} + 2x + c$

x = 0, y = 0

0 = 0 + 0 + c $\Rightarrow$ c = 0

$\int_{-\frac{1}{2}}^{\frac{1}{2}} \left(\frac{x^4}{4} + 2x\right)dx$

$$\frac{x^5}{20}+x^2\Bigg|_{-\frac{1}{2}}^{\frac{1}{2}}=\frac{1}{20}\left(\frac{1}{32}+\frac{1}{32}\right)=\frac{1}{320}$$

$k^{-1} = 320$

Question ID : 1929

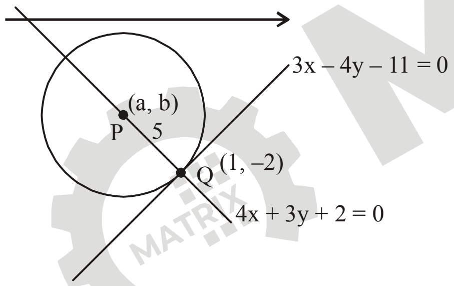
Circle

29. Let the circle C of radius 5 lie below the x-axis. The line $L_1 : 4x + 3y + 2 = 0$ passes through the centre P of the circle C and intersects the line $L_2 : 3x - 4y - 11 = 0$ at Q. The line L_2 touches C at the point Q. Then the distance P from the line $5x - 12y + 51 = 0$ is ______.

माना त्रिज्या 5 का एक वृत्त C, x-अक्ष के नीचे है। रेखा $L_1 : 4x + 3y + 2 = 0$ वृत्त C के केन्द्र P से होकर जाती है तथा रेखा $L_2 : 3x - 4y - 11 = 0$ को बिंदु Q पर काटती है और L_2 बिंदु Q पर C की स्पर्श रेखा है। तो P की रेखा $5x - 12y + 51 = 0$ से दूरी है ______

Ans. Official Answer NTA (11)

Sol.



$a = 1 + 5\cos\theta$

$a = 1+\left(\frac{3}{5}\right)5 = 4$

$b = -2 + 5\sin\theta$

$= -2+5\left(\frac{-4}{5}\right)$

$= -6$

$\tan\theta = \frac{-4}{3}$

Req distance $=\left|\frac{5(4)-12(-6)+51}{13}\right|=11$

Question ID : 1930

Probability

30. Let $S = \{E_1, E_2, ..., E_8\}$ be sample space of a random experiment such that $P(E_n) = \frac{n}{36}$ for every n = 1, 2,......,8. Then the number of elements in the set $\left\{A \subseteq S : P(A) \geq \frac{4}{5}\right\}$ is _______.

माना एक यादृच्छिक परीक्षण की प्रतिदर्श समष्टि $S = \{E_1, E_2, ..., E_8\}$ है तथा प्रत्येक n = 1, 2,......,8 के लिए $P(E_n) = \frac{n}{36}$ है। तो समुच्चय $\left\{A \subseteq S : P(A) \geq \frac{4}{5}\right\}$ में अवयवों _______

Ans. Official Answer NTA (19)

Sol. $A \subseteq S$ $\Rightarrow$ No of elements A can have 0, 1, 2, ... 8

$n(A) = 0,$ $\{\}$

$n(A) = 1,$ $\{E_1\}\ \{E_2\}$ .... $\{E_8\}$

$n(A) = 2,$ $\{E_1, E_2\}$ .... $\{E_7, E_8\}$

$\vdots$

$n(A) = 8,$ $\{E_1, E_2, ...\ E_8\}$

$n(A) > 4$ as, $P(E_5) + P(E_6) + P(E_7) + P(E_8)$

$$= \frac{5+6+7+8}{36} = \frac{26}{36} < \frac{4}{5}$$

for n = 5: $\frac{n_1 + n_2 + ... + n_5}{36} \geq \frac{4}{5}$, $n_1 + ... n_5 \geq 28.8$, possible ways = 2

n = 6: $n_1 + ... + n_6 \geq 28.8$, $n_1 + ... n_8 \geq 28.8$, possbile ways = 9

n = 7: $n_1 + ... + n_7 \geq 28.8$, possible ways = 4

n = 8: $n_1 + ... + n_8 \geq 28.8$, possible ways = 1

Hence Total ways = 19